A New Fuzzy Method for Trip Distribution Forecasting in Multi-commodity Transportation Networks

Mir B. Aryanezhad; F. Zandi

ABSTRACT

In this paper, a new method is proposed for trip distribution forecasting in multi-commodity transportation networks. In this new approach, real trip distributions at present are modeled by approximation functions for each commodity. In large-scale multi-commodity transportation networks, these functions could be obtained by parts of network as training data, without losing generality of the algorithm. Of course, it will have an influence on the approximation. Then, a fuzzy model for trip distribution forecasting calibrates these functions. This model considers all adjustments simultaneously. In order to show the application of the new method, a numerical example is given and solved.

KEYWORDS
Fuzzy, Multi-commodity, Approximation Function, Trip Distribution.

1. INTRODUCTION

Trip distribution constitutes the second stage in the transportation planning process. Trip distribution models are used to determine the number of trips between pairs of zones when the number of trips generated-attracted by particular zones is known. Thus, trip distribution forecasting identifies roadways where the volume of traffic will exceed the capacity of the roadway. Continual utilization of this model will enable the county to update and identify future capital needs. Generally, such information is essential to a variety of operational tasks failure analysis to capacity planning and traffic engineering.

The literature review presents a survey of theories, algorithms, techniques, and methods used for trip distribution forecasting. Before the 1970s, trip distributions were obtained via statistical surveys, such as home interviews and roadside surveys. The emphasis on transportation system management in the early 1970s increased the need for trip distribution forecasting.

Different models have been developed since then. A survey of these models may be divided into the following types:

In gravity-based models, regression techniques are applied to calibrate the parameters. The models are divided into linear (Low 1972, Holm et al. 1976, Gaudry and Lamarre 1978,) and nonlinear (Robillard 1975, Hogberg 1976) regression models.

In entropy models, the probability of a particular trip distribution occurring is assumed to be proportional to the number states of the system. The pioneers of these models are Willumsen (1978) and Zuylen (1978, 1979).

Statistical models include the constrained generalized least squares model (McNeil 1983) and constrained maximum-likelihood models (Geva 1983, Spiess 1987, Waitting and Maher 1992, Waitting and Grey 1991). Statistical models take into account the stochastic nature of the data and the problem. However, they have not been adequately tested. In addition, the stochastic theory used itself sometimes makes the problem more complicated for practical purposes.

Another group has been developed based on sort computing techniques such as fuzzy logic and neural network (Kalic et al. 1997, Mozolin et al. 2000). They compared their model based on multilayer perception neural network with maximum-likelihood doubly-constrained models. There still exists a need for further testing these models. Then, Kalic et al. (2003) attempted to develop a technique for modeling trip distribution. The model developed represents application of Genetic Algorithm (GA) for trip distribution forecasting.

All the above-mentioned models forecast the trip distribution for single-commodity transportation networks. But, no-body has addressed the simultaneous calibration of trip distribution forecasting in multi-commodity transportation networks.

1 Mir B. Aryanezhad and F. Zandi, are with the Department of Industrial Engineering, Iran University of Science and Technology, Tehran. Iran (e-mail: zandi@just.ac.ir).
In order to overcome this weakness, we will formulate the problem as follows:
If we consider a multi-commodity transportation network, how do we forecast trip distribution for future?
To answer this question, we reached to a new approach for trip distribution forecasting which saves time with the following contributions:
1) In the proposed method, trip distribution at present is modeled by the approximation functions for each commodity (these functions will be able to generalized, adapt, and learn trip distribution based on the real trip distribution at present).
2) In large multi-commodity transportation networks, it is proposed that the approximation functions be obtained by parts of networks as training data.
3) A fuzzy model calibrates the approximation functions for each commodity simultaneously.
4) Parameters of the multi-commodity network are defined imprecisely. Then, possibility distributions are defined for them.
Therefore, the basic research task in this article is to develop the trip distribution forecasting from single-commodity to multi-commodity. This paper is organized into three sections. The second section contains the new formulation followed by a new algorithm and dimension consideration of the problems in the case of large scale. A numerical example is given. Conclusions are presented in the third section.

2. NEW FORMULATION
Let us first define parameters of multi-commodity networks:
Suppose that zones \( i \) and \( j \) are the trip origin and trip destination of commodity \( k \), respectively:
\[
T^k_{ij} = \text{Number of interchanged trips of commodity } k \text{ between } i \text{ and } j.
\]
\[
P^k_i = \text{Number of trips produced of commodity } k \text{ at zone } i.
\]
\[
A^k_j = \text{Number of trips attracted of commodity } k \text{ at zone } j.
\]
\[
t^k_{ij} = \text{Travel time of commodity } k \text{ between } i \text{ and } j.
\]
\[
L^k_{ij} = \text{Optional adjustment factor for interchanges between zones } i \text{ and } j \text{ for commodity } k.
\]
\(i\) = An origin zone
\(j\) = A destination zone
Since these parameters of transportation networks are obtained by samples and statistical methods, they are imprecise. Therefore, possibility distributions must be defined for every parameter. Let us assume that the above parameters have possibility distributions \([8]-[13]\) with trapezoid functions (1):
\[
\begin{align*}
\Pi^k_i &= (P^k_{i1}, P^k_{i2}, P^k_{i3}, P^k_{i4}) \\
\tau^k_{ij} &= (t^k_{ij1}, t^k_{ij2}, t^k_{ij3}, t^k_{ij4}) \\
A^k_i &= (A^k_{i1}, A^k_{i2}, A^k_{i3}, A^k_{i4}) \\
L^k_{ij} &= (L^k_{ij1}, L^k_{ij2}, L^k_{ij3}, L^k_{ij4}) \\
T^k_{ij} &= (T^k_{ij1}, T^k_{ij2}, T^k_{ij3}, T^k_{ij4})
\end{align*}
\]
Now, the proposed method is described as follows:
STEP1: Defuzzify the parameters of network
These numbers are converted to crisp numbers by the Fuller method [12], that is:
\[
P^k_i = \frac{1}{3}[P^k_{i1} + P^k_{i2} + \frac{1}{2}(P^k_{i3} + P^k_{i4})]
\]
\[
t^k_{ij} = \frac{1}{3}[t^k_{ij1} + t^k_{ij2} + \frac{1}{2}(t^k_{ij3} + t^k_{ij4})]
\]
\[
A^k_i = \frac{1}{3}[A^k_{i1} + A^k_{i2} + \frac{1}{2}(A^k_{i3} + A^k_{i4})]
\]
\[
T^k_{ij} = \frac{1}{3}[T^k_{ij1} + T^k_{ij2} + \frac{1}{2}(T^k_{ij3} + T^k_{ij4})]
\]
And let: \(X^k_{ij} = [P^k_i, A^k_j, L^k_{ij}, t^k_{ij}]\)
Then, for normalizing the elements of \(X^k_{ij}\), we have:
\[
P^k_i (s) = \frac{P^k_i}{\max_i P^k_i} \quad A^k_j (s) = \frac{A^k_j}{\max_j A^k_j}
\]
\[
L^k_{ij} (s) = \frac{L^k_{ij}}{\max_{i,j} L^k_{ij}} \quad t^k_{ij} (s) = \frac{t^k_{ij}}{\min_{i,j} t^k_{ij}}
\]
Therefore, \(X^k_{ij}\) becomes:
\[
\hat{X}^k_{ij} (s) = [\hat{P}^k_i (s), \hat{A}^k_j (s), \hat{L}^k_{ij} (s), \hat{t}^k_{ij} (s)]
\]
STEP2: Calculate the approximation function
Since \(T^k_{ij}\) is a function of \(P^k_i, A^k_j, L^k_{ij}, t^k_{ij}\), That is:
\[
\hat{T}^k_{ij} = f_k (\hat{P}^k_i (s), \hat{A}^k_j (s), \hat{L}^k_{ij} (s), \hat{t}^k_{ij} (s))
\]
where: \(\hat{T}^k_{ij}\) estimates \(T^k_{ij}\).
For estimating of $T_{ij}^{k}$, Sum of squares error between $T_{ij}$ and $\hat{T}_{ij}$ must be minimized:

$$\text{SSE} = \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} (T_{ij}^{k} - \hat{T}_{ij}^{k})^2 = \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \left| T_{ij}^{k} - f_k(P_i(s), A_i^k(s), L_i^k(s), t_i^k(s)) \right|^2$$

(6)

It is proven based on regularization theory that the approximation function (7) minimizes SSE [7],[12]:

$$\hat{T}_{ij}^{k} = \sum_{n=1}^{N} w_{nj}^{k} G(X_{ij}^{k}(s), X_{[i]}^{k}(s))$$

(7)

where:

$$G(X_{ij}^{k}(s), X_{[i]}^{k}(s)) = e^{-\frac{(X_{ij}^{k}(s) - X_{[i]}^{k}(s))^2}{\delta_{ij}^{k}(s)}}$$

(8)

And $X_{[i]}^{k}(s)$ is one of $X_{ij}^{k}(s)$ that selected as the center of function $G$.

Clearly, we can rewrite the (7) in matrix notation:

$$T_{ij}^{k} = G^k W^k$$

where:

$$T_{ij}^{k} = [T_{i1}^{k}, T_{i2}^{k}, ..., T_{iN}^{k}]$$

(9)

$$G^k = \begin{bmatrix}
G(X_{11}^{k}, X_{11}^{k}) & \cdots & G(X_{1N}^{k}, X_{11}^{k}) \\
\vdots & \ddots & \vdots \\
G(X_{N1}^{k}, X_{11}^{k}) & \cdots & G(X_{NN}^{k}, X_{11}^{k}) 
\end{bmatrix}$$

(10)

$$W^k = [W_1^k, W_2^k, ..., W_N^k]$$

(11)

$M$: Number of the Arcs.

$N$: Number of selected arcs for learning approximation function.

Now, for finding the weight vector $W^k$, we may have these cases:

CASE1: $M > N$ (for large multi-commodity transportation networks, it is proposed that the above approximation functions be calculated by parts of multi-commodity network as far as the training data are available):

$$W^k = (G^T G^k)^{-1} G^T \vec{T}^k$$

(12)

where: $(G^T G^k)^{-1} G^T$ represents the pseudoinverse $G^T$.

Then, go to step 3.

CASE2: $M = N$ (For small multi-commodity transportation networks):

$$W^k = (G^k)^{-1} \vec{T}^k$$

Then, go to step 4.

STEP3: Formulate the Fuzzy mathematical model

For calibrating the above approximation function, the following proposed model is formulated, which is one of the main contributions of this paper:

A. Objective Functions 1: Based on conservation of flow [1], the sum of trips produced for $k$-th commodity in zone $i$ should be equal to $P_i^k$:

$$\sum_{j=1}^{m} \hat{T}_{ij}^{k} \equiv P_i^k \quad i = 1, ..., n$$

(14)

But the sum of trips produced of commodity $k$ in zone $i$ do not necessarily equal to $P_i^k$. Therefore, $\hat{T}_{ij}^{k}$ must be calibrated by adjustment coefficients $x_i^k, x_j^k$:

$$\sum_{j=1}^{m} \hat{T}_{ij}^{k} x_i^k x_j^k \equiv P_i^k$$

(15)

B. Objective Functions 2:

Similarly, the sum of trips attracted of commodity $k$ to zone $j$ should be equal to $A_j^k$:

$$\sum_{i=1}^{n} \hat{T}_{ij}^{k} x_i^k x_j^k \equiv A_j^k$$

(16)

C. Objective Functions 3:

Also, the calculated trip-time frequency distribution for commodity $k$ should be equal to the trip-time frequency distribution of real trip distribution [3],[5], i.e.,

$$\sum_{i} \hat{T}_{ij}^{k} x_i^k x_j^k - \sum_{i} T_{ij}^{k} \equiv 0$$

(17)

D. Constraints:

Let $u_0$ represents the upper limit of the sum of all commodities in arc $(i, j)$ of the multi-commodity network. Then, we will have:

$$\sum_k \hat{T}_{ij}^{k} x_i^k x_j^k \leq u_0$$

(18)

The above objective functions and the constraints could make a fuzzy multi-objective decision making model. It should be mentioned that in references [13],[14],[16],[8] a fuzzy objective function has been converted to the constraint
a linear programming. Now, we have utilized this idea to propose the following new model:

Find $x^k_i, x^k_j$

St:

\[
\begin{align*}
    f^k_i &= P^k_i - \sum_{i=1}^{n} \tilde{T}^k_{ij} x^k_i x^k_j \geq 0 \\
    f^k_j &= A^k_j - \sum_{i=1}^{n} \tilde{T}^k_{ij} x^k_i x^k_j \geq 0 \\
    f^k_k &= \sum_{i=1}^{n} \tilde{T}^k_{ij} x^k_i x^k_j - \sum_{i=1}^{n} T^k_{ij} \geq 0 \\
    \sum_{k=1}^{n} \tilde{T}^k_{ij} x^k_i x^k_j &\leq u_j \\
    x^k_i, x^k_j &\geq 0
\end{align*}
\]  

(19)

Obviously, the fuzzy model (19) calibrates all of the approximation functions simultaneously.

Now, according to Zimmermann's approach [8], the optimal solution of the above fuzzy model can be obtained by solving the following model:

\[
\begin{align*}
    \text{Minimize} &\left[ \text{Min} \left[ \mu_i(\tilde{T}^k_{ij}), \mu_j(\tilde{T}^k_{ij}) \right], \mu_i(\tilde{T}^k_{ij}) \right] \\
    \mu_i(\tilde{T}^k_{ij}) &= \begin{cases} 
    1 + \frac{f^k_i}{r_j} & -r_j \leq f^k_i \leq 0 \\
    1 - \frac{f^k_i}{r_j} & 0 \leq f^k_i \leq r_j \\
    0 & \text{otherwise}
\end{cases} \\
    \mu_j(\tilde{T}^k_{ij}) &= \begin{cases} 
    1 + \frac{f^k_j}{r_i} & -r_i \leq f^k_j \leq 0 \\
    1 - \frac{f^k_j}{r_i} & 0 \leq f^k_j \leq r_i \\
    0 & \text{otherwise}
\end{cases}
\end{align*}
\]  

(20)

where $q_i, r_j$ and $v_i$ are the tolerances of the objective functions $f^k_i, f^k_j$ and $f^k_k$. It should be mentioned that the amount of them are given initially.

By assuming:

\[
\text{Min}[\mu_i(\tilde{T}^k_{ij}), \mu_j(\tilde{T}^k_{ij}), \mu_i(\tilde{T}^k_{ij})] = \lambda
\]  

(22)

This satisfies:

\[
\lambda \leq \mu_i(\tilde{T}^k_{ij}), \lambda \leq \mu_j(\tilde{T}^k_{ij}), \lambda \leq \mu_i(\tilde{T}^k_{ij})
\]  

(23)

From (21), we obtain:

\[
\begin{align*}
    \lambda &\leq 1 + \frac{f^k_i}{q_i}, \lambda \leq 1 - \frac{f^k_i}{q_i}, \\
    \lambda &\leq 1 + \frac{f^k_j}{r_j}, \lambda \leq 1 - \frac{f^k_j}{r_j}, \\
    \lambda &\leq 1 + \frac{f^k_k}{v_i}, \lambda \leq 1 - \frac{f^k_k}{v_i}
\end{align*}
\]  

(24)

(\lambda - 1)q_i \leq f^k_i \leq (1 - \lambda)q_i, \\
\left(\lambda - 1\right)r_j \leq f^k_j \leq \left(1 - \lambda\right)r_j, \\
\left(\lambda - 1\right)v_i \leq f^k_k \leq \left(1 - \lambda\right)v_i.
\]  

(25)

In conclusion, by referring to (19), (20) and (25) the proposed new model (P) will be obtained:

\[
\begin{align*}
    \text{Min} \lambda &\quad \text{(Model P)} \\
    \text{St:} &\quad \left(\lambda - 1\right)r_j \leq A^k_j - \sum_{i=1}^{n} \tilde{T}^k_{ij} (I) \leq \left(1 - \lambda\right)r_j
\end{align*}
\]
\[(\lambda - 1)q_i \leq P_{i}^{k} - \sum_{j=1}^{m} \hat{T}_{ij}^{k}(I) \leq (1 - \lambda)q_i,\]

\[(\lambda - 1)v_i \leq \sum_{j=1}^{n} \hat{T}_{ij}^{k}(I) - \sum_{j=1}^{n} T_{ij}^{k} \leq (1 - \lambda)v_i,\]

\[\sum_{k} \hat{T}_{ij}^{k}(I) \leq u_{ij},\]

\[\hat{T}_{ij}^{k}(I) = x_{i}^{k} x_{j}^{k},\]

\[x_{i}^{k}, x_{j}^{k}, \lambda, \hat{T}_{ij}^{k}(I) \geq 0\]

Therefore, the model (P) calibrates the approximation functions (7). Fortunately, since most of the constraints in model (P) are of quadratic form, this model can be handled by Lingo software. It is worth mentioning that the quadratic model is convex [9].

STEP4: Forecast trip distribution

Suppose that parameters of the multi-commodity network in the future are \(P_{ij}^{k}, A_{ij}^{k}, L_{ij}^{k}, t_{ij}^{k}\). Trip distribution in the future \((\hat{T}_{ij}^{k})\) is calculated by (26):

\[X_{ij}^{k} = [P_{ij}^{k}, A_{ij}^{k}, L_{ij}^{k}, t_{ij}^{k}] \rightarrow X_{ij}^{k}(s) = [P_{ij}^{k}(s), A_{ij}^{k}(s), L_{ij}^{k}(s), t_{ij}^{k}(s)],\]

\[t_{ij}^{k}(s) \rightarrow \hat{T}_{ij}^{k} = f_{k}(P_{ij}^{k}(s), A_{ij}^{k}(s), L_{ij}^{k}(s), t_{ij}^{k}(s)) = \sum_{n} W_{h}^{k} G(X_{ij}^{k}(s), X_{[n]}^{k}(s)) x_{i}^{k} x_{j}^{k}.\]

Then, it will be calibrated by proposed model (F):

\[\text{Min } \lambda^{\prime}\]

\[(\text{Model F})\]

\[\text{St:}\]

\[(\lambda - 1)r_{ij}^{\prime} \leq A_{ij}^{k} - \sum_{j=1}^{m} \hat{T}_{ij}^{k}(I) \leq (1 - \lambda)r_{ij}^{\prime},\]

A. Dimension Consideration

If we consider the multi-commodity transportation network with \(n\) nodes, \(r\) arcs and \(m\) commodities:

1. Maximum size of the model \(P\) will be \((n+r)rm\) constraints and \(nm\) variables which could be handled in the case of large scale.

2. Size of the model \(F\) will be \(nm\) constraints and variables.

It is worth mentioning that the main contribution of this research is to develop two new optimization problems (P) and (F). The optimal solution of model (F) can be obtained by Lingo software.

Now, the proposed model is illustrated by a numerical example.

B. Numerical Example

Consider the following two-commodity network. Trips produced, trips attracted, the travel time and trip distribution at present between zones for each commodity are given in Table 1.
Figure 1: Two-commodity transportation network

TABLE 1
PARAMETER VALUES AT PRESENT (GIVEN DATA)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{T}_{15}^1$</td>
<td>(230,240,245,250)</td>
<td>$\tilde{T}_{15}^2$</td>
<td>(410,420,425,430)</td>
</tr>
<tr>
<td>$\tilde{T}_{16}^1$</td>
<td>(150,160,165,170)</td>
<td>$\tilde{T}_{16}^2$</td>
<td>(270,280,285,290)</td>
</tr>
<tr>
<td>$\tilde{T}_{25}^1$</td>
<td>(150,160,165,170)</td>
<td>$\tilde{T}_{25}^2$</td>
<td>(270,280,285,290)</td>
</tr>
<tr>
<td>$\tilde{T}_{26}^1$</td>
<td>(50,60,65,70)</td>
<td>$\tilde{T}_{26}^2$</td>
<td>(50,60,65,70)</td>
</tr>
<tr>
<td>$\tilde{T}_{35}^1$</td>
<td>(230,240,245,250)</td>
<td>$\tilde{T}_{35}^2$</td>
<td>(410,420,425,430)</td>
</tr>
<tr>
<td>$\tilde{T}_{45}^1$</td>
<td>(110,120,125,130)</td>
<td>$\tilde{T}_{45}^2$</td>
<td>(270,280,285,290)</td>
</tr>
<tr>
<td>$\tilde{T}_{56}^1$</td>
<td>(30,40,45,50)</td>
<td>$\tilde{T}_{56}^2$</td>
<td>(30,40,45,50)</td>
</tr>
<tr>
<td>$\tilde{T}_{65}^1$</td>
<td>(90,100,105,110)</td>
<td>$\tilde{T}_{65}^2$</td>
<td>(90,100,105,110)</td>
</tr>
<tr>
<td>$\tilde{T}_{76}^1$</td>
<td>(90,100,105,110)</td>
<td>$\tilde{T}_{76}^2$</td>
<td>(90,100,105,110)</td>
</tr>
</tbody>
</table>
If parameter values change in the future as follows (see table 2). Find trip distribution between the zones for two-commodity network in the future (assuming $L_{ij}^1 = L_{ij}^2 = 1$).

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Parameter Values in the Future (Given Data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{t}<em>{13} = \tilde{t}</em>{24} = (4.5,5.5,5.6,6.5)$</td>
<td>$\tilde{P}^{1}_{4} = (140,150,155,160)$</td>
</tr>
<tr>
<td>$\tilde{t}_{56} = (4.5,5.5,5,6)$</td>
<td>$\tilde{P}^{2}_{4} = (240,250,255,260)$</td>
</tr>
<tr>
<td>$\tilde{t}<em>{12} = \tilde{t}</em>{15} = \tilde{t}<em>{34} = \tilde{t}</em>{26} = \tilde{t}_{45} = (2.4,2.5,2.55,2.6)$</td>
<td>$\tilde{A}^{1}_{4} = (890,900,905,910)$</td>
</tr>
<tr>
<td>$\tilde{t}<em>{14} = \tilde{t}</em>{33} = \tilde{t}<em>{16} = \tilde{t}</em>{25} = \tilde{t}_{35} = (3.4,4.5,5.5)$</td>
<td>$\tilde{A}^{2}_{4} = (690,700,705,710)$</td>
</tr>
<tr>
<td>$\tilde{t}_{46} = (3.4,5.5)$</td>
<td>$\tilde{A}^{2}_{6} = (1090,1100,1105,1110)$</td>
</tr>
</tbody>
</table>

Solution:

In this example, we consider $M=N$, then, the construction and solution of (model F) will be required. Here is the model F. 

Min $\lambda$

(Model F)

\[
-5(1-\lambda) \leq 600 - (\tilde{T}_{15}^1(I) + \tilde{T}_{16}^1(I)) \leq 5(1-\lambda)
\]

\[
-5(1-\lambda) \leq 900 - (\tilde{T}_{15}^1(I) + \tilde{T}_{16}^1(I)) \leq 5(1-\lambda)
\]

\[
-5(1-\lambda) \leq 600 - (\tilde{T}_{25}^1(I) + \tilde{T}_{26}^1(I)) \leq 5(1-\lambda)
\]

\[
-5(1-\lambda) \leq 500 - (\tilde{T}_{25}^1(I) + \tilde{T}_{26}^1(I)) \leq 5(1-\lambda)
\]

\[
-5(1-\lambda) \leq 500 - (\tilde{T}_{35}^1(I) + \tilde{T}_{36}^1(I)) \leq 5(1-\lambda)
\]

\[
-5(1-\lambda) \leq 500 - (\tilde{T}_{35}^1(I) + \tilde{T}_{36}^1(I)) \leq 5(1-\lambda)
\]

\[
-5(1-\lambda) \leq 150 - (\tilde{T}_{45}^1(I) + \tilde{T}_{46}^1(I)) \leq 5(1-\lambda)
\]

\[
-5(1-\lambda) \leq 150 - (\tilde{T}_{45}^1(I) + \tilde{T}_{46}^1(I)) \leq 5(1-\lambda)
\]

\[
-5(1-\lambda) \leq 250 - (\tilde{T}_{45}^1(I) + \tilde{T}_{46}^1(I)) \leq 5(1-\lambda)
\]

\[
-5(1-\lambda) \leq 900 - (\tilde{T}_{13}^1(I) + \tilde{T}_{14}^1(I) + \tilde{T}_{16}^1(I) + \tilde{T}_{35}^1(I)) \leq 5(1-\lambda)
\]

\[
-5(1-\lambda) \leq 700 - (\tilde{T}_{13}^1(I) + \tilde{T}_{14}^1(I) + \tilde{T}_{16}^1(I) + \tilde{T}_{35}^1(I)) \leq 5(1-\lambda)
\]

\[
-5(1-\lambda) \leq 950 - (\tilde{T}_{26}^1(I) + \tilde{T}_{36}^1(I) + \tilde{T}_{16}^1(I) + \tilde{T}_{46}^1(I)) \leq 5(1-\lambda)
\]

So:

\[
-5(1-\lambda) \leq 1100 - (\tilde{T}_{26}^1(I) + \tilde{T}_{36}^1(I) + \tilde{T}_{16}^1(I) + \tilde{T}_{46}^1(I)) \leq 5(1-\lambda)
\]
\[ \hat{T}_{15}(I) = 318.709x_1, x_2\]
\[ \hat{T}_{16}(I) = 263.204x_1, x_2\]
\[ \hat{T}_{25}(I) = 281.775x_1, x_2\]
\[ \hat{T}_{26}(I) = 300.404x_1, x_2\]
\[ \hat{T}_{35}(I) = 235.419x_3, x_4\]
\[ \hat{T}_{36}(I) = 256.766x_3, x_4\]
\[ \hat{T}_{45}(I) = 69.095x_5, x_6\]
\[ \hat{T}_{46}(I) = 53.881x_5, x_6\]
\[ \hat{T}_{55}(I) = 424.804x_5, x_6\]
\[ \hat{T}_{56}(I) = 289.287x_5, x_6\]
\[ \hat{T}_{65}(I) = 167.019x_5, x_6\]
\[ \hat{T}_{66}(I) = 232.302x_5, x_6\]
\[ \hat{T}_{55}(I) = 48.816x_5, x_6\]
\[ \hat{T}_{56}(I) = 68.080x_5, x_6\]
\[ \hat{T}_{65}(I) = 118.736x_5, x_6\]
\[ \hat{T}_{66}(I) = 80.398x_5, x_6\]

The optimal solution of model (F) is given in Table 3 (solved by Lingo software):

TABLE 3

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Forecasting</th>
<th>Commodity</th>
<th>Survey model</th>
</tr>
</thead>
<tbody>
<tr>
<td>404.3265</td>
<td>420</td>
<td>240</td>
<td>T15</td>
</tr>
<tr>
<td>490.6735</td>
<td>280</td>
<td>160</td>
<td>T16</td>
</tr>
<tr>
<td>145.1731</td>
<td>277.2482</td>
<td>160</td>
<td>T25</td>
</tr>
<tr>
<td>359.8269</td>
<td>327.7518</td>
<td>240</td>
<td>T26</td>
</tr>
<tr>
<td>44.47258</td>
<td>224.0430</td>
<td>40</td>
<td>T35</td>
</tr>
<tr>
<td>110.5274</td>
<td>270.9570</td>
<td>60</td>
<td>T36</td>
</tr>
<tr>
<td>111.0278</td>
<td>83.12359</td>
<td>120</td>
<td>T45</td>
</tr>
<tr>
<td>133.9722</td>
<td>71.87641</td>
<td>80</td>
<td>T46</td>
</tr>
</tbody>
</table>
By referring to Table 3, we could see the forecasted trip distribution between the zones for some commodity gets higher (i.e. T16 for commodity 2 which is 280 at present gets 490.67 in the future). If we consider the Table 3 in more details, we could find more similar cases. This trip distribution forecasting identifies that the volume of traffic will exceed the capacity of paths. Thus, the capacity planning and traffic engineering are required.

3. CONCLUSION

In this research, the trip distribution forecasting has been formulated in multi-commodity transportation networks by a fuzzy multi-objective optimization model. Then, this multi-objective model has been converted to single objective model by Zimmermann's approach. The special structures of these two models are very close to quadratic programming, where its convexity is confirmed. Therefore, there was no difficulty applying Lingo software to solve these models. In both models, the amounts of tolerances are given by prior-information.

4. ACKNOWLEDGEMENT

The authors gratefully acknowledge the anonymous referees for their useful comments on the original version of this paper.

5. REFERENCES


