

# Optimum Structural Design of Spillway Radial Gate by Finite Element Methods

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## Abstract

*In structural optimization, the design variables are normally, representative of the properties as: a) the mechanical or physical properties of the material, b) the topology of the structure, c) the configuration or geometric layout of structure, and d) the cross sectional dimensions of members.*

*However, a successful procedure, mostly, is provided through the use of linearizing a fully nonlinear problem optimizing substructures in such a complex structure. Optimization of substructures is often based on a double iteration method in the space of reduced number of variables. Later, sub-optimization that certainly are based on less design formulas or simple optimization procedures, is simply led to a overall optimization through the structure.*

*The dimensions of gate analyzed here as basic sample are 15 meters width ( $l$ ), 18.3 meters height ( $h$ ), and inner radius of skin plate ( $r$ ) 22 meters.*

*Finite element method is employed as a tool and ease to analyze the samples several times upon different loading conditions.*

*Optimization took place based on minimizing the weight and keeping deformations under  $l/1000$ . Plenty of local stiffener plates are employed to guarantee the plates local buckling.*

## Keywords

## Introduction

Crest type radial gates are generally designed for each spillway opening to regulate the reservoir water level during the flood season. The gates normally, include arms, trunnion bearings, trunnion brackets supports, side guide rollers, seals, fittings and some other necessary appurtenances for their operation and such side equipment as dogging device and etc.. The design of gate must be such that to operate smoothly and without vibration at any head, up to the normal maximum water level and at any position between fully open and fully closed. The gate shall also be designed to close at the rated speeds under the action of its dead weight, with the reservoir at the normal maximum water level. It must be noted that down-pull force as one of the fluctuating hydrodynamic effects on gates is produced by the variation of pressures while the gate is moving up or down in the flow. This hydrodynamic force is primarily of concern to the designer of gate-hoisting equipment. Down-pull may be many times greater than the weight of a gate, and under some conditions it may become negative, indicating uplift. Usually, radial gate shall be of welded construction and consist of a structural steel frame formed by two main vertical radial-shaped girders and between which horizontal beams are spaced according to their individual capacity and the design water pressure. Secondary beams and supplementary bracing shall be provided, where necessary. The vertical girders shall transmit the design loads to the arms without warping or undue deflection. The upstream face of the frame shall be covered with a curved heavy steel skin

plate.

The arms of gate shall be in the form of structural steel box members which transfer the load from the vertical girders to the trunnion bearings.

Recently, most of radial gate hoisting systems are hydraulically operated. Generally, two hydraulic cylinder connections shall be provided on the bottom member on the gate. Cylinder forces on the gate shall be transmitted to the connections through welded joints, vertical members and associated stiffeners. The pin for connections of the hoist cylinders shall be corrosion-resistant steel and the bushings must be a first class, durable and self-lubricant type. To make a higher degree of freedom against the existing deviation of tolerances, a spherical self-lubricant plane bearing is recommended.

The gate shall be sealed by contact of the bottom wedge type elastomeric seal with the sill beam. The bottom edge of the skin plate shall be machined to make a near water-tight contact with the sill beam. The side seals shall normally be the molded single stem 'J' type. The seals shall be positive in action so there is no leakage when the gate is in its position. The seals shall also not unduly interfere with the operation of the gates. Seals are preferred to be in continuous lengths. Where joints are necessary, these shall be oblique, rather than perpendicular, to the line of contact and a minimum of 300 mm. from a corner. All joints must be either hot-vulcanized or joined by an appropriate compound.

The most important point in geometry of gate which to be considered in manufacturing is the tolerance of a line through the center of trunnion bearings which shall not be offset from the theoretical center line by more than 1.5 mm. in any direction.

The role of automated numerical optimization in the overall design process of complex structures must be divided into some stages as: a) Formulation of functional requirements, b) The conceptual design, c) Optimization, d) Detailing. A structural system can be described by a set of quantities, some of which are viewed as variables during the optimization process. Those quantities defining a structural system that are fixed during the automated design as pre-assigned parameters, though they are not varied by the optimization algorithm.

### Optimal Design of Wide Flange Sections

The objective function to be minimized for a typical wide flange section as shown in Figure 1-a is the weight of substructure.

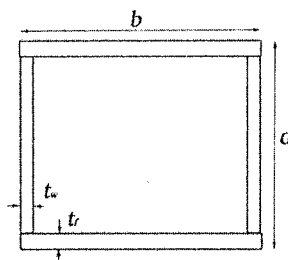


Figure (1-a) Cross section of column.

The constraints in this example can be based on any code such as AISC design specifications [10], but it should be emphasized that other requirements can be employed. All the constraints are expressed in the form:

$$g_j(d, t_w, b, t_r) \leq 0 \quad j = 1, \dots, m \quad (1)$$

m is the number of constraints. Accordingly, the following limitations have been considered in the present formulation:

**a-Shear stress constraints:**

$$g_v \equiv \begin{cases} f_v - \frac{C_v}{2.89} F_y \leq 0 & \text{if } C_v/2.89 < 0.4 \\ f_v - 0.4F_y \leq 0 & \text{if } C_v/2.89 \geq 0.4 \end{cases}$$

(2)

$$C_v = \begin{cases} \frac{240,300}{(d/t_w)^2 F_y} & \text{if } C_v \leq 0.8 \\ \frac{190}{(d/t_w)} \sqrt{\frac{5.34}{F_y}} & \text{if } C_v > 0.8 \end{cases}$$

$F_y$  is the specified yield stress of the steel and  $f_v$  is the computed shear stress.

**b-Bending Geometrical constraints:**

- (1)  $L_b \leq 76b/\sqrt{F_y}$
- (2)  $L_b \leq 20,000bt_F/(dF_y)$
- (3)  $(d/t_w) \leq 412/\sqrt{F_y}$
- (4)  $412/\sqrt{F_y} < (d/t_w) \leq 760/\sqrt{F_y}$
- (5)  $760/\sqrt{F_y} < (d/t_w) \leq 760/\sqrt{f_{bi}^U}$
- (6)  $(d/t_w) > 760/\sqrt{f_{bi}^U}$
- (7)  $b/(2t_F) \leq 52.2/\sqrt{F_y}$
- (8)  $52.2/\sqrt{F_y} < b/(2t_F) \leq 95/\sqrt{F_y}$
- (9)  $95/\sqrt{F_y} < b/(2t_F) \leq 176/\sqrt{F_y}$
- (10)  $b/(2t_F) > 176/\sqrt{F_y}$
- (11)  $L_b/r_G \leq \sqrt{510,000C_b/F_y}$
- (12)  $L_b/r_G > \sqrt{510,000C_b/F_y}$

$L_b$  = distance between sections braced

$r_G$  = radius of gyration of a section

$C_b$  = a constant by the specifications

$f_{bi}^U$  = smallest allowable bending stress

The bending stress constraints are

$$g_b \leq 0$$

$g_b$  is given as follows:

$$\begin{aligned}
(1),(2),(3),(7) \quad & f_b - 0.66F_y \leq 0 \\
(1),(2),(3),(8) \quad & f_b - F_y(0.733 - 0.0014(b_f/2t_f)\sqrt{F_y}) \leq 0 \\
(1),(2),(3),(9) \quad & f_b - 0.6F_y(1.415 - 0.00437(b_f/2t_f)\sqrt{F_y}) \leq 0 \\
(1),(2),(3),(10) \quad & f_b - 20,000/(F_y(b/2t_f)^2) \leq 0 \\
(1),(2),(4),(7) \quad & f_b - F_y(0.7309 - 0.000172(d/t_w)\sqrt{F_y}) \leq 0 \\
(11) \quad & f_b - F_y(2/3 - F_y(L_b/r_G)^2/(1,530,000C_b)) \leq 0 \\
(12) \quad & f_b - 170,000C_b/(L_b/r_G)^2 \leq 0 \\
(5) \quad & f_b - 0.6F_y \leq 0 \\
(6) \quad & f_b - f_{bi}^U \left( 1 - 0.0005 \left( \frac{dt_w}{bt_f} \right) \left( \frac{d}{t_w} - \frac{760}{\sqrt{f_{bi}^U}} \right) \right) \leq 0
\end{aligned}$$

$f_b$  is the computed bending stress.

### c- Bending stress constraints:

$$\left. \begin{aligned}
g_1 &\equiv d/t_w - 14,000/\sqrt{F_y(F_y + 16.5)} \leq 0 \\
g_2 &\equiv d/t_w - 260 \leq 0 \\
g_3 &\equiv b - b^U \leq 0 & g_4 &\equiv b^L - b \leq 0 \\
g_5 &\equiv t_w - t_w^U \leq 0 & g_6 &\equiv t_w^L - t_w \leq 0 \\
g_7 &\equiv d - d^U \leq 0 & g_8 &\equiv d^L - d \leq 0 \\
g_9 &\equiv t_f - t_f^U \leq 0 & g_{10} &\equiv t_f^L - t_f \leq 0
\end{aligned} \right\} \quad (3)$$

$b^U, b^L, t_w^U, t_w^L, d^L, t_f^U, t_f^L$ , are given upper and lower bounds. The above AISC specifications covered here contain discontinuities in two places. The value of  $d/t_w=412/\sqrt{F_y}$  is reference value. For  $d/t_w \leq 412/\sqrt{F_y}$  the allowable bending stress is  $0.66F_y$ , while for  $d/t_w \geq 412/\sqrt{F_y}$  the bending stress is limited to  $0.6F_y$ . To avoid discontinuity the linearized transition constraint of fifth case for  $f_b$  in equations (3) was added. Another discontinuity occurs in dealing with lateral buckling, where the allowable stress jumps from  $0.6F_y$  to  $0.66F_y$ . The best suggestion is a continuous transition between two values.

A method based on a sequence of linear program is employed the nonlinear programming with inequality constraints as the general form of:

$$Z=F(\{X\}) \rightarrow \text{minimum} \quad (4)$$

$$\text{While: } g_j(\{X\}) \leq 0, \text{ for } j=1,2,\dots,m \quad (5)$$

Sequential linear programming method is based on successive linearization of the constraints and the objective function. The use of Taylor series expansion of  $F$  and  $g_j$  about a point  $\{X^*\}$  up to linear terms, equations (4) and (5) can be approximated by:

$$F(\{X\}) \approx F^* + \{\nabla F^*\}^T (\{X\} - \{X^*\}) \quad (6)$$

$$g_j(\{X\}) \approx g_j^* + \{\nabla g_j^*\}^T (\{X\} - \{X^*\}), j=1,2,\dots,m \quad (7)$$



$F^*$ ,  $\{\nabla F^*\}$ ,  $g_j^*$ , and  $\{\nabla g_j^*\}$  are the values of  $F$ ,  $\{\nabla F\}$ ,  $g_j$ , and  $\{\nabla g_j\}$  respectively, at the point  $\{X^*\}$ . The simplest approach for replacing the nonlinear programming problem by a sequence of linear programming problems is to choose an initial point  $\{X^*\}$  and finding  $\{X\}$  by solving the following equations:

$$Z = F^* + \{\nabla F^*\}^T (\{X\} - \{X^*\}) \quad (8)$$

$$g_j^* + \{\nabla g_j^*\}^T (\{X\} - \{X^*\}) \leq 0, j=1,2,\dots,m \quad (9)$$

This linear programming equation can be solved repeatedly, redefining  $\{X^*\}$  each time as the optimal solution of the preceding problem. This process, however, may not always converge to the optimum. Accordingly, the original nonlinear program is locally approximated by linear terms, permitting the solution of non-convex programming problem. To ensure that the approximation is adequate, the variation of  $\{X\}$  is limited in linearized computation as follows:

$$-\{\Delta X^L\} \leq \{X\} - \{X^*\} \leq \{\Delta X^U\} \quad (10)$$

$\{\Delta X^L\}$  and  $\{\Delta X^U\}$  are the move limits assumed as chosen vectors of positive constraints.

Now, a starting point  $\{X^*\}$  and linearize the objective function and the constraints in the neighborhood of  $\{X^*\}$  based on equations (6) and (7). The simulated linearized problem of equations (8), (9), and (10) is solved and redefining of  $\{X^*\}$  for the optimum solution is proceeded. The nonlinear functions are relinearized about the new  $\{X^*\}$ , and the process is repeated until either no significant improvement occurs in the solution or successive solutions start to oscillate between the vertices of the feasible region. In the latter event the values of the boundaries  $\{\Delta X^L\}$  and  $\{\Delta X^U\}$  may be reduced and continue the procedure.

In general, finite element analysis requires a fine mesh to obtain acceptable results. Since the analysis phase usually involves considerable computing work, it is to perform most iterations on a simplified mesh, with only a few refined analysis to correct the approximate results.

A corrective ratio named  $\alpha_i$  characterizes the maximum level of the discrepancy existing within region I between stress in the real structure  $\sigma_i$  and stress computed by a course mesh  $\sigma_{gi}$  as follows:

$$\alpha_i(\{X\}) = \sigma_i(\{X\})/\sigma_{gi}(\{X\}) \quad (11)$$

Assuming that all quantities in equation (11) are functions of  $\{X\}$ , the value of  $\alpha_i$  provides sufficiently accurate maximum stresses through gross mesh analysis. The variation of cross section and region geometry as irregularity of shapes and the mesh layout of members affect slightly variation of  $\alpha_i$ . However, to ease the solution some of the stated parameters can be pre-assigned and not permitted to vary during the optimization process.  $\sigma_i(\{X\})$  must certainly be computed using a fine mesh analysis, and the last computed stress named by  $\sigma_{fi}(\{X\})$ , equation (11) is as follows:

$$\alpha_i(\{X\}) = \sigma_{fi}(\{X\})/\sigma_{gi}(\{X\}) \quad (12)$$

The following steps lead the numerical solution to optimum design:

### 1-Assume initial design.

2-Perform one course and one fine mesh analysis on the structure.

3-Compute  $\alpha_i$  from equation (12).

4-Redesign the structure by using at each step of the iteration a course mesh analysis corrected by the  $\alpha_i$  to estimate  $\sigma_{fi}$ .

Step 4 represents the primary iteration phase. Since the  $\alpha_i$  only approximately correct the analysis, the design obtained in step 4 might be not optimal and feasible. However, it can be used as a new starting point for a similar sequence, therefore, the process is repeated until convergence. This introduces a higher level of iteration as secondary iteration. The primary level involves a high number of simple analysis, while each secondary iteration implies that a very fine analysis is performed. A simple stress-ratio redesign method is employed to obtain fully stress design.

The modules of employed computation are schematically shown in Figure 1-b.

## System of Structures

In design history of radial gates, three structural systems were common as the name of main substructures were employed to sustain gate leaf as:

a- Vertical beams,

b-Horizontal beams, and

c-Frame system. Since the time that numerical methods such as finite element method was employed for analysis of such a big structures instead of hand calculations, naturally, the first two structural systems were disposed. While the structure is divided into several small plate elements, as a skin plate, vertical and horizontal main and secondary stiffeners, and flange elements, etc., the stresses are distributed in natural case according to existing stiffness of any even small members take share at a joint.

One of the most common type of elements may be used to map the body of radial gate is four noded quadrilateral plate bending isoparametric elements including membrane strength. In this analysis, to benefit of symmetry, half of the gate body is modeled. Three columns at each side support the main gate frame which mainly composed of two vertical curved girders.

Most of the plate thicknesses were controlled by minimum plate thickness allowed by client through tender documents minus 1.5 mm. thickness for corrosion protection.

Except for few frame elements presenting the effects of bracing members and also few solid elements for two solid pivot shafts, the rest of the gate body is mapped with plate bending-membrane shell elements.

To analyze the gate against hydrostatic (NB) [6] loading, half of its body has been employed upon symmetrical both geometrically and load wise. Figure 2 shows the finite element mesh. The ideal boundary conditions provide the stability of half gate also shown in Figure 2.

According to loading conditions, as well as the effect of hydrostatic loading, the loading conditions upon hoisting the gate by hydraulic jacks and also loading while the gate stands on two dogging devices must be checked by separate analysis. In the latter case, the wind pressure must be applied on skin plate once towards concave surface and second time on opposite direction separately.

Figure 3-a shows hydrostatic loading plus hoisting loads applied to both eyes at start of gate raising. Figure 3-b shows a typical iteration history in primary and secondary stages.

To control stresses in two and three dimensional media, Von-Mises theory of failure was employed as the elastic limit. Therefore, combined stresses must be compared with the values recommended in DIN19704[6], for ST-37 and ST-52. Box sections are made of ST-52 steel plate used for column to sustain higher pressure and make enough radius of gyration against

global buckling. The local buckling is controlled by adding diaphragm and some longitudinal internal members at certain distances. The pivot housings is to be made from ST-52 steel due to the high stress concentration. Also, trunnion beams to support the pivots are to be made of ST-52.

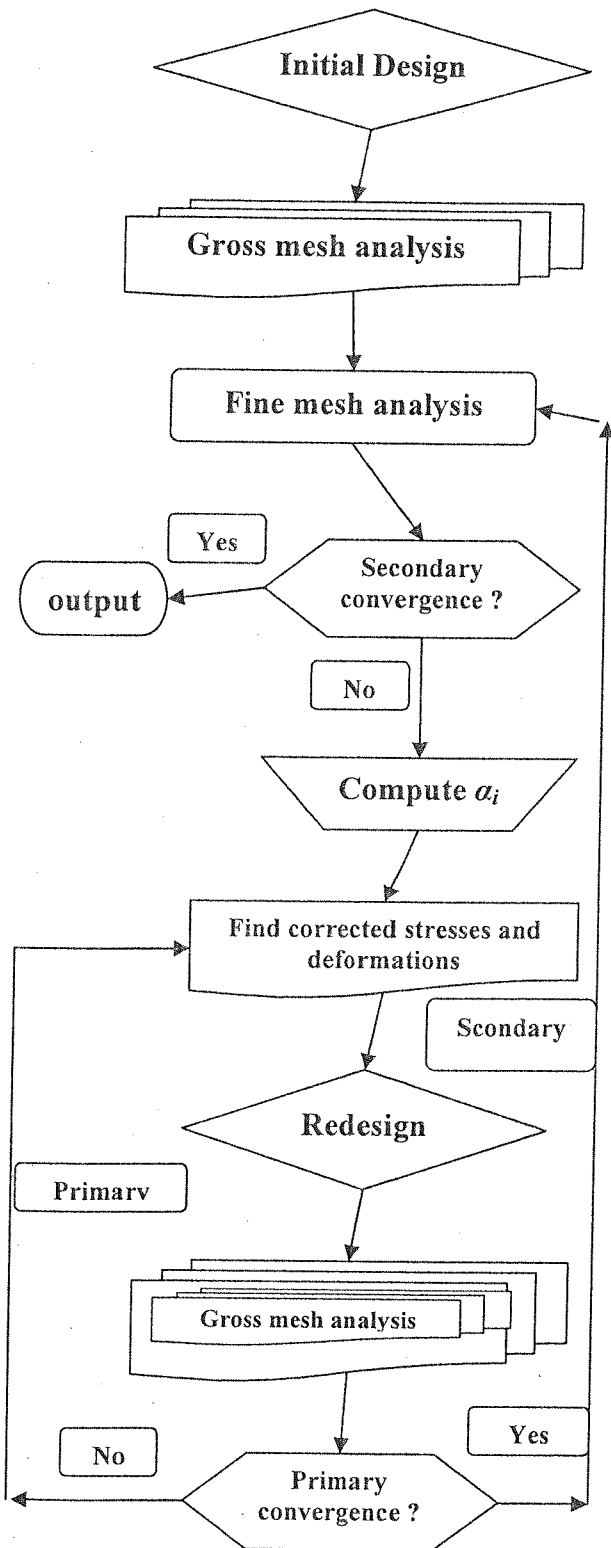


Figure (1-b) Flow chart of approximate optimum.



The rest of gate parts including the gate leaf, bracings, seal belts excluding bolts and nuts shall be made of ST-37. The use of ST-37 is preferred for gate leaf to provide thicker plates, lower stress, and higher gyration radius against local buckling. Bolts and nuts are used for The deformations all through the skin plate are controlled not to be more than  $\ell/1000$ , where  $\ell$  is the gate width. Furthermore, to control the gate movements due to loading and unloading post-tensioned foundations by strand tendons are used to keep trunnion beams mostly unmovable.

### **Minimum Weight of Designed Gate**

The stress distribution in a radial gate including about 30000 members, is quite complex and the structural members are normally modeled as assemblage of at least 10000 elements. The finite element method, has to be used as the analysis tools to ease the repeatedly computations. The skin plate thickness is fixed and predefined to 15 mm. All through member thicknesses must be more than 10 mm. To optimize this structure one load condition (NB) is assumed and there are 35 design variables, including 8 thicknesses, 28 geometric variables. Upper and lower limits were imposed on design variables to satisfy practical considerations as well as mandatory regulations. The behavior constraints are represented by limitations on the shear equivalent and buckling stresses in plates.

The main objective function which used in this optimization is the weight of the structure. It must be noted that a change in the cross sections normally affects the finite element mesh. A change in thickness of plate element, forces to change the dimension of members upon the code limitations, but may be conserved to be applied in final run. However, in the latter case a suitable mesh must be provided at each iteration, but having the minimum geometry changes will help saving time by fixed predefined sizes.

The methods of optimization used is the sequence of linear programs, upon which, moving limits are applied to any of the design variables. Furthermore, constraint accumulation is used whereby the optimization model is allowed to build up a piecewise linear approximation to the true nonlinear constraints.

The behavior constraints are represented by limitations on the combined stress in plate elements, and mid-deformation of the gate leaf.

The objective function used in this example is the weight of the structure.

After, optimization of base numerical model, geometry the gate assumed as global variables such as height and width the gate leaf are considered to provide some general information for estimating the gate weigh before accurate analysis.

The provided finite element model is based on first system of gate body, including three columns at each side, two vertical main girders and three horizontal main beams.

Two main limitations as allowable Von-Mises stress and allowable deformation of gate are considered to provide the gate minimum weight. Accordingly, different applicable height (13 to 24 m) and gate width (10 to 26 m) are supposed. Figure 4-a, b, and c present minimum obtained weights upon both controls of deformation and combined stress to be less than allowable value for different gate width and 13-16 m., 17-20 m., and 21-24 m. respectively. Upon accepting elastic behavior of material, any needed gate having certain width and height may have a certain minimum weight. Consequently, the minimum weight of radial gate restricting either deformation or stress, and other code limitation for buckling is obtained.

### **Conclusion**

A thin wall structure, mainly, composed of steel plates was analyzed and optimized to have





the minimum weight and also ease of manufacture. The gate structure is built based on having three columns at each sides and two vertical main girders supporting three main horizontal beams and several vertical and horizontal secondary stiffeners to sustain curved skin plate.

The minimum weight of gate upon different geometry, mainly, based on change of gate width and height, and also several times computer running are carried out.

Two main design characteristics, namely deformation and combined (Von-Mises) stress controls are considered. The variations of minimum weight versus gate width and height which are the most common geometrical characteristics of gate use are also presented.

These results, can obviously be used as a general guidance to quote the needed steel for manufacturing radial gate before analysis and design.

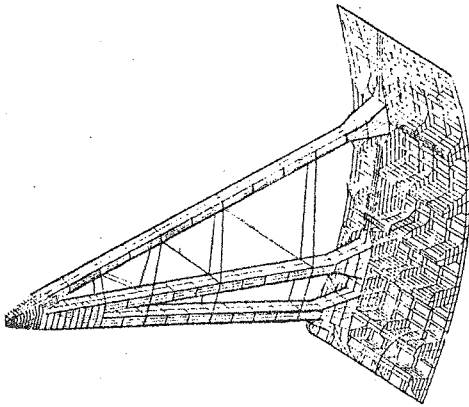


Figure 2 Finite element mesh

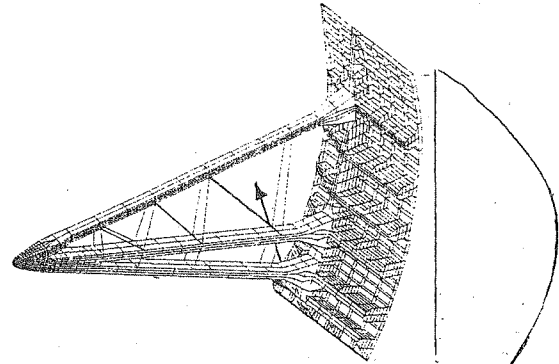


Figure 3-a Hydrostatic and hoisting loads

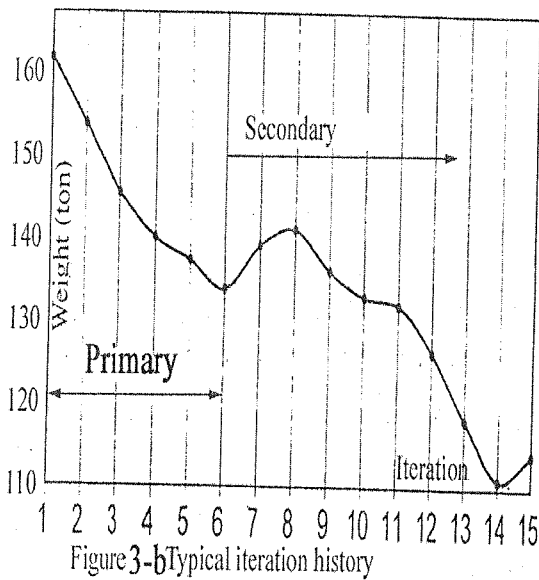


Figure 3-b Typical iteration history

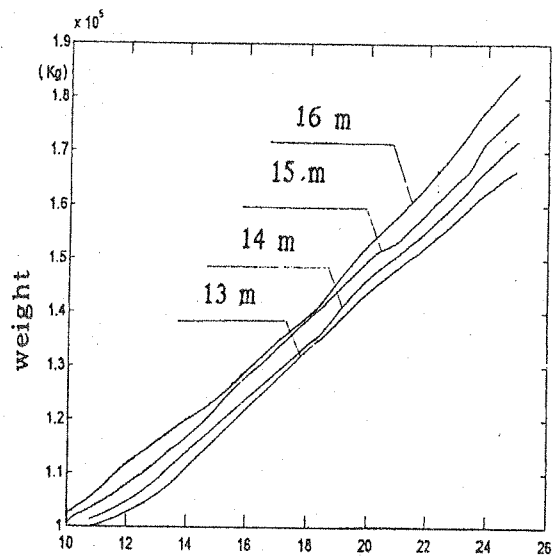


Figure 4-a Optimized weight width (m) (gate height 13-16 m.)



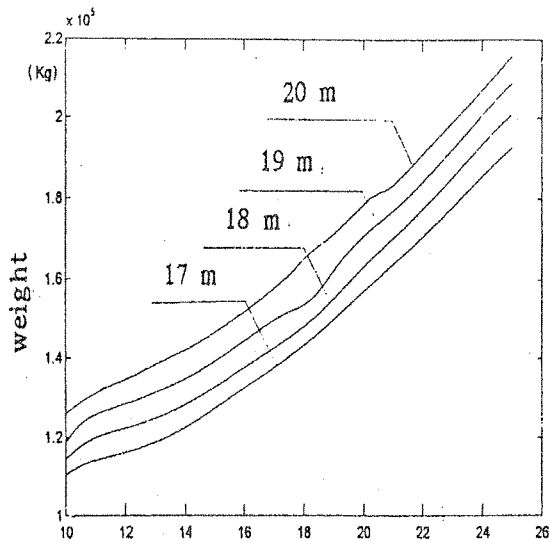


Figure 4-b Optimized weightwidth (m)  
(gate height 17-20 m.)

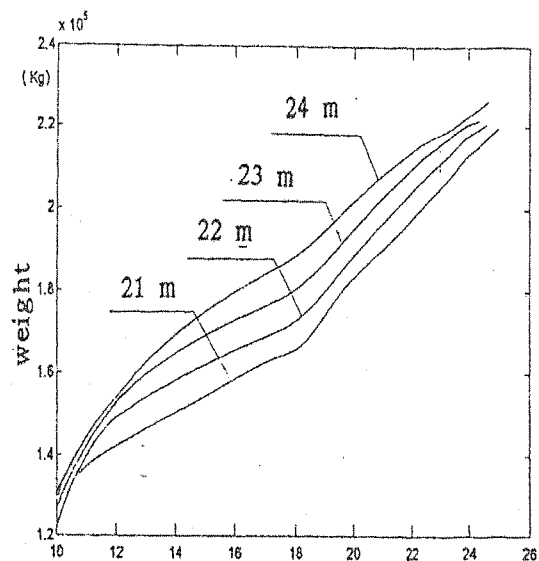


Figure 4-c Optimized weight width (m)  
(gate height 21-24 m.)

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