Effects of Yarn Bending Rigidity and Fabric Structure on the Bending Rigidity of Warp Knitted Fabrics

S.Ajeli [i], Ali.A.A.Jeddi [ii] and A.Rastgo [iii]

ABSTRACT

The relationship between the flexural rigidity of standard warp knitted fabrics and fabric and yarn parameters, such as wale and course spacing, underlap length of the front and back guide bars, yarn bending rigidity, etc., is discussed. In this study, a mechanical model for the bending behaviour of the warp knitted fabrics is also presented. In theoretical analysis,the knitted fabric loop structure is assumed to consist of a series of straight and skew yarn simulating legs and underlaps. The estimation of fabric bending rigidity is based on summing up the bending rigidity of the straight and skew yarns that found by energy method with taking into account of rigid region lying in the direction of bending. The estimated values from this model are compared with experimental results obtained on a developed automatic cyclic pure bending tester. Comparision with exprimental and theoritical data shows resonable agreement between pridicted and mesured flexural rigidity of warp knitted fabrics.

KEYWORDS:

Warp knitted fabric, Bending rigidity, Energy method, Fabric structure, Yarn bending properties and Fabric density.

1. INTRODUCTION

Fabric bending rigidity is one of the most important factor influencing handle and comfort of apparel, and hence the bending behavior of fabrics has received considerable attention in the literature. The bending behavior of fabrics has been studied quite extensively, beginning with Peirce[1]. Abbott *et al.*[2] supposed that because of the pressures at the cross-over points, the yarn in the woven fabric is composed essentially of alternate rigid and flexible sections and assumed that the yarn lie in a straight line. In 1964, Livesey and Owen [3] proposed a mathematical formula to show the relationship between cloth flexural rigidity and single fiber flexural rigidity. They considered the yarn twist and crimp in the fabric to be a collection of independent non-interacting helices. Later, Grosberg [4] suggested that the bending behavior of cloth is nonlinear and is separated into two components; a flexural rigidity and frictional resistance.

A fabric model was proposed by Abbott *et al.* [5].They assumed that fabric made from a large number of long and thin plates and shear effect during bending can be neglected and then investigated the bending of a series of parallel plates. In 1973, they developed two models of a plain set and unset woven fabric [6]. A theoretical analysis was used to obtain the predicted relationship between the applied couple and the curvature of the fabric. They also showed disagreement with the previous work of Abbott in 1960[2]. Later, Hamilton and Postle [7] extended the Livesey's idea by applying it to plain knitted fabrics, where they assumed that each wale in the fabric behaved as a pair of double helices. However they found a large difference between the experimental and estimated values in their studies, due to assumption of the rigid joint at the interlocking point. Moreover, it is noteworthy that the energy analysis proposed by de Jong and Postle [8] simulated a direct comparison of different woven and knitted fabric constructions in terms of normalized dimensionless parameters. Leaf *et al.* [9] considered a model using strain energy and Costigliano's theorem and showed the relationship between the flexural rigidity of a plain woven fabric and the fabric and yarn parameters such as thread spacing and crimp, yarn bending rigidity etc. In this model, they assumed that yarns are incompressible, inextensible and perfectly elastic and fabric is set and consequently before deformation there is no force between warp and weft in the intersection

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regions. In the recent studies, Alimaa *et al.* [10] presented a straight parallel yarns model to analyze the effect of yarn bending properties and fabric structure on the bending rigidity and frictional bending moment of the plain and rib weft knitted fabrics. In 2004, Kang *et al.* [11] suggested a mathematical model that considered the bending behavior of fabric is non-linear.

Although there are some experimental studies [12]-[15] on the bending properties of weft and warp knitted fabrics, there has been comparatively little study of fabric flexural rigidity in terms of the flexural properties of the constituent yarns. Therefore our purpose in this paper is to clarify, in detail, the interrelation of warp knitted fabric with yarn bending behavior and fabric structure and density. To explain the bending behavior of these structures in terms of constituent yarn bending properties and geometry, a structural model for the bending properties of full-set threading two guide bars warp knitted fabric is proposed. In addition the validity of our approach is verified by the experimental data.

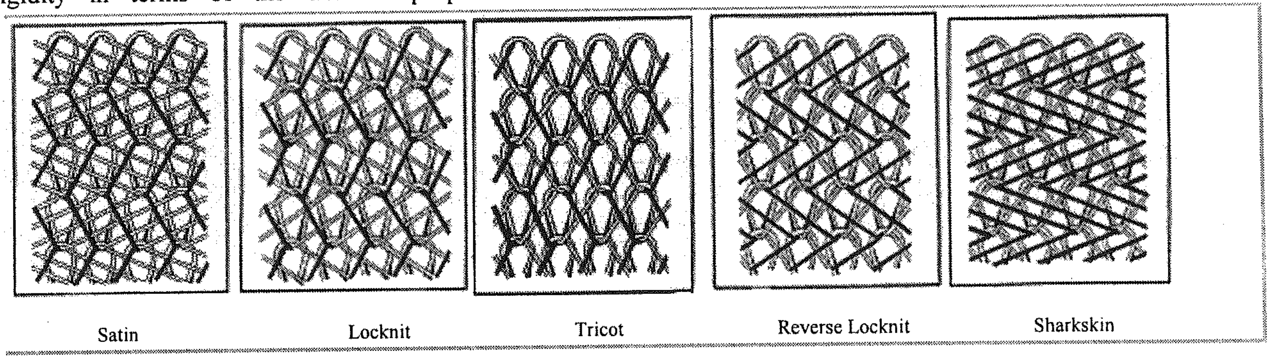

Figure 1: Schematic diagrams of the warp knitted fabric structures

2. EXPERIMENTAL

A. Knitting Details and Relaxation Treatment

Seven of the most common structure of two guide bars warp knitted fabrics i.e., Tricot, Locknit, three & four needles Satin, Reveres locknit, three and four needles sharkskin were produced as shown in Figure 1.All were knitted from the same 75 denier(8.3 tex)/36 filament flat polyester yarn.

A positive let-off system warp-knitting machine (Liba) with a 28 gauge was used. This feeding system was used to minimize the yarn-tension variations. The constructional details of the fabrics are given in TABLE 1. In this work, to study the influence of the fabric density on the bending rigidity, all fabric knitted structures were produced in three different densities.

The fabric samples were then left in a room atmosphere (251 °C & 30%3 *RH*) for a few days. They were washed for 30 minutes at a temperature of 30 °C in a Winch washing machine with a detergent. Fabrics were relaxed on a flat surface and left to be conditioned over night in the same room atmosphere. After relaxation, the samples were ironed gently to be a completely flat form.

B. Cyclic Bending Tests

Wale wise and course wise bending parameters of the fabrics were measured using a modified cyclic bending tester based on Livesey and Owen's apparatus [3].

Figure 2 shows a schematic photo of the instrument. The specimen is held at one end in a rotatable clamp C which rotates with a DC motor very gently and its position (angle() is registered by a shaft encoder system. Other end of the specimen is attached to an arm system. This arm system includes of a floating clam, detected arm, and dead weight which under the action of gravity provides the bending couple in specimen. Livesey and Owen suggested that the center of gravity of the arm system should be as far from the specimen as possible. Consequently, it is made from a light rigid material. The position of the arm (angle®) is detected with a rotate-able laser sensor. By knowing that the curvature is proportional to the angle (-® and the couple to *sin*®, the bending hysteresis curvature is obtained. Figure 3 illustrates a typical bending hysteresis curve generally obtained for our warp knitted samples.

TABLE 1: The fabric characteristics.

Fabric structure	density	Fabric code	Number of underlap		Run-in (*cm/rack**)		CPC	WPC	Fabric mass (g/m^2)
			FB	BB	FB	BB			
Tricot	loose	Tl	1	1	202	189	12.0	13.2	98.25
	medium	Tm	1	1	165	145	16.2	14.0	112.87
	tight	Tt	1	1	141	129	20.6	13.6	141.9
Locknit	loose	Ll	2	1	225	192	11.8	13.0	115.77

	medium	Lm	2	1	182	145	16.2	16.6	149.72
	tight	Lt	2	1	160	131	22.0	15.8	178.59
Satin 3	loose	S3l	3	1	261	198	11.6	14.0	123.23
	medium	S3m	3	1	224	150	16.4	16.4	162.98
	tight	S3t	3	1	209	130	19.8	16.4	179.01
Satin 4	loose	S4l	4	1	306	195	12.2	14.8	139.38
	medium	S4m	4	1	261	151	16.8	15.8	174.88
	tight	S4t	4	1	245	132	21.6	15.2	207.58
Reveres locknit	loose	RLl	1	2	201	221	12.0	13.2	113.04
	medium	RLm	1	2	160	180	16.6	15.2	138.73
	tight	RLt	1	2	142	162	21.0	14.2	165.59
Sharkskin3	loose	SH3l	1	3	205	256	12.2	13.2	116.52
	medium	SH3m	1	3	162	216	17.2	13.6	143.43
	tight	SH3t	1	3	144	211	21.2	13.4	174.19
Sharkskin4	loose	SH4l	1	4	210	306	11.0	12.4	124.24
	medium	SH4m	1	4	161	261	17.0	13.2	163.18
	tight	SH4t	1	4	150	252	22.0	12.6	185.16

FB, front guide bar; BB, back guide bar; CPC, course per cm; WPC, wale per cm

* *1 rack=480 courses*

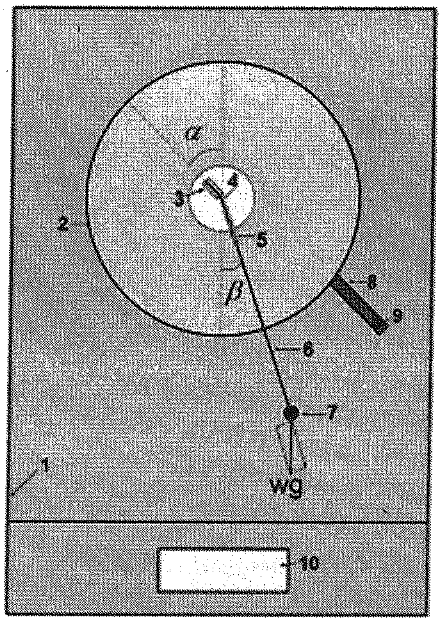

Figure 2: A schematic diagram of an automatic cyclic bending tester. (1,main frame;2,rotateble disk;3,rotatable clamp;4,specimen;5,floating clamp;6,pointer arm;7,weight;8,sensor arm;9,Laser sensor;10,Keyboard)

The following equations that were explained by Livesey and Owen [3] give the curvature and bending moment:

$$\kappa = L(\alpha - \beta) \qquad (1)$$

$$M = w.g.(l + 0.35).\sin\beta$$

Where, κ is curvature, L is the specimen's length, M is the bending moment, $w.g$ is the weight and l is the length of arm.

It should be noted that, the effective specimen length between the clamps is 0.5 *cm* and the specimen width (perpendicular to the plane of bending) is 2.5 *cm*. Four specimens of each sample were normally tested, two being placed in the clamp with the face to the right and two with the face to the left, so that any face-to-back differences are eliminated. The bending test of samples was done at 271 °C & 25%3 *RH*.

Yarn bending properties was also measured using the developed cyclic bending tester. To measure the bending properties of the yarn, both ends of a parallel set of constituent straight yarn after relaxation were attached to adhesive tape and then the yarn sample was positioned between the clamps. The tapes were positioned perpendicular to the yarns and very narrow gaps were left between yarns.

3. RESULTS

A. Bending Hysteresis Curves and Parameters

Typical bending hysteresis curves in two principal refers to direction of wale and course are shown in Figure 3. According to these figurers, the bending hysteresis loop is asymmetric and is displaced from the origin, as a result of curling tendency. In order to analyze the experimental results, the slopes of the curve at linear regions (at curvatures of 1 to 3 and -1 to -3 cm^{-1}) of each hystersis curve were obtained and therefore, two bending rigidities calculated.

Subscripts w and c are used to denote the case where the plane of the bending is parallel to the wale and courses, respectively (the reverse description with Hamilton and Postle [7]); and subscript f corresponds to the flexural rigidity situation where the technical face of the fabric is on the outer or convex side of the bent fabric and it is not related to the sign of curvature point and b corresponds to opposite situation for the technical back. In this case our subscript description is a little different with Hamilton and Postle. Consequently, four different fabric bending rigidities $B_{w,f}$, $B_{w,b}$, $B_{c,f}$, $B_{c,b}$ are obtained for each fabric construction. Superscripts ′ and ″ are used to denote that the experimental test is beginning on the technical face and technical back, respectively. Then, it should be noted that for measuring each fabric flexural rigidity, as shown in Figure 4 we accounted the means of bending curve slope as below:

$$B_{w,f} = (B'_{w,f} + B''_{w,f})/2 \qquad (2)$$

$$B_{w,b} = (B'_{w,b} + B''_{w,b})/2$$

Table 2 shows the values of the bending parameters for all warp-knitted fabric structures. All numerical values of the flexural rigidities are quoted per *cm* width of samples

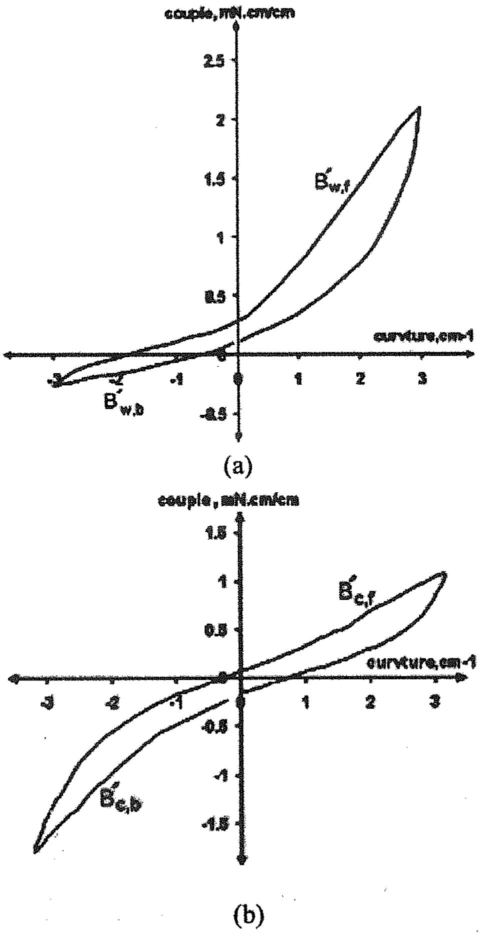

Figure 3: Typical bending hysteresis curves for a warp-knitted fabric. (a: wale wise bending b:course wise bending)

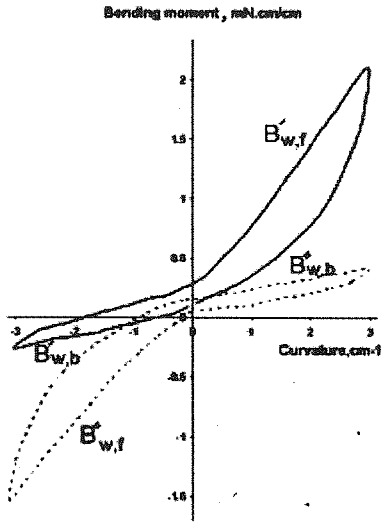

Figure 4: Fabric bending hysteresis curves beginning with two different sides. __ start test with technical face, --- start test with technical back

B.Effect of Density and Structural Characteristics

According to the bending hystersis curves, the following results are evident for all warp-knitted fabrics:

$$B_{w,f} \rangle B_{w.b}, \qquad B_{c,f} \langle B_{c.b}$$

It is obvious from Table.2, that the $B_{w,f}$ values are four times more than $B_{w,b}$ and in course direction it is reversed. However ,the values of $B_{w,f}/B_{w,b}$ and $B_{c,b}/B_{c,f}$ show that the differences between $B_{c,f}$ and $B_{c,b}$ is less than the difference between $B_{w,f}$ and $B_{w,b}$.

These results are analogous to those described by Hamilton and Postle [7] for bending of plain weft knitted fabrics.

But, the presence of underlaps in the warp-knitted fabric has an additional effect on the mechanisms of bending about an axis parallel to the wale and course direction.

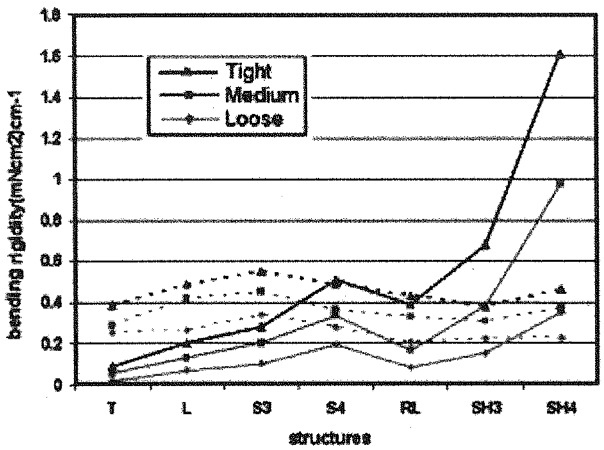

Figure 5: Fabric bending rigidity __ course wise bending, --- wale wise bending

Bending rigidity and density: Figure 5 illustrates that the effect of density factor on B_w and B_c values. The corresponding slopes show that there is considerably increase of bending rigidity with increasing density factor for both wale and course direction. The large values of bending properties for tighter samples are understandable, since increasing fabric density means decreasing space for stitch movements as well as increasing friction between yarns.

Bending rigidity and construction: A comparison of the results by warp-knitted structures revel a much lower bending rigidity in low underlap size of fabric than in high underlap size in wale and course direction, but there isn't a clear distinction of the bending rigidity with direction between Bw and Bc. Here Bw is the mean value of Bw,f and Bw,b, and Bc is the mean value of Bc,f and Bc,b.

TABLE 2: EXPERIMENTAL Bending Rigidity parameters of FABRICS (mN.cm2) cm-1.

Fabric code	$B_{w,f}$	$B_{w,b}$	B_w	$B_{w,f}/B_{w,b}$	$B_{c,f}$	$B_{c,b}$	B_c	$B_{c,b}/B_{c,f}$	B_w/B_c
Tl	0.403	0.098	0.250	4.123	0.009	0.032	0.020	0.281	12.500
Tm	0.467	0.086	0.277	5.406	0.023	0.084	0.053	0.274	5.226
Tt	0.630	0.144	0.387	4.385	0.042	0.137	0.090	0.307	4.300
Ll	0.424	0.101	0.262	4.202	0.032	0.113	0.073	0.283	3.589
Lm	0.708	0.147	0.427	4.810	0.052	0.207	0.129	0.251	3.310
Lt	0.826	0.144	0.485	5.753	0.090	0.315	0.203	0.286	2.389
S3l	0.574	0.106	0.340	5.413	0.042	0.163	0.102	0.258	3.333
S3m	0.780	0.118	0.449	6.600	0.096	0.310	0.203	0.310	2.212
S3t	0.961	0.138	0.550	6.951	0.139	0.423	0.281	0.329	1.957
S4l	0.480	0.074	0.277	6.477	0.109	0.279	0.194	0.391	1.428
S4m	0.628	0.098	0.363	6.420	0.206	0.467	0.336	0.441	1.080
S4t	0.833	0.096	0.465	8.642	0.368	0.648	0.508	0.568	0.915
RLl	0.325	0.084	0.204	3.883	0.039	0.125	0.082	0.312	2.488
RLm	0.559	0.093	0.326	6.000	0.100	0.232	0.166	0.431	1.964
RLt	0.733	0.117	0.425	6.263	0.241	0.535	0.388	0.450	1.095
SH3l	0.362	0.085	0.223	4.274	0.100	0.203	0.151	0.493	1.477
SH3m	0.502	0.110	0.306	4.582	0.252	0.512	0.382	0.492	0.801
SH3t	0.635	0.100	0.367	6.356	0.511	0.853	0.682	0.599	0.538
SH4l	0.376	0.136	0.256	2.777	0.275	0.434	0.354	0.634	0.723
SH4m	0.632	0.107	0.370	5.910	0.604	1.339	0.971	0.451	0.381
SH4t	0.789	0.110	0.449	7.161	1.091	2.130	1.611	0.512	0.279

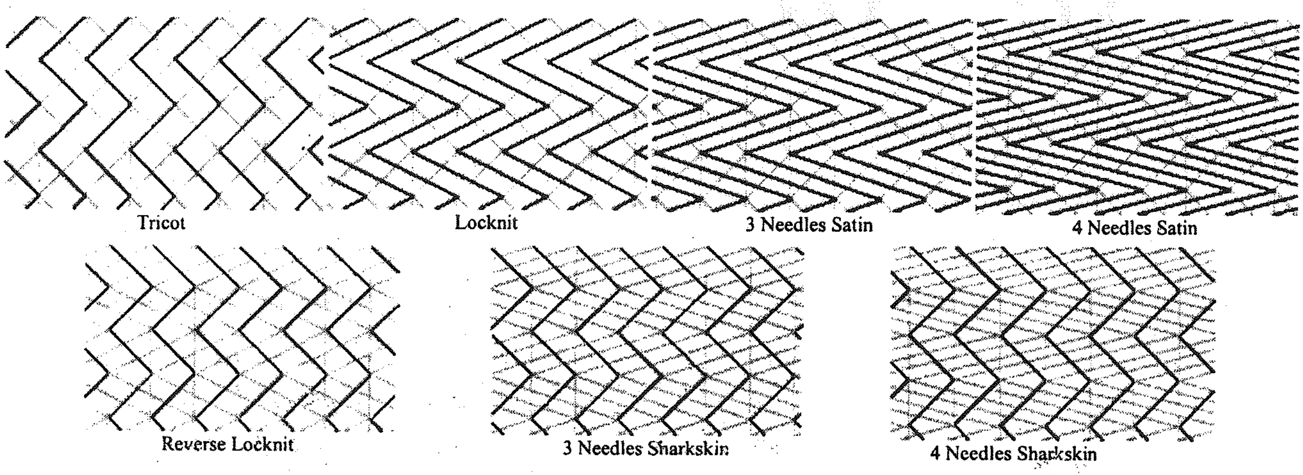

Figure 6: Simple models of warp knitted fabric structures

4. THEORTICAL CONSIDRATIONS

A. Bending model for warp knitted structure

The simple wale wise and course wise bending rigidity of warp knitted fabrics are mainly sum of the bending resistance of the contained yarns lined in the direction of the bending. In this paper, the warp knitted loop is assumed to consist of a series of linear elastic straight and skew yarns in a plane. The geometry of knitted fabrics is relatively simple, as illustrated in Figure 6 for all knitted structures.

In addition, as Grosberg [4] expressed, we assumed that the yarns in fabric pass through two regions, one region (in which the yarns are under pressure) is in the intersection of the roots and overlap; this region is assumed completely rigid. The second region is the flexible region that the yarns pass from one intersection to the next, where underlaps of the front and back guide bars and the legs of front and back loops are located (the friction between the legs and two underlaps in this region is ignored). Hence, we expect that the knitted fabric bending rigidity is the sum of the resistance of these straight and skew yarns. Base of these consideration, we propose a structural model for bending of warp knitted fabric as shown in Figure 7.

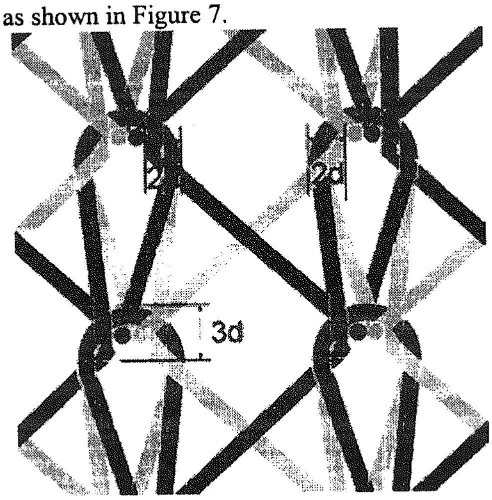

Figure 7: warp knitted fabric loops

B. Bending Rigidity in wale direction

Each wale in the warp knitted fabrics is considered to be composed of four straight yarns (legs of the loops) and two skew yarns (underlaps) that have rigid region at the intersection of the roots and overlaps.

As shown in Figure 7, the bending rigidity in the wale direction can be calculated by summing up the bending rigidity of these straight and skew yarns (see appendix A). On the basis of these assumptions, the bending rigidity of the flexible region of the fabric in the wale direction can be expressed as;

$$Bf_{fabric,w} = (4 + \frac{\sin\theta_{f,w}}{\sin^2\theta_{f,w} + a\cos^2\theta_{f,w}}.n_f + \frac{\sin\theta_{b,w}}{\sin^2\theta_{b,w} + a\cos^2\theta_{b,w}}.n_b).WPC.B_{yarn} \quad (3)$$

Where, $Bf_{fabric,w}$ is the bending rigidity of the flexible region of the fabric in wale direction; B_{yarn} is the bending rigidity of the yarn; n_f and n_b are the number of yarns in the wale space as the same as the number of underlap for the front and back guide bar, respectively; $\theta_{f,w}$ and $\theta_{b,w}$ are the angle between front and back underlaps and the course direction, respectively .as follows;

$$\theta_{f,w} = \sin^{-1}(\frac{c}{\sqrt{c^2 + n_f^2 w^2}}) \quad (4)$$

$$\theta_{b,w} = \sin^{-1}(\frac{c}{\sqrt{c^2 + n_b^2 w^2}})$$

Where, c is the course spacing and w is the wale spacing.

Considering the rigid region into the Eq.3 as describe in appendix B, we have

$$\frac{1}{B_{fabric,w}} = \frac{1-\alpha_w}{Bf_{fabric,w}} \quad (5)$$

Where, $B_{fabric,w}$ is the bending rigidity of the fabric in wale direction; the index α_w from the Figure.7 is expressed as follows;

$$\alpha_w = \frac{3d}{c} \quad (6)$$

Here, d is the yarn diameter.

C. Bending Rigidity in course direction

In the same way, if we assume that each course can be replaced by two skew yarns (underlaps), the bending rigidity in the course direction can be calculated as follows;

$$Bf_{fabric,c} = (\frac{\sin\theta_{f,c}}{\sin^2\theta_{f,c} + a\cos^2\theta_{f,c}}.n_f + \frac{\sin\theta_{bc}}{\sin^2\theta_{b,c} + a\cos^2\theta_{b,c}}.n_b).CPC.B_{yarn} \quad (7)$$

Where, $Bf_{fabric,c}$ is the bending rigidity of the flexible region of the fabric in course direction; $\theta_{f,c}$ and $\theta_{b,c}$ are the angle between front and back underlaps and the wale direction, respectively .as follows;

$$\theta_{f,c} = \sin^{-1}(\frac{n_f w}{\sqrt{c^2 + n_f^2 w^2}}) \quad (8)$$

$$\theta_{b,c} = \sin^{-1}(\frac{n_b w}{\sqrt{c^2 + n_b^2 w^2}})$$

Considering the rigid region into the Eq.7, we have;

$$\frac{1}{B_{fabric,c}} = \frac{1-\alpha_c}{Bf_{fabric,c}} \quad (9)$$

Where, $B_{fabric,c}$ is the bending rigidity of the fabric in course direction; the index α_c from the Figure.7 is expressed as follows;

$$\alpha_c = \frac{4d}{w} \quad (10)$$

5. Discussion

The estimated bending rigidity of warp knitted fabrics are obtained with theoretical model using yarn bending rigidity and fabric structure parameters. The predicted and experimental results values for all fabric structure and densities are shown in Figure 8, for both wale and course directions.

According to the wale wise results, the difference between density factors of fabrics is more effective on the bending rigidity than the difference of structure. It occurs because of the underlaps (two skew yarns) are less important in wale direction of bending and the main part of bending rigidity value depends on legs (four straight yarns).

In wale direction the estimated values of the bending rigidity have a high degree of correlation with experimental measured values as shown in Figure 9.
On the other hand, in course wise of bending the effects of structure is considered as well as the density factor. Since, increasing the underlap size of fabric and density increased the bending rigidity.

As shown in Figure 8, a disagreement is observed between the last three structures (reverse locknit and sharkskins) and densities with the bending rigidity in course direction. This result may be due to the assumption of ignoring friction between underlaps which gives an inadequate approximation for these groups of warp knitted structures.

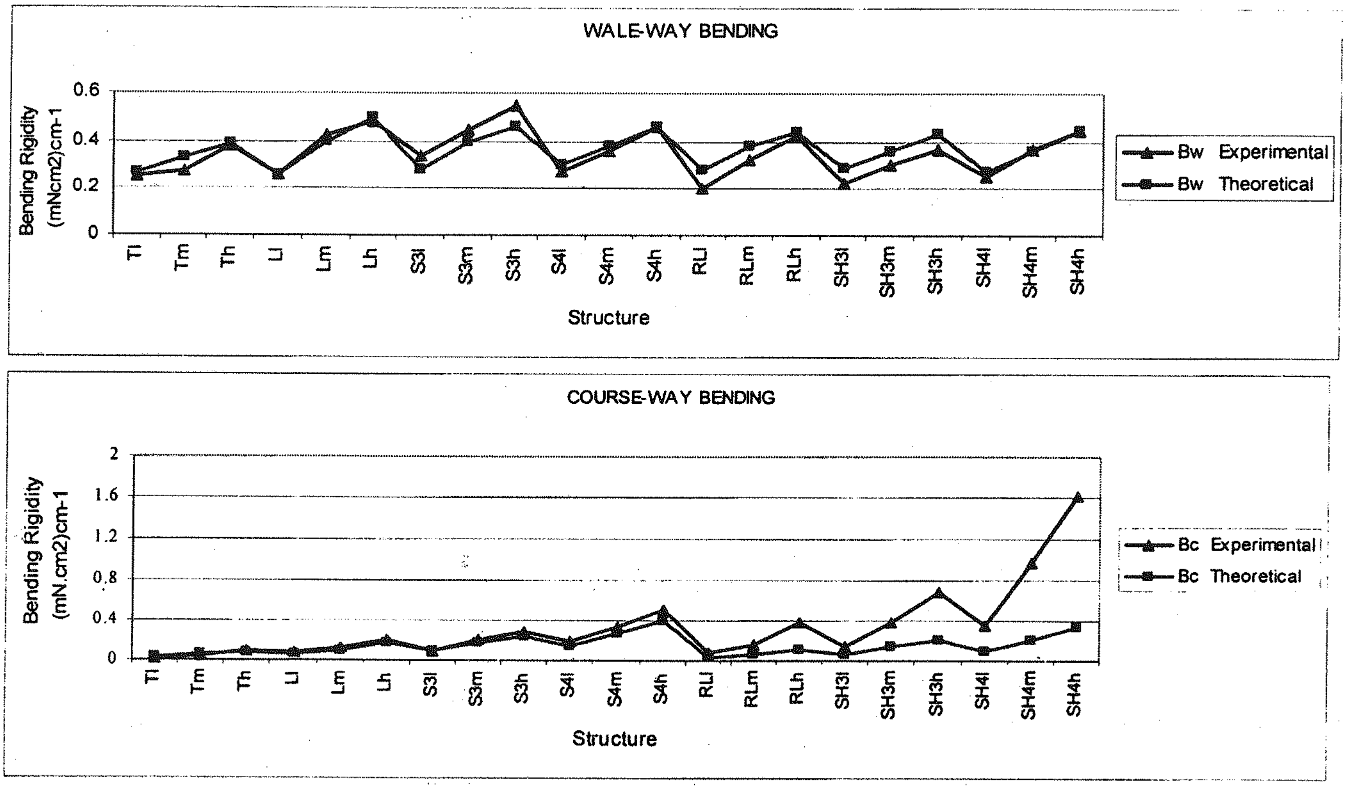

Figure 8: Theoretical and experimental bending rigidity of fabrics

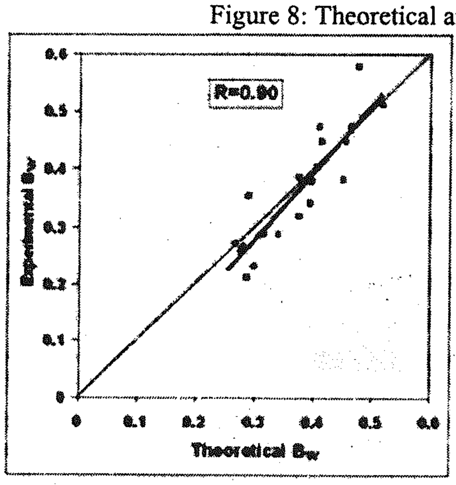

Figure 9: Wale wise correlation of the theoretical and experimental values.

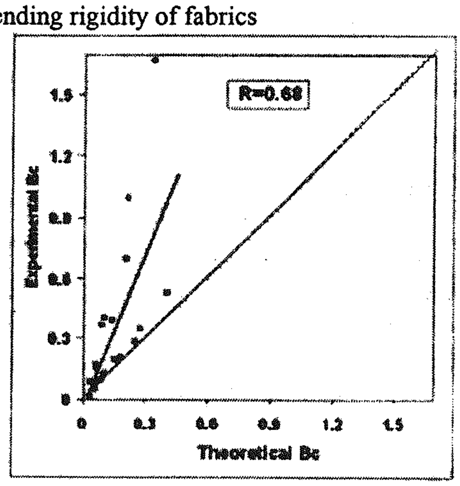

Figure 10: Course wise correlation of the theoretical and experimental values for all fabric's structure.

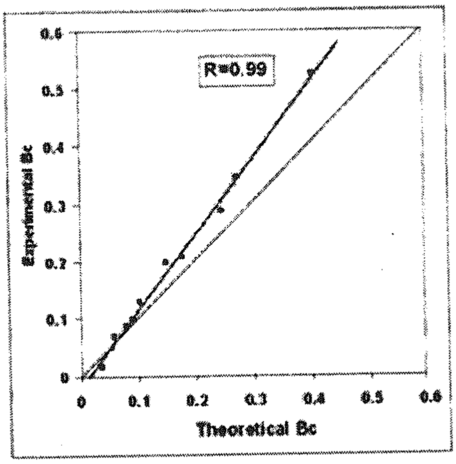

Figure 11: Course wise correlation of the theoretical and experimental values one underlap length of the back guide bar.

Figure 10 showed the correlation value of course wise bending rigidity between theoretical and experimental results. If the results of last three group of fabrics that underlap length of the back guide bar are more than one, were omitted, as shown in Figure 11, a high degree of correlation were found between theory and experiment .

6. CONCLUSION

We have investigated the bending rigidity of warp knitted fabrics in term of the bending properties of the constituent yarns. The experimental results of relaxed fabrics show that the bending rigidity is largely determined by the fabric density in both wale and course direction. On the other hand, the relation between the fabric structure and bending properties show that the effect of fabric structure on the bending rigidity in course direction is more significant than in wale direction.

We have also made theoretical analysis of the effects of yarn bending rigidity and fabric structure on the bending rigidity of warp knitted fabrics by means of the straight and skew yarns model with considering the rigid region. We have shown that based on the bending parameters of the constituent yarns, our straight and skew yarn model can reasonably estimate the bending rigidity of relaxed knitted fabric in both wale and course directions.

7. APPENDIX

A. Flexural Rigidity of the Skew Straight bar

According to Figure 12a, case of straight bar bent into a curve by this component of the coupleis investigated. The basic differential equation relating the deflection angle Π to bending moment in an elastic rod whose cross section is symmetrical about the perpendicular plane of loading is:

$$M = B\frac{d\varphi}{ds} \quad (1)$$

Where, $B=EI$ is termed the flexural rigidity that E is called the modulus of elasticity and I represent the moment of inertia. Getting integration of this equation, gives:

$$\varphi = ML\frac{1}{B} \quad (2)$$

The skew straight bar at the same natural and geometry boundary condition is shown in Figure 12b. Considering the Figure 12b; the torque M on skew bar is subjected two loads which at any cross section produce a twisting moment as well as a bending moment.

The bending and twisting moments at any section are, respectively:

$$M_b = M\sin\theta$$
$$M_t = M\cos\theta \quad (3)$$

Expressions for the strain energy in bending U_b and torsion U_t are shown as follows:

$$U_b = \frac{1}{2}\int\frac{M_b^2}{EI}ds$$
$$U_t = \frac{1}{2}\int\frac{M_t^2}{GJ}ds \quad (4)$$

Where, GJ is termed the torsional rigidity of the rod that G is called shearing modulus of elasticity and J represents the polar moment of inertia of the cross-sectional area.

The total strain energy with substituting quantities of M_b and M_t is as follows:

$$U = \frac{M^2}{2}\int_0^{L'}\left(\frac{\sin^2\theta}{EI}+\frac{\cos^2\theta}{GJ}\right)ds \quad (5)$$

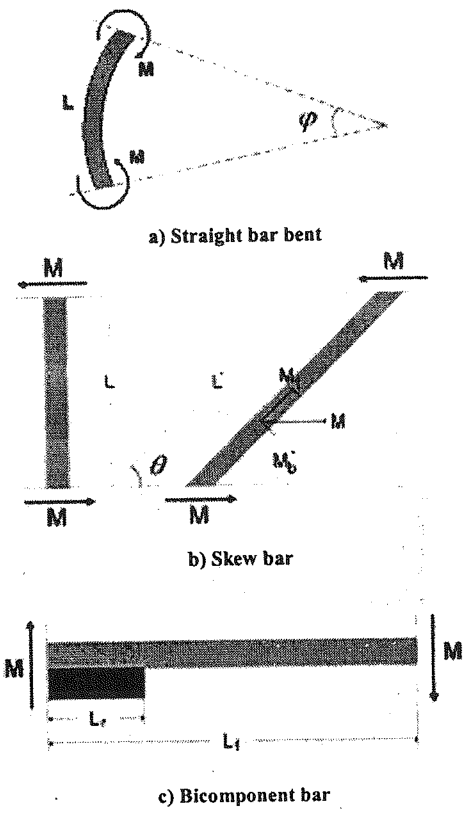

Figure 12: Pure bending of a bar model

Eq.5 becomes after integration;

$$U = \frac{M^2 L'}{2}\left(\frac{\sin^2\theta}{EI} + \frac{\cos^2\theta}{GJ}\right) \quad (6)$$

Upon application of Castigliano's theorem $\varphi = \partial U / \partial M$, it is found that;

$$\varphi' = ML'\left(\frac{\sin^2\theta}{EI} + \frac{\cos^2\theta}{GJ}\right) \quad (7)$$

From Figure 12b;

$$L' = \frac{L}{\sin\theta} \quad (8)$$

And for circular cross-section;

$$J = 2I \quad (9)$$

On the other hand, if the shearing modulus of elasticity is related to the modulus of elasticity E as follows:

$$a = \frac{E}{2G} \quad (10)$$

Then, carrying Eqs.8, 9 and 10 into Eq.7, yield;

$$\varphi' = ML\left(\frac{\sin^2\theta + a\cos^2\theta}{EI\sin\theta}\right) \quad (11)$$

With comparing Eq.11 and Eq.2, we have;

$$B' = B\left(\frac{\sin\theta}{\sin^2\theta + a\cos^2\theta}\right) \quad (12)$$

Where, B' is the flexural rigidity of the skew bar.

B. Flexural Rigidity of the Bicomponent bar

According to Figure 12c, a straight flexible bar of length L_f, coupling with rigid section of length L_r .suppose that the Proportion of the rigid section with flexible section is given by;

$$\alpha = \frac{L_r}{L_f} \quad (13)$$

Again from the basic Eq.1, the total angle of through with the structure is bent under bending couple M is;

$$\varphi = M\int\frac{ds}{B(s)} \quad (14)$$

Where, $B(s)$ is the bending rigidity of the structure along the length .then, becomes after integration;

$$\varphi = M\left(\frac{L_r}{B_r + B_f} + \frac{L_f - L_r}{B_f}\right) \quad (15)$$

Where, B_r and B_f are the bending rigidity of the rigid and flexural section of the bar, respectively.

Then, carrying Eq.13 into Eq.15, yield;

$$\varphi = ML\left(\frac{\alpha}{B_r + B_f} + \frac{1-\alpha}{B_f}\right) \quad (16)$$

With comparing Eq.16 and Eq.2 and knowing that $\alpha<1$ and $B_f \ll B_r$, so $\alpha/(B_r+B_f)\approx 0$, then we have;

$$\frac{1}{B} = \frac{1-\alpha}{B_f} \quad (17)$$

Where, B is the bending rigidity of the bicomponent bar.

8. ACKNOWLEDGMENT

We are indebted to the warp-knitted workshop of the Isfahan University of Technology for assistance towards carrying out this work. We would also like to acknowledge the help of Mr.H.Moradi who modifies the bending tester and Mr.J.Madahian for his assistance in preparing fabrics.

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