Low-Velocity Impact Response of Biaxial Pre-stressed Composite Sandwich Plates Using New Analytical Procedure

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ABSTRACT

A new computational procedure based on two Spring-Mass-Damper-Dashpot (SMDD) and Spring-Mass-Damper (SMD) models representing contact behavior between the impactor and the plate are presented to study the low velocity impact phenomenon of sandwich plates comprising of a transversely flexible core and laminated composite face-sheets. The interaction between the impactor and the plate is modeled with the help of a new system having three-degrees of-freedom, consisting of spring-mass-damper-dashpot or spring-mass-damper. The plate may be subjected to initial in-plane biaxial normal and shear stresses along the edges of the plate. Impact is assumed to occur normally over the top or the bottom face-sheets, at arbitrary location. The boundary conditions of the top and the bottom face-sheets are independent. The effects of transverse flexibility of the core and structural damping of the plate are considered in the proposed models. Based on these models, the analytic functions describing the contact force, the transverse deflections of the impactor and the plate are derived. The effects of some parameters such as plate aspect ratio, boundary conditions, initial in-plane biaxial stresses, and impact location are examined in details.

KEYWORDS

Sandwich Plate, Biaxial Pre-stresses, Low-velocity Impact, Soft Flexible Core, New Procedure.

1. INTRODUCTION

The time and cost associated with testing and the need for more rational design motivates development of methods to predict the effects of impact loads. The use of composite sandwich structures is hampered by their poor resistance to formation of impact damage, which may severely reduce the structural strength and stability [1],[2]. Extensive research has been dedicated to the problem of impact on composite laminates [3]-[6], while the work on sandwich structures is somewhat more limited. In this context, the work of Ambur and Cruz [7] may be mentioned in which a local-global analysis was carried out to determine the contact force and displacement of the plate. The effects of initial in-plane stresses on transverse impact response of single layer unidirectional composite plates were studied by Khalili *et al.* [8]. A two degree-of-freedom spring-mass model was improved with the use of implementing the structural damping to determine the contact force between the target and the impactor during the impact, by Gong and Lam, [9]. Recently, a complete review of impact on sandwich structures was performed by Abrate [10]. Analytical models for the wave controlled impact response that were caused by relatively small impactors were given in Ref. [11]. Analytical solutions for the transient deformation response of the sandwich plates were presented by Hoo Fatt *et al.* [12]. Olsson [13] also gave an engineering method for predicting the impact response and damage in sandwich plates. Various approaches have been proposed for the analysis of impact response of sandwich plates [14-17]. Most recently, the Higher-order Sandwich Plate Theory (HSAPT) has been proposed [18]-[20]. For a soft core, the vertical flexibility of the core must be taken into account since this flexibility of the core influences the stress and displacement fields in the face sheets. This vertical flexibility affects stress and displacement fields in the face sheets and leads to

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nonlinear in-plane and vertical displacement patterns in the core. First order Shear Deformation Theory (FSDT) and HSDT do not consider the effects of transverse flexibility and as well as interaction between face- sheets and the soft flexible core. Recently, Malekzadeh *et al.* [21]-[23] introduced FSDT for face-sheets and a non-linear variation of vertical acceleration in the core, instead of linear variation as is commonly assumed. With this modification, solutions were obtained for damped and undamped free vibrations of simply supported sandwich plates based on Improved Higher-order Sandwich Plate Theory (IHSAPT). As noted above, much effort has been made to analyze composite plates subjected to low-velocity impact using a discrete spring-mass dynamic system. However, none of those studies considered the effects of transverse flexibility and structural damping of the core of sandwich plates on their transient response. Therefore, the new analytical analysis procedure includes the effects of the above parameters. In this paper, new Three-Degree-Of-Freedom (TDOF) Springs-Masses-Damper (SMD) and Springs-Masses-Damper-Dashpot (SMDD) models are proposed to predict the contact force history for composite sandwich plates with transversely flexible core.

2. FORMULATION AND ASSUMPTIONS

The rectangular sandwich plate studied in this paper is composed of two FRP composite laminated and symmetric face sheets and a core, as shown in Fig. 1 where coordinates and sign conventions are also shown. The assumptions used in the present analysis are those encountered in linear elastic small deformation theories [18-23]. The stacking sequences of laminate in top and bottom face sheets are assumed to be symmetric. The core behavior follows the assumption which has been adopted by many researchers for the honeycomb type of core [20]. The core is assumed to behave in a linear elastic manner with small deformations, although its height may change and its transverse plane takes a nonlinear shape after deformation. The impactor is assumed to have a spherical/hemispherical shape and is made of an elastic material with high stiffness in comparison with the transverse stiffness of plate. The friction between plate and impactor is assumed to be negligible. In this work, the composite sandwich plate can be simply supported *(SSSS)* or fully clamped *(CCCC)* around all four edges of top and bottom face sheets. Alternatively, the boundary conditions of the top and bottom face-sheets may be prescribed separately. The effects of secondary contact loadings are assumed to be negligible. Various stages of the problem formulation are discussed below.

A. The modified Hertzian contact stiffness of impacted Sandwich plates with elastic flexible Core

The contact force between impactor and the target has often been assumed to be known. However, in practice, the contact force is the result of contact deformation between the impactor and the target and should be evaluated. The effective stiffness K_c can be obtained by the experimental static load-indentation test [12], or it can also be estimated by the following modified analytical methods. Note that the solution suggested below is only applicable prior to fiber fracture, core crushing (plastic deformation) or the penetration of the plate. Let $\Delta_1(t), \Delta_2(t)$ and $\Delta_0(t)$ represent the transverse displacements at the load point of the sandwich flat plate in the impacted top face sheet, bottom face sheet (see Fig. 2) and that of the impactor, at any time t during impact, respectively. The contact deformation is:

$$\alpha(t) = \Delta_0(t) - \Delta_1(t) \tag{1}$$

The contact force between the impactor and the sandwich plate during the impact is assumed to be governed by the nonlinear Hertzian contact law of the form:

$$F_c(t) = K_c \alpha^P \tag{2}$$

where K_c and P can be obtained by static indentation tests or they can also be estimated by Hertzian contact theory [9]. As the equation (2) can be highly nonlinear, seeking an analytic solution for the contact force might poses a formidable task. The present approach, as in Ref. [9], employs effective contact stiffness K_c^* and the assumption of linear relationship between equivalent contact force and contact deformation. This relationship is given below:

$$F_c^*(t) = K_c^* \alpha = K_c^*[\Delta_0(t) - \Delta_1(t)] \tag{3}$$

K_c^* is the effective stiffness and can be estimated analytically [9, 23].

B. The contact stiffness of impacted Sandwich plates with Rigid plastic flexible core in contact zone

Many analytical methods for determining the local deformation assume a Hertzian contact [9]-[13]. However, the local deformation considered here causes transverse deflections of the entire top face sheet and core crushing, so that the Hertzian contact laws are inappropriate for finding the local indentation response. Hoo Fatt and Park [12] divided the whole impact indentation process into three stages and applied three different mathematical models to the corresponding stages. These stages are: (I) plate on an elastic foundation; (II) plate on a rigid-plastic foundation; (III) Membrane on a rigid-plastic foundation. The load-indentation response has been obtained for the stage (II) by using the principle of minimum potential energy and is given as follows [12]:

$$F = \frac{32}{\sqrt{255}}\sqrt{2D_1\delta q} + \pi q R^2 + \{(\frac{65536}{33075})(\overline{N}_{xx} + \overline{N}_{yy}) + 2\overline{N}_{xy}\}\delta \tag{4}$$

D_{ij} (i, j=1, 2, 6) are coefficients of bending stiffness matrix of the plate. δ, q and R are respectively the deflection under the impactor, the crushing strength of the core and the radius of the impactor , and D_1 is the bending stiffness of the orthotropic face sheet [12, 23].

The first nonlinear term of equation (4) is the resistance due to the face sheet bending and crushing of honeycomb/foam outside the contact area of indentor, while the second term is due to crushing of core under the indentor (see Ref. [12]). The effective stiffness K_c^* can be obtained by linearizing the equation (4). The third term of equation (4) shows linear load-indentation behavior. Increasing the in-plane initial tensile forces increases the contact indentation force while an increase of the compressive in-plane forces decreases indentation force.

C. The proposed enhanced effective Structural Damping of Sandwich composite Beam

In this paper, the effective structural loss factor for two-layer composite beam that was proposed by Gong and Lam[9], is adopted and enhanced for calculation of the effective structural loss factor for three-layer sandwich composite beam. The energy dissipating effects of the material damping are considered. Young's modulus for the top and bottom face sheets and the damping layer (core) are complex quantities that are denoted by a superscript asterisk. The strain of the beam is [9]:

$$\varepsilon = \frac{z d\theta}{dx} = \frac{z}{j\omega_{11}}\frac{\partial}{\partial x}(\frac{\partial\theta}{\partial t}) \tag{5}$$

where $j = \sqrt{-1}$ and ω_{11} is the fundamental frequency and θ is the section rotation angle in pure bending of the sandwich beam. Using equation (5) and doing the same procedure indicate in section III of Ref. [9], the neutral axis position (δ_{neut}) of three layers sandwich beam section can be derived as follows:

$$\delta_{neut} = \frac{1}{2}\frac{[E^*{}_b h^2{}_b + E^*{}_c h^2{}_c - E^*{}_t h^2{}_t + 2E^*{}_b h_b h_c]}{[E^*{}_b h_b + E^*{}_t h_t + E^*{}_c h_c]} \tag{6}$$

where $E^*{}_t, E^*{}_b$ and $E^*{}_c$ are given as follows, respectively:

$$E^*{}_t = \overline{E}_t(1+\eta_t j),\ E^*{}_b = \overline{E}_b(1+\eta_b j), \tag{7}$$

$$E^*{}_c = \overline{E}_c(1+\eta_c j)$$

where $\overline{E}_t, \overline{E}_b, \overline{E}_c$ *are respectively the effective elastic Young's modulus of the top face sheet, the bottom face sheet and the core layer in length direction of beam and for symmetric laminate can be written as follows:*

$$\overline{E}_t = \frac{A^t{}_{11}A^t{}_{22} - A^t{}_{12}}{A^t{}_{22}h_t},\quad \overline{E}_b = \frac{A^b{}_{11}A^b{}_{22} - A^b{}_{12}}{A^b{}_{22}h_b},\ \overline{E}_c = E_c \tag{8}$$

The complex function of bending stiffness can be calculated by:

$$B^* = B_{st}(1+\eta_{st}j) = \int_{-(h_c+h_b-\delta_{neut})}^{-(h_c-\delta_{neut})} E^*{}_b z^2 dz + \int_{-(h_c-\delta_{neut})}^{\delta_{neut}} E^*{}_c z^2 dz + \int_{\delta_{neut}}^{(h_t+\delta_{neut})} E^*{}_t z^2 dz \tag{9}$$

Because the loss factor η_t and η_b in the basic elastic top and bottom face sheets are much lower than that in the damping thick core layer ($\eta_t, \eta_b << \eta_c$), the effects of η_t and η_b can be ignored and thus with taking both sides of equation (9) to be equal in real and imaginary components yields:

$$B_{st} = \frac{\Theta_6 - \Theta_7}{4\Theta_4},\quad \eta_{st} = \frac{\Theta_8 + \Theta_9}{\Theta_6 + \Theta_7} \tag{10}$$

where

$$\Theta_1 = \overline{E}_b h_b{}^2 + 2\overline{E}_b h_b h_c - \overline{E}_t h_t{}^2$$
$$\Theta_2 = \overline{E}_b h_b{}^3 + \overline{E}_t h_t{}^3 + \overline{E}_c h_c{}^3$$
$$\Theta_3 = \overline{E}_b h_c h_b(h_c + h_b)$$
$$\Theta_4 = (\overline{E}_b h_b + \overline{E}_t h_t + \overline{E}_c h_c)^2 + (\overline{E}_c h_c \eta_c)^2$$
$$\Theta_5 = \overline{E}_b h_b + \overline{E}_t h_t$$
$$\Theta_6 = 4\Theta_4(\frac{1}{3}\Theta_2 + \Theta_3) - \Theta_1{}^2(\Theta_5 + h_c\overline{E}_c) - 2h_c{}^2\Theta_1\Theta_5\overline{E}_c$$
$$\Theta_7 = -\Theta_5 h_c{}^4\overline{E}_c{}^2(1-\eta_c{}^2) - h_c{}^5\overline{E}_c{}^3(1+\eta_c{}^2) - 2h_c{}^3\Theta_1\overline{E}_c{}^2(1+\eta_c)^2$$
$$\Theta_8 = \eta_c(\frac{4}{3}\Theta_4 h_c{}^3\overline{E}_c + \Theta_1{}^2 h_c\overline{E}_c - 2\Theta_5 h_c{}^4\overline{E}_c{}^2 - 2h_c{}^2\Theta_1\Theta_5 E_c)$$
$$\Theta_9 = -h_c{}^5\overline{E}_c{}^3\eta_c(1+\eta_c{}^2)$$

Equation (10) shows that the effective structural loss factor η_{st} and the effective bending stiffness B_{st} can be obtained in terms of the loss factor of the elastic core layer, Young's modulus and the thicknesses of the core and the face sheets. The effective viscous damping coefficient C_{ef} can be obtained as follows [9]:

$$C_{ef} = \eta_{st} \frac{K_{sand}}{\omega_{11}} \tag{11}$$

D. The proposed Low velocity impact analysis Procedure

In Figures 2a and 2b, the core is assumed elastic and rigid plastic, respectively. When the indentation is small and core crushing is elastic (i.e., transverse strains is less than the elastic limit ε_c^e), the impact response is found by considering the model I (Fig. 2a). As the face sheet indentation becomes larger and core crushing is rigid plastic, impact response is found using the model II (Fig.2b).

1): Low elocity impact response of Sandwich plate with elastic flexible core (SMD-Model)

The equations of motion of the three-degree-of-freedom system (model I- Fig.2a) are as follows:

$$\begin{aligned} &M_I\ddot{\Delta}_0 + K_c^*(\Delta_0 - \Delta_1) = 0 \\ &\overline{M}_{face}\ddot{\Delta}_1 + K_{face}\Delta_1 + K_{core}(\Delta_1 - \Delta_2) + K_c^*(\Delta_1 - \Delta_0) = 0 \\ &\overline{M}_{sand}\ddot{\Delta}_2 + K_{sand}\Delta_2 + K_{core}(\Delta_2 - \Delta_1) + C_{ef}\dot{\Delta}_2 = 0 \end{aligned} \tag{12}$$

The equivalent stiffness of impacted face sheets for impact at centre or off-centre can be calculated from static analysis of laminated composite face sheets with arbitrary boundary conditions by using the analytical method [17] based on the first shear deformation theory (FSDT). In the special case of central impact, the equivalent stiffness of face sheets can be obtained as follows:

$$K_{face} = \omega_{11}^{\ 2}\overline{M}_{face} \tag{13}$$

where ω_{11} is the fundamental natural frequency parameter of face sheets that can be calculated from free vibration analysis of laminated composite plates [17]. It is clear that the magnitudes of natural frequencies and status of the shape modes of the top face-sheet depend on boundary conditions along the edges of the top face sheet.

The equivalent stiffness of the impacted sandwich plate with soft flexible core with an arbitrary location of impact point can be calculated from static/free vibration analysis of laminated composite sandwich plates with simply supported or fully clamped boundary conditions based on improved higher-order sandwich plate theory (IHSAPT) [21], [22]. In special case of impact at centre, the equivalent stiffness of sandwich plate can be obtained as follows:

$$K_{sand} = \omega_{11}^{\ 2}\overline{M}_{sand} \tag{14}$$

where ω_{11} is the fundamental natural frequency parameter of sandwich plate that can be calculated from the free vibration analysis of sandwich composite plate [22].

The effective mass of impacted face sheet can be determined from Ref. [9]:

$$\overline{M}_{face} = \rho_t h_t \iint_{A_P} W^2(x,y)dA_P \tag{15}$$

in which dA_P is the differential surface area of the target face sheet. The system of ordinary differential equation (12) can be solved analytically using the following initial conditions:

$$\begin{aligned} &\Delta_0(t=0) = 0, \Delta_1(t=0) = 0, \Delta_2(t=0) = 0 \\ &\dot{\Delta}_0(t=0) = V_0, \dot{\Delta}_1(t=0) = 0, \dot{\Delta}_2(t=0) = 0 \end{aligned} \tag{16}$$

Withapplying complex stiffness damping as a method of incorporating energy dissipation [9], the eigenvalue equation can be determined. Therefore:

$$\begin{aligned} &(M_I\overline{M}_{face}\overline{M}_{sand})\lambda^{*3} - [K_{gbc}\overline{M}_{face}M_I + K_{gcc}\overline{M}_{sand}M_I \\ &+ K_c^*\overline{M}_{face}\overline{M}_{sand}]\lambda^{*2} + [M_I(K_{gcc}K_{gbc} - K_{core}^{\ 2}) \\ &+ K_c^*(K_{gbc}\overline{M}_{face} + K_{gcc}\overline{M}_{sand}) - K_c^{*2}\overline{M}_{sand})]\lambda^* \\ &+ [K_c^{*2}K_{gbc} - K_c^*(K_{gcc}K_{gbc} - K_{core}^{\ 2})] = 0 \end{aligned} \tag{17}$$

where

$$K_{gcc} = K_{face} + K_{core} + K_c^*$$

$$K_{gbc} = K_{sand}(1 + \eta_{st}j) + K_{core} \quad , \quad j = \sqrt{-1}$$

The above eigen value equation has complex coefficients. The equation (21) can be solved analytically or numerically. $\lambda^* = (\omega^*)^2$ are the complex eigenvalues [23]. By solving equation (17) the complex eigen-values $\lambda^* = \lambda' + i\lambda''$ are obtained. After the complex eigen-values have been obtained, the circular frequency ω and the modal loss factor η of the plate can be calculated for a given mode of vibration from the following formulae:

$$\omega = \sqrt{\lambda'} \, , \qquad \eta = \frac{\lambda''}{\lambda'} \tag{18}$$

Then, the analytic functions of dynamic deflections and impact force can be obtained as follows:

$$\Delta_0 = c_1\phi^1{}_0 \sin(\omega_1 t) + c_2\phi^2{}_0 \sin(\omega_2 t) + c_3\phi^3{}_0 \sin(\omega_3$$
$$\Delta_1 = c_1 \sin(\omega_1 t) + c_2 \sin(\omega_2 t) + c_3 \sin(\omega_3 t)$$
$$\Delta_2 = c_1\phi^1{}_2 \sin(\omega_1 t) + c_2\phi^2{}_2 \sin(\omega_2 t) + c_3\phi^3{}_2 \sin($$
$$F^*{}_c(t) = K^*{}_c[c_1(\phi^1{}_0 - 1)\sin(\omega_1 t) + c_2(\phi^2{}_0 - 1)\sin($$
$$+ c_3(\phi^3{}_0 - 1)\sin(\omega_3 t)] \quad (19)$$

where coefficients $\phi^i{}_0$, $\phi^i{}_2$ and c_i $(i = 1,2,3)$ can be calculated using equations (16), [23].

2): LowV elocity Impact response of Sandwich Plate with Rigid plastic Core in contact Zone (SMDD-Model)

The quasi-static force-indentation response of a rigidly supported plate can be described in terms of a nonlinear spring force and a constant force dashpot. The response of this model is given by Eq. (4). The effective stiffness K_c^* can be obtained by linearizing the equation (4). For impact loading, additional resistance will be produced by the inertia of the impactor and deforming face sheets and the core [12], [23]. The constant force dashpot represents the dynamic crushing resistance of the core and is given by:

$$\overline{F}_{dash} = \pi R^2 q_d \quad (20)$$

The equations of motion of the SMDD system (model II) are as follows:

$$M_I\ddot{\Delta}_0 + K_c^*(\Delta_0 - \Delta_1) = 0$$
$$\overline{M}_{face}\ddot{\Delta}_1 + (K_{face} + K_c^*)\Delta_1 - K_c^*\Delta_0 + \overline{F}_{dash} = 0 \quad (21)$$
$$\overline{M}_{sand}\ddot{\Delta}_2 + K_{sand}\Delta_2 - \overline{F}_{dash} + C_{ef}\dot{\Delta}_2 = 0$$

The dynamic response of the impacted plate is described by the system of equations (21). The third equation is decoupled and independent, but the first two equations are coupled. The first two equations of the set of equations (21) can be rewritten as follows:

$$\Delta_1 = (\frac{M_I}{K_c^*})\ddot{\Delta}_0 + \Delta_0$$
$$(\overline{M}_{face}M_I)\ddddot{\Delta}_0 + [(K_{face} + K_c^*)M_I + \overline{M}_{face}K_c^*]\ddot{\Delta}_0 \quad (22)$$
$$+ (K_{face}K_c^*)\Delta_0 + K_c^*\overline{F}_{dash} = 0$$

Using the equations (16) and (22), the above stated fourth order ordinary differential equation can be solved analytically with the following initial conditions:

$$\Delta_0(0) = 0, \dot{\Delta}(0) = V_0, \ddot{\Delta}_0(0) = 0, \dddot{\Delta}_0(0) = -\frac{V_0}{M_I}K_c^* \quad (23)$$

Therefore, the impactor deflection function is obtained analytically as follows:

$$\Delta_0(t) = c_1 e^{a_1 t} + c_2 e^{a_2 t} + c_3 e^{a_3 t} + c_4 e^{a_4 t} - \frac{\overline{F}_{dash}}{K_{face}} \quad (24)$$

where $a_i (i = 1,2,3,4)$ are the analytical roots of following polynomial equation:

$$(\overline{M}_{face}M_I)a^4 + [(K_{face} + K_c^*)M_I$$
$$+ \overline{M}_{face}K_c^*]a^2 + (K_{face}K_c^*) = 0 \quad (25)$$

and using the equations (23) to (25), the coefficients $c_i(i = 1,2,3,4)$ can be obtained analytically [23].

Then, the general contact force function can be obtained as follows:

$$F_c^*(t) = -M_I\ddot{\Delta}_0(t) = -M_I(c_1 a_1{}^2 e^{a_1 t}$$
$$+ c_2 a_2{}^2 e^{a_2 t} + c_3 a_3{}^2 e^{a_3 t} + c_4 a_4{}^2 e^{a_4 t}) \quad (26)$$

The proposed spring-mass-damper-dashpot (SMDD) and spring-mass-damper (SMD) models facilitate the general formulation of the analytic impact force functions for various categories of impact, when the flexible core deforms according to elastic or rigid plastic behaviors.

3. Verification of Results and Discussions

The performance characteristics of proposed models in predicting the transient impact response of orthotropic, laminated composite sandwich plates are evaluated.

In this section, two examples were analyzed. Material data and geometrical properties of the plates used for the two examples are shown in Table 1. The results are compared with other analytical and experimental results on sandwich plates with edge support to verify the accuracy of the procedure.

A. Dynamic response of Sandwich plate subjected to low-velocity Large mass impact (Example 1)

The plate is simply supported and made of AS4/3501-6 carbon/epoxy face sheets and Nomex honeycomb core (HRH 10 1/8-4.0) with crushing strength $q = 3.83MPa$. The local indentation before damage initiation was about 0.7 mm which is small compared to the face sheet thickness which is approximately 2 mm.

The value of critical strain ε_c^e at which the Nomex core begins to exhibit nonlinear elastic behavior is 0.02 mm/mm [12]. Therefore, the 12.7 mm thick core should deform more than 0.24 mm before it can be modeled as plastic. In impact analysis, the dynamic crushing strength of Nomex honeycomb q_d is equal to $1.1q$ [12]. The material and geometrical properties of impactor are as follow: Geometry: Hemispherical-nosed cylinder (case-hardened steel):

$V_0 = 1.42 m/s$, $d = 25.4 mm$, $M_I = 3.48 Kg$.

The material properties of the core and the face sheets for example 1 are shown in Table 1. The static load-indentation and impact force curves of simpply supported composite sandwich plate are obtained based on present method, and compared with experimental results of Ref. [12]. The stacking sequences of 32 plies in the top and the bottom face-sheets are $[0/90]_{8s}$. The static deflection of the top face-sheet is computed from equation (4) and is compared with the experimental results of Ref. [12] in Fig. 3. Also, the effects of in-plane initial stresses on indentation force are shown, using the proposed modified equation (4). Figure 3 shows the increase of indentation force with increasing tensile in-plane initial stresses. A possible reason for discrepancy in Fig. 3 can be the neglecting of the effects of loading rate in both the honeycomb constitutive model and in the linear elastic model used for the face-sheets and bond layer. The effects of in-plane initial tensile stresses on contact force history based on SMD model are shown in Fig. 4. It is found that the higher initial stresses tend to intensify the contact force while simultaneously reducing the total contact time. The contact force history is compared with the analytical and experimental results in Fig. 4. Also, the effects of boundary conditions on contact force history are shown in Fig. 4. There is a negligible difference in the phase and magnitude of contact force results for a fully clamped and simply-supported boundary conditions. Further, the contact force result for a sandwich plate with top and bottom face sheets clamped on all edges (CCCC) is the same as the contact force result for a sandwich plate with fully clamped boundary in top face sheet and simply supported in bottom face sheet (CTSB). Therefore, the effects of the boundary conditions of bottom face sheet on contact force history are negligible. These observations indicate that the impact response of a sandwich plate is a very localized phenomenon as compared to that of a monolithic plate. For the clamped case, contact force magnitude is a little larger, and duration is smaller than for the simply supported case. Contact force history for simply supported plate obtained from the present method is in good agreement with that obtained from the experiments [12]. The theoretical predictions based on SMDD model, shown in Fig.4, indicate that the largest error in the value of maximum contact force obtained from the present analysis vis-a-vis the measured experimental values [12] is about 4.76% which corresponds to the impactor velocity, $V_0 = 1.42 m/\sec$. A possible reason for this small discrepancy may be the non-inclusion of the effects of loading rate in case of both the honeycomb constitutive model as well as the linear elastic model used for the face sheets and the bond layer.

B. Dynamic Response of Simply supported Composite Sandwich Plate subjected to Low-Velocity Medium mass Impact (Example 2)

The staking sequence at the top face sheet and the bottom face sheet are $[(45/0/-45/90/0)_2/90]_s$. The details of this sandwich construction are presented in Table 1. The plate that results in a 114.3 mm wide by 241.3 mm long exposed area were centrally or non-centrally impacted by means of a dropped weight with 12.7 mm diameter hemispherical tip $(M_I / M_{sand} = 1.602)$. The sandwich plate was then impacted at energy level of 2.7138 J below the damage threshold levels to generate contact force results that are used for validating the analysis approach [7]. Contact-force results for a sandwich plate subjected to 2.7138 J of impact energy are presented in Fig. 5. The present analytical contact force history obtained, are compared with the experimental and analytical results of Ref. [7] with the face sheet treated as a plate on an elastic foundation. The contact force results, based on present SMD model (model I) are in good agreement with the experimental results. The correlation of results is very good, because the through-thickness deformation and transverse flexibility of plate are important and are considered in the present analysis.

Figures 6a and 6b show the effect of impact location on contact force and central deflection of the plate. These figures show that the contact force increases, while the central deflection and the contact time decreases when the location of impact point moves closer to the edges or corners of the plate. It is because the maximum bending moment is obtained when the impact point is central and it decreases as the impact location moves away from the center towards the edge supports. The variations of contact force and central deflection of the plate by moving the impact point location around of center of the plate are small. Therefore, the impact response for sandwich plates is a local phenomenon. Figures 6c and, 6d show the effects of plate aspect ratio a/b with $(a/h = 10.618, h_c/h = 0.4191, M = 1kg, V_0 = 2.33 m/s)$ on contact force and central deflection of the plate. Figures show that when a/b increases, the impact time increases and maximum contact force decreases slightly. Also when the a/b increases, the central deflection of the top face sheet of the plate decreases.

3. CONCLUSIONS

The new equivalent Three Degree-Of-Freedom (TDOF) Spring-Mass-Damper-Dashpot and Spring-Mass-Damper (SMDD and SMD) models and a new procedure in impact analysis had presented and used to predict the low-velocity impact response of composite sandwich plates with elastic flexible core.

As it can be seen, the present results are in excellent agreement with the analytical solutions and experimental test results. Since the impact occurs within a very short time, damping of the structure is usually neglected in the low-velocity impact analysis and has small effects on impact response of sandwich plate. The solutions are based on closed-form expressions which have been implemented in a spreadsheet. The effects of the initial in-plane biaxial stresses and the transverse flexibility of the core have been considered in the proposed models. The present approach can be linked with the standard optimization programs and it can be used in the iteration process of the structural optimization. Although the initial tensile stress results in a higher contact force, its greater stiffening effect reduces the deflection as well as the stresses in the plate. In order to determine all components of the displacements, stresses and strains in the face-sheets and the core, a commercial FEM software can be employed only for modeling the layered sandwich plate (without the impactor), whereas the presented force function can be implemented to handle the contact force between the impactor and the plate. Therefore, the problem of impact on the sandwich structures can be simplified to solve a standard structural response equation of motion for a known impact loading.

5. NOMENCLATURE

a, b : length and width of the plate, respectively

C_{ef} : effective viscous damping coefficient of the panel

E_P : effective elastic modulus of the sandwich panel

E_I : effective elastic modulus of the impactor

$\overline{F}_{dash}$: dashpot force representing the dynamic crushing resistance of the core

$F_c(t), F_c^*(t)$: contact forces based on the Hertzian and linearized Hertzian contact laws, respectively

h_t, h_b, h_c, h : thicknesses of the top and the bottom face-sheets, the core and the panel, respectively

K_c, K_c^* : coefficients of the Hertzian and linearized Hertzian contact laws, respectively

$K_{face}, K_{core}, K_{sand}$: effective stiffnesses of the impacted face-sheet, the core and the panel at the impact location

M_I : mass of impactor

$\overline{M}_{sand}$: effective mass of the sandwich panel

$\overline{M}_{face}$: effective mass of the impacted face-sheet of the panel

N_{xx}, N_{yy}, N_{xy} : in-plane forces per unit length of the edge

P : exponent in the Hertzian contact law

q, q_d : static and dynamic crushing strengths of the core

R_I : impactor radius

V_0 : transverse velocity of the impactor at contact

$W(x, y)$: mode shape corresponding to the fundamental natural frequency of the impacted face-sheet

(x_0, y_0) :point of impact on the face-sheet,

z_t, z_b, z_c :normal coordinates in the mid- plane of the top and the bottom face-sheets and the core

$\alpha(t)$: contact indentation

$\alpha_{\max}$:maximum contact indentation

$\Delta_0(t), \Delta_1(t), \Delta_2(t)$: transverse displacement functions in discrete dynamic systems (SMD and SMDD models)

η_{st} : effective structural loss factor of the panel

ν_P : effective Poisson's ratio of the sandwich panel

ν_I : effective Poisson's ratio of the impactor

ρ_t, ρ_b, ρ_c : material densities of the face-sheets and the core

ω_{11} : fundamental natural frequency (rad/s) of the panel

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TABLE 1
MATERIAL PROPERTIES OF THE SANDWICH PLATES

Examples	**Example. 1**		**Example. 2**	
Properties	**Face**	**core**	**Face**	**core**
$E_{11}(Gpa)$	144.8	0.0804	153	0.00068
$E_{22}(Gpa)$	9.7	0.0804	8.96	0.00068
$E_{33}(Gpa)$	9.7	1.005	8.96	0.4756
$G_{12}(Gpa)$	7.1	0.0322	5.1	0.00027
$G_{13}(Gpa)$	7.1	0.1206	3.79	0.2754
$G_{23}(Gpa)$	3.76	0.0758	3.3	0.0965
ν_{12}	0.3	0.25	0.29	0.25
ν_{13}	0.3	0.02	0.29	0.03
ν_{23}	0.3	0.02	0.29	0.03
$\rho(kg/m^3)$	1632	64	1650	88.14
$h_c(mm)$	-	12.7	-	9.525
$t_f(mm)$	0.0635	-	0.3	-
$a(mm)$	178	178	241.3	241.3
$b(mm)$	178	178	114.3	114.3

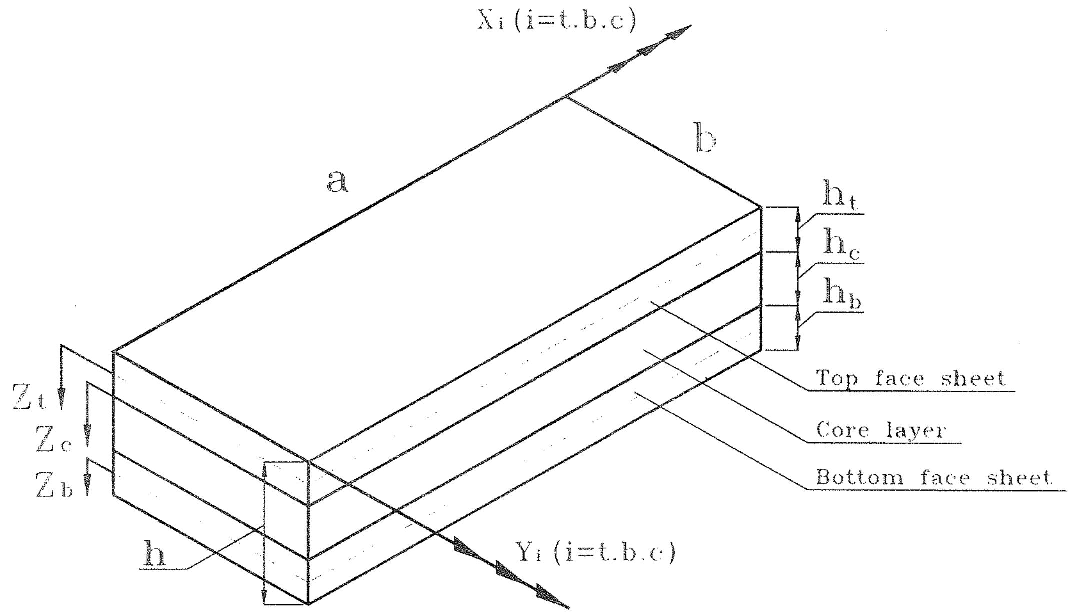

Figure 1:Sandwich composite plate with laminated face sheets. Plate coordinates and plate dimensions are also shown.

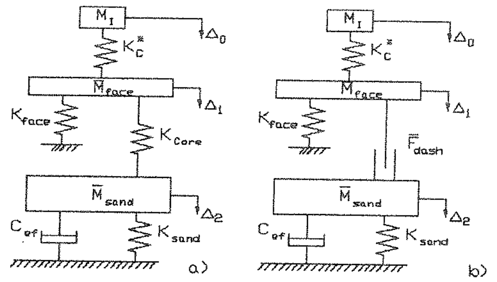

Figure 2: The equivalent three degree-of-freedom models of the plate and the impactor systems. a) Model: I(SMD model); b) Model: II (SMDD model).

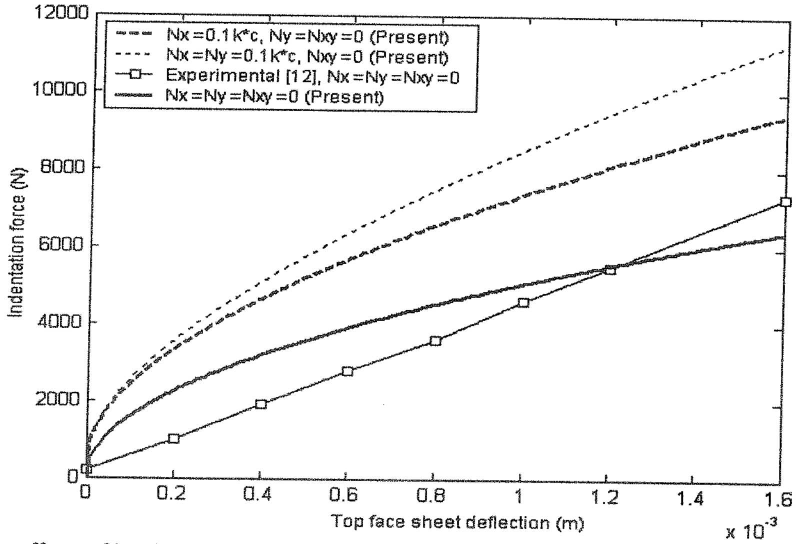

Figure 3: The effects of in-plane initial stresses on indentation force of impacted composite sandwich plate.

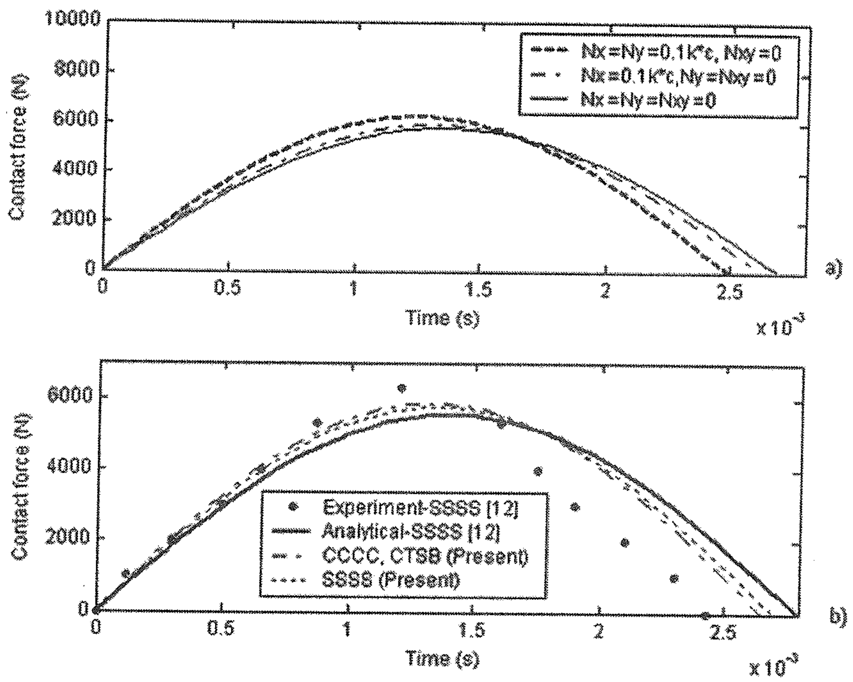

Figure 4: The contact force histories of the impacted composite sandwich plate: a) The effects of in-plane initial stresses on the contact force history, b) The effects of boundary conditions on contact force history.

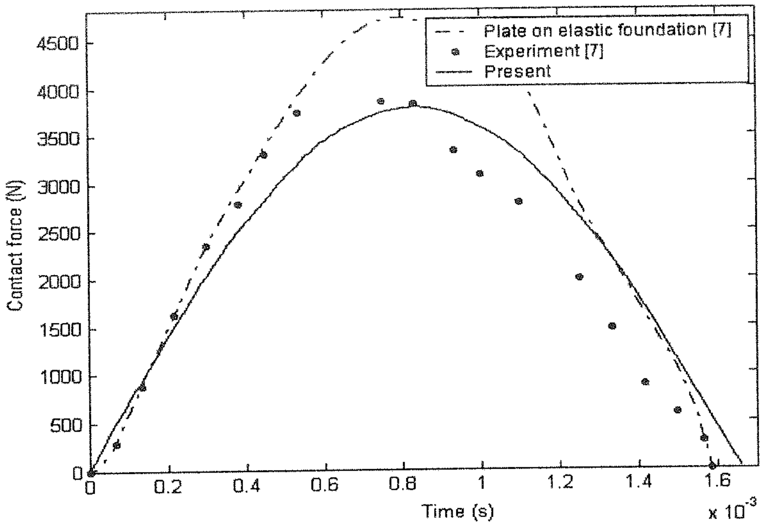

Figure 5: Comparison of the present contact-force results of the sandwich plate subjected to 2.7138J of impact energy in center of the plate with the analytical and experimental results.

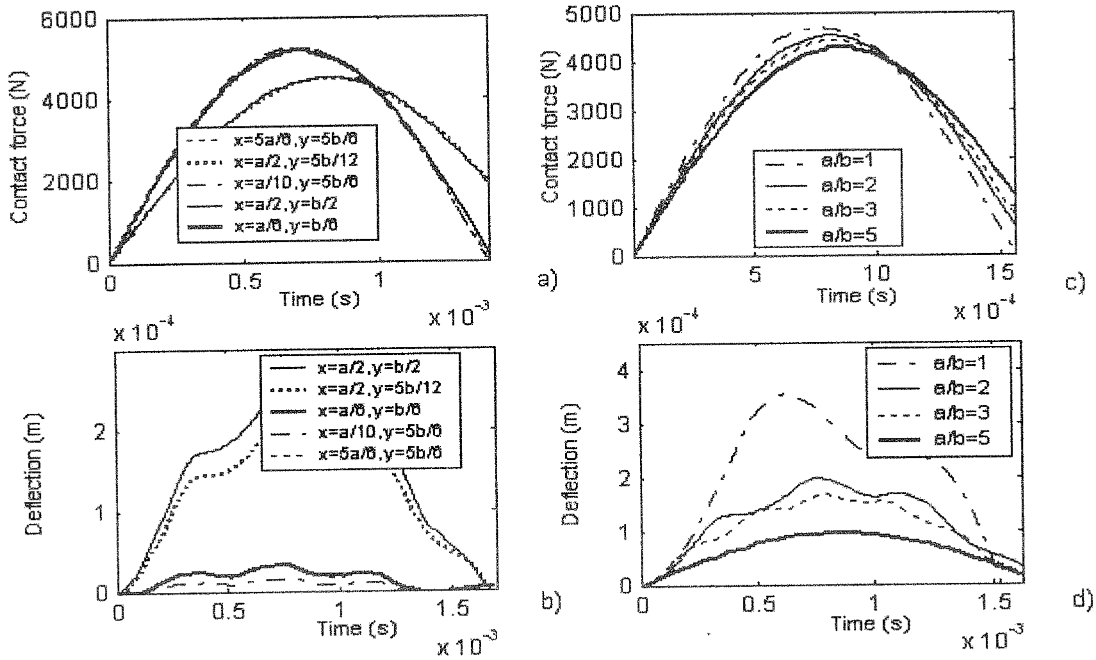

Figure 6: Effects of the plate aspect ratio a/b and the location of impact point on contact force and central top face sheet deflection of the plate, a) Effect of the impact location on the contact force, b) Effect of the impact location on central top face sheet deflection of the plate, c) Effect of the plate aspect ratio on contact force, d) Effect of the plate aspect ratio on the central top face sheet deflection of the plate.