A New Fuzzy Method for Trip Distribution Forecasting in Multi-commodity Transportation Networks

Mir B. Aryanezhad; F. Zandi[1]

ABSTRACT

In this paper, a new method is proposed for trip distribution forecasting in multi-commodity transportation networks. In this new approach, real trip distributions at present are modeled by approximation functions for each commodity. In large-scale multi-commodity transportation networks, these functions could be obtained by parts of network as training data, without losing generality of the algorithm. Of course, it will have an influence on the approximation. Then, a fuzzy model for trip distribution forecasting calibrates these functions. This model considers all adjustments simultaneously. In order to show the application of the new method, a numerical example is given and solved.

KEYWORDS
Fuzzy, Multi-commodity, Approximation Function, Trip Distribution.

1. INTRODUCTION

Trip distribution constitutes the second stage in the transportation planning process. Trip distribution models are used to determine the number of trips between pairs of zones when the number of trips generated-attracted by particular zones is known. Thus, trip distribution forecasting identifies roadways where the volume of traffic will exceed the capacity of the roadway. Continual utilization of this model will enable the county to update and identify future capital needs. Generally, such information is essential to a variety of operational tasks failure analysis to capacity planning and traffic engineering.

The literature review presents a survey of theories, algorithms, techniques, and methods used for trip distribution forecasting. Before the 1970s, trip distributions were obtained via statistical surveys, such as home interviews and roadside surveys. The emphasis on transportation system management in the early 1970s increased the need for trip distribution forecasting.

Different models have been developed since then. A survey of these models may be divided into the following types.

In gravity-based models, regression techniques are applied to calibrate the parameters. The models are divided into linear (Low 1972, Holm *et al.* 1976, Gaudry and Lamarre 1978,) and nonlinear (Robillard 1975, Hogberg 1976) regression models.

In entropy models, the probability of a particular trip distribution occurring is assumed to be proportional to the number states of the system. The pioneers of these models, are Willumsen(1978) and Zuylen (1978, 1979).

Statistical models include the constrained generalized least squares model (McNeil 1983) and constrained maximum-likelihood models (Geva 1983, Spiess 1987, Walting and Maher 1992, Walting and Grey 1991).Statistical models take into account the stochastic nature of the data and the problem. However, they have not been adequately tested. In addition, the stochastic theory used itself sometimes makes the problem more complicated for practical purposes. Another group has been developed based on sort computing techniques such as fuzzy logic and neural network (Kalic *et al.* 1997, Mozolin *et al.* 2000). They compared their model based on multilayer perception neural network with maximum-likelihood doubly-constrained models. There still exists a need for further testing these models. Then, Kalic *et al.* (2003) attempted to develop a technique for modeling trip distribution. The model developed represents application of Genetic Algorithm (GA) for trip distribution forecasting.

All the above-mentioned models forecast the trip distribution for single-commodity transportation networks. But, no-body has addressed the simultaneous calibration of trip distribution forecasting in multi-commodity transportation networks.

[1] Mir B. Aryanezhad and F. Zandi, are with the Department of Industrial Engineering, Iran University of Science and Technology, Tehran , Iran (e-mail:zandi@iust.ac.ir).

In order to overcome this weakness, we will formulate the problem as follows:

If we consider a multi-commodity transportation network, how do we forecast trip distribution for future?

To answer this question, we reached to a new approach for trip distribution forecasting which saves time with the following contributions:

1) In the proposed method, trip distribution at present is modeled by the approximation functions for each commodity (these functions will be able to generalized, adapt, and learn trip distribution based on the real trip distribution at present).

2) In large multi-commodity transportation networks, it is proposed that the approximation functions be obtained by parts of networks as training data.

3) A fuzzy model calibrates the approximation functions for each commodity simultaneously.

4) Parameters of the multi-commodity network are defined imprecisely. Then, possibility distributions are defined for them.

Therefore, the basic research task in this article is to develop the trip distribution forecasting from single-commodity to multi-commodity. This paper is organized into three sections. The second section contains the new formulation followed by a new algorithm and dimension consideration of the problems in the case of large scale. A numerical example is given. Conclusions are presented in the third section.

2. NEW FORMULATION

Let us first define parameters of multi-commodity networks:

Suppose that zones i and j are the trip origin and trip destination of commodity k, respectively:

T_{ij}^{k} = Number of interchanged trips of commodity k between i and j.

P_i^k = Number of trips produced of commodity k at zone i.

A_j^k = Number of trips attracted of commodity k at zone j.

t_{ij}^k = Travel time of commodity k between i and j.

L_{ij}^k = Optional adjustment factor for interchanges between zones i and j for commodity k.

i =An origin zone

j =A destination zone

Since these parameters of transportation networks are obtained by samples and statistical methods, they are imprecise. Therefore, possibility distributions must be defined for every parameter. Let us assume that the above parameters have possibility distributions [8]-[13] with trapezoid numbers (1):

$$\begin{cases} \tilde{P}_i^k = (P_{i1}^k, P_{i2}^k, P_{i3}^k, P_{i4}^k) \\ \tilde{t}_{ij}^k = (t_{ij1}^k, t_{ij2}^k, t_{ij3}^k, t_{ij4}^k) \\ \tilde{A}_j^k = (A_{j1}^k, A_{j2}^k, A_{j3}^k, A_{j4}^k) \\ \tilde{L}_{ij}^k = (L_{ij1}^k, L_{ij2}^k, L_{ij3}^k, L_{ij4}^k) \\ \tilde{T}_{ij}^k = \left(T_{ij1}^k, T_{ij2}^k, T_{ij3}^k, T_{ij4}^k\right) \end{cases} \tag{1}$$

Now, the proposed method is described as follows:

STEP1: Defuzzify the parameters of network

These numbers are converted to crisp numbers by the Fuller method [12], that is:

$$\begin{aligned} P_i^k &= \frac{1}{3}[P_{i1}^k + P_{i3}^k + \frac{1}{2}(P_{i2}^k + P_{i4}^k)] \\ t_{ij}^k &= \frac{1}{3}\left[t_{ij1}^k + t_{ij3}^k + \frac{1}{2}(t_{ij2}^k + t_{ij4}^k)\right] \\ A_j^k &= \frac{1}{3}[A_{j1}^k + A_{j3}^k + \frac{1}{2}(A_{j2}^k + A_{j4}^k)] \\ T_{ij}^k &= \frac{1}{3}\left[T_{ij1}^k + T_{ij3}^k + \frac{1}{2}(T_{ij2}^k + T_{ij4}^k)\right] \\ \tilde{L}_{ij}^k &= \frac{1}{3}[L_{ij1}^k + L_{ij3}^k + \frac{1}{2}(L_{ij2}^k + L_{ij4}^k)] \end{aligned} \tag{2}$$

And let: $\underline{X}_{ij}^k = [P_i^k, A_j^k, L_{ij}^k, t_{ij}^k]$

Then, for normalizing the elements of $\underline{X}_{ij}^k$, we have:

$$\begin{aligned} P_i^k(s) &= \frac{P_i^k}{\max_i P_i^k}, \quad A_j^k(s) = \frac{A_j^k}{\max_j A_j^k} \\ L_{ij}^k(s) &= \frac{L_{ij}^k}{\max_{i,j} L_{ij}^k}, \quad t_{ij}^k(s) = \frac{\min_{i,j} t_{ij}^k}{t_{ij}^k} \end{aligned} \tag{3}$$

Therefore, $\underline{X}_{ij}^k$ becomes:

$$\underline{X}_{ij}^k(s) = [P_i^k(s), A_j^k(s), L_{ij}^k(s), t_{ij}^k(s)] \tag{4}$$

STEP2: Calculate the approximation function

Since T_{ij}^k is a function of $P_i^k(s)$, $A_j^k(s)$, $L_{ij}^k(s)$, $t_{ij}^k(s)$, That is:

$$\hat{T}_{ij}^k = f_k(P_i^k(s), A_j^k(s), L_{ij}^k(s), t_{ij}^k(s)) \tag{5}$$

where: $\hat{T}_{ij}^k$ estimates T_{ij}^k.

For estimating of T_{ij}^k, Sum of squares error between T_{ij}^k and $\hat{T}_{ij}^k$ must be minimized:

$$SSE = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m}(T_{ij}^k - \hat{T}_{ij}^k)^2 =$$

$$\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m}[T_{ij}^k - f_k(P_i^k(s), A_j^k(s), L_{ij}^k(s), t_{ij}^k(s))]^2 \quad (6)$$

It is proven based on regularization theory that the approximation function (7) minimizes SSE [7],[12]:

$$\hat{T}_{ij}^k = \sum_{h=1}^{N} w_h^k G(\underline{X}_{ij}^k(s), \underline{X}_{[h]}^k(s)) \quad (7)$$

where:

$$G(\underline{X}_{ij}^k(s), \underline{X}_{[h]}^k(s)) = e^{-(\|\underline{x}_{ij}^k(s) - \underline{x}_{[h]}^k(s)\|)^2} \quad (8)$$

And $\underline{X}_{[h]}^k(s)$ is one of $\underline{X}_{ij}^k(s)$ that selected as the center of function G.

Clearly, we can rewrite the (7) in matrix notation:

$$\underline{T}^k = G^k . \underline{W}^k$$

where:

$$\underline{T}^k = [T_{[1]}^k, T_{[2]}^k, ..., T_{[N]}^k]' \quad (9)$$

$$G^k = \begin{bmatrix} G(\underline{X}_{11}^k, \underline{X}_{[1]}^k) & \cdots & G(\underline{X}_{1n}^k, \underline{X}_{[N]}^k) \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ G(\underline{X}_{m1}^k, \underline{X}_{[1]}^k) & \cdots & G(\underline{X}_{mn}^k, \underline{X}_{[N]}^k) \end{bmatrix} \quad (10)$$

$$\underline{W}^K = [W_1^k, W_2^k, ..., W_N^k] \quad (11)$$

M: Number of the Arcs.

N: Number of selected arcs for learning approximation function.

Now, for finding the weight vector $\underline{W}^k$, we may have these cases:

CASE1: $M > N$ (for large multi-commodity transportation networks, it is proposed that the above approximation functions be calculated by parts of multi-commodity network as far as the training data are available):

$$\underline{W}^k = (G^{k'} G^k)^{-1} G^{k'} \underline{T}^k \quad (12)$$

where: $(G^{k'} G^k)^{-1} G^{k'}$ represents the pseudoinverse G^k.

Then, go to step 3.

CASE2: $M = N$ (For small multi-commodity transportation networks):

$$\underline{W}^k = (G^k)^{-1} . \underline{T}^k \quad (13)$$

Then, go to step 4.

STEP3: Formulate the Fuzzy mathematical model

For calibrating the above approximation function, the following proposed model is formulated, which is one of the main contributions of this paper:

A. Objective Functions 1:

Based on conservation of flow [1], the sum of trips produced for k-th commodity in zone i should be equal to P_i^k:

$$\sum_{j=1}^{m} \hat{T}_{ij}^k \cong P_i^k \qquad i = 1, ..., n \quad (14)$$

But the sum of trips produced of commodity k in zone i do not necessarily equal to P_i^k. Therefore, $\hat{T}_{ij}^k$ must be calibrated by adjustment coefficients x_i^k, x_j^k:

$$\sum_{j=1}^{m} \hat{T}_{ij}^k . x_i^k . x_j^k \cong P_i^k \quad (15)$$

B. Objective Functions 2:

Similarly, the sum of trips attracted of commodity k to zone j should be equal to A_j^k:

$$\sum_{i=1}^{n} \hat{T}_{ij}^k . x_i^k . x_j^k \cong A_j^k \quad (16)$$

C. Objective Functions 3:

Also, the calculated trip-time frequency distribution for commodity k should be equal to the trip-time frequency distribution of real trip distribution [3]-[5], i.e.,

$$\sum_{t} \hat{T}_{ij}^k . x_i^k . x_j^k - \sum_{t} T_{ij}^k \cong 0 \quad (17)$$

D. Constraints:

Let u_{ij} represents the upper limit of the sum of all commodities in arc *(i ,j)* of the multi-commodity network. Then, we will have:

$$\sum_{k} \hat{T}_{ij}^k . x_i^k . x_j^k \le u_{ij} \quad (18)$$

The above objective functions and the constraints could make a fuzzy multi-objective decision making model. It should be mentioned that in references [13],[14],[16],[8] a fuzzy objective function has been converted to the constraint a linear programming. Now, we have utilized this idea to propose the following new model:

Find x_i^k, x_j^k

St:

$$f_i^k = P_i^k - \sum_{j=1}^{m} \hat{T}_{ij}^k . x_i^k . x_j^k \cong 0$$

$$f_j^k = A_j^k - \sum_{i=1}^{n} \hat{T}_{ij}^k . x_i^k . x_j^k \cong 0 \qquad (19)$$

$$f_t^k = \sum_t \hat{T}_{ij}^k . x_i^k . x_j^k - \sum_t T_{ij}^k \cong 0$$

$$\sum_k \hat{T}_{ij}^k . x_i^k . x_j^k \le u_{ij}$$

$$x_i^k, x_j^k \ge 0$$

Obviously, the fuzzy model (19) calibrates all of the approximation functions simultaneously.

Now, according to Zimmermann's approach [8], the optimal solution of the above fuzzy model can be obtained by solving the following model:

$$Minimize[Min[\mu_i(\hat{T}_{ij}^k), \mu_j(\hat{T}_{ij}^k), \mu_t(\hat{T}_{ij}^k)]] \qquad (20)$$

where the membership functions $\mu_i(\hat{T}_{ij}^k), \mu_j(\hat{T}_{ij}^k), \mu_t(\hat{T}_{ij}^k)$ are defined as follows:

$$\mu_j(\hat{T}_{ij}^k) = \begin{cases} 1 + \dfrac{f_j^k}{r_j} & -r_j \le f_j^k \le 0 \\ 1 - \dfrac{f_j^k}{r_j} & 0 \le f_j^k \le r_j \\ 0 & otherwise \end{cases} \qquad (21)$$

$$\mu_t(\hat{T}_{ij}^k) = \begin{cases} 1 + \dfrac{f_t^k}{v_t} & -v_t \le f_t^k \le 0 \\ 1 - \dfrac{f_t^k}{v_t} & 0 \le f_t^k \le v_t \\ 0 & otherwise \end{cases}$$

$$\mu_i(\hat{T}_{ij}^k) = \begin{cases} 1 + \dfrac{f_i^k}{q_i} & -q_i \le f_i^k \le 0 \\ 1 - \dfrac{f_i^k}{q_i} & 0 \le f_i^k \le q_i \\ 0 & otherwise \end{cases}$$

where q_i, r_j and v_t are the tolerances of the objective functions f_i^k, f_j^k and f_t^k. It should be mentioned that the amount of them are given initially.

By assuming:

$$Min[\mu_j(\hat{T}_{ij}^k), \mu_i(\hat{T}_{ij}^k), \mu_t(\hat{T}_{ij}^k)] = \lambda \qquad (22)$$

This satisfies:

$$\lambda \le \mu_j(\hat{T}_{ij}^k), \ \lambda \le \mu_i(\hat{T}_{ij}^k), \ \lambda \le \mu_t(\hat{T}_{ij}^k) \qquad (23)$$

From (21), we obtain:

$$\lambda \le 1 + \frac{f_i^k}{q_i}, \lambda \le 1 - \frac{f_i^k}{q_i},$$

$$\lambda \le 1 + \frac{f_j^k}{r_j}, \lambda \le 1 - \frac{f_j^k}{r_j}, \qquad (24)$$

$$\lambda \le 1 + \frac{f_t^k}{v_t}, \lambda \le 1 - \frac{f_t^k}{v_t}$$

$$(\lambda - 1)q_i \le f_i^k \le (1-\lambda)q_i,$$

$$(\lambda - 1)r_j \le f_j^k \le (1-\lambda)r_j, \qquad (25)$$

$$(\lambda - 1)v_t \le f_t^k \le (1-\lambda)v_t$$

In conclusion, by referring to (19), (20) and (25) the proposed new model (P) will be obtained:

Min λ (Model P)

St:

$$(\lambda - 1)r_j \le A_j^k - \sum_{i=1}^{n} \hat{T}_{ij}^k(I) \le (1-\lambda)r_j$$

$$(\lambda-1)q_i \le P_i^k - \sum_{j=1}^{m} \hat{T}_{ij}^k(I) \le (1-\lambda)q_i$$

$$(\lambda-1)v_t \le \sum_t \hat{T}_{ij}^k(I) - \sum_t T_{ij}^k \le (1-\lambda)v_t$$

$$\sum_k \hat{T}_{ij}^k(I) \le u_{ij}$$

$$\hat{T}_{ij}^k(I) = \hat{T}_{ij}^k . x_i^k . x_j^k$$

$$x_i^k, x_j^k, \lambda, \hat{T}_{il}^k(I) \ge 0$$

Therefore, the model (P) calibrates the approximation functions (7). Fortunately, since most of the constraints in model (P) are of quadratic form, this model can be handled by Lingo software. It is worth mentioning that the quadratic model is convex [9].

STEP4: Forecast trip distribution

Suppose that parameters of the multi-commodity network in the future are $P_i^{k'}$, $A_j^{k'}$, $L_{ij}^{k'}$, $t_{ij}^{k'}$. Trip distribution in the future ($T_{ij}^{k'}$) is calculated by (26):

$$\begin{aligned}
&\underline{X}_{ij}^{k'} = [P_i^{k'}, A_j^{k'}, L_{ij}^{k'}, t_{ij}^{k'}] \rightarrow \\
&\underline{X}_{ij}^{k'}(s) = [P_i^{k'}(s), A_j^{k'}(s), L_{ij}^{k'}(s), \\
&t_{ij}^{k'}(s)] \longrightarrow \hat{T}_{ij}^{k'} = \\
&f_k(P_i^{k'}(s), A_j^{k'}(s), L_{ij}^{k'}(s), t_{ij}^{k'}(s)) = \\
&\sum_h W_h^k G(\underline{x}_{ij}^{k'}(s), \underline{X}_{[h]}^{k'}(s)).x_i^{k'}.x_j^{k'}
\end{aligned} \tag{26}$$

Then, it will be calibrated by proposed model (F):

Min λ (Model F)

St:

$$(\lambda-1)r_j' \le A_j^{k'} - \sum_{i=1}^{n} \hat{T}_{ij}^{k'}(I) \le (1-\lambda)r_j'$$

$$(\lambda-1)q_i' \le P_i^{k'} - \sum_{j=1}^{m} \hat{T}_{ij}^{k'}(I) \le (1-\lambda)q_i'$$

$$\sum_k \hat{T}_{ij}^{k'}(I) \le u_{ij}'$$

$$\hat{T}_{ij}^{k'}(I) = \hat{T}_{ij}^{k'} . x_i^{k'} . x_j^{k'}$$

$$x_i^{k'}, x_j^{k'}, \lambda, \hat{T}_{ij}^{k'}(I) \ge 0$$

A. Dimension Consideration

If we consider the multi-commodity transportation network with n nodes, r arcs and m commodities:

1. Maximum size of the model P will be $(n+r).m$ constraints and $n.m$ variables which could be handled in the case of large scale.
2. Size of the model F will be $n.m$ constraints and variables.

It is worth mentioning that the main contribution of this research is to develop two new optimization problems (P) and (F). The optimal solution of model (F) can be obtained by Lingo software.

Now, the proposed model is illustrated by a numerical example.

B. Numerical Example

Consider the following two-commodity network. Trips produced, trips attracted, the travel time and trip distribution at present between zones for each commodity are given in Table 1.

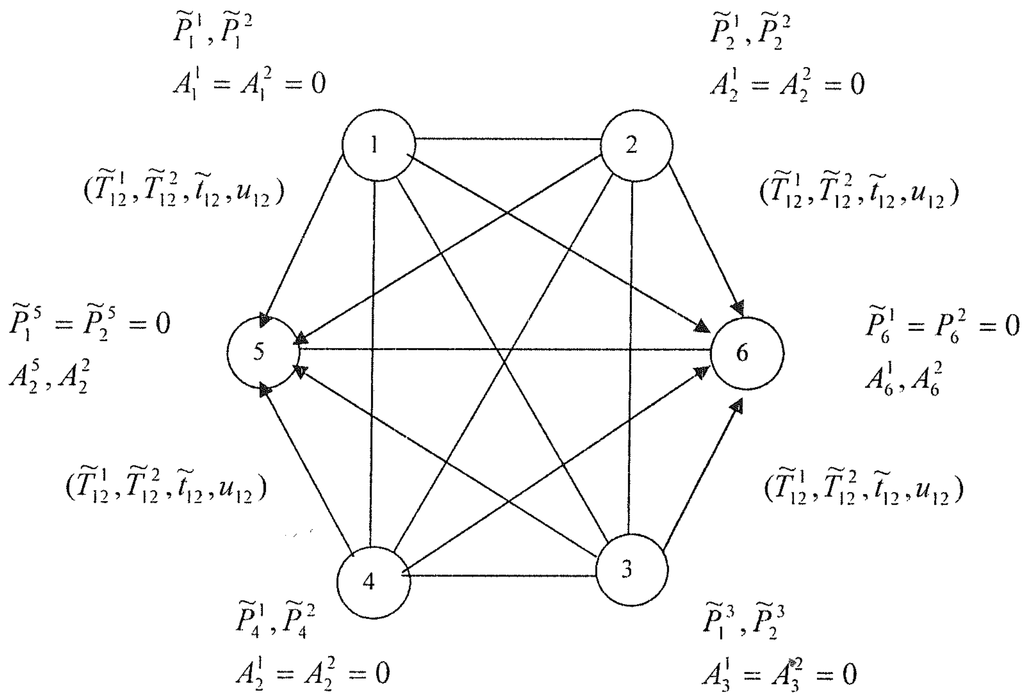

Figure 1: Two-commodity transportation network

TABLE 1
PARAMETER VALUES AT PRESENT (GIVEN DATA)

$\widetilde{T}_{36}^2 = (50,60,65,70)$	$\widetilde{T}_{15}^1 = (230,240,245,250)$	$\widetilde{P}_1^1 = (390,400,405,410)$
$\widetilde{T}_{45}^1 = (50,60,65,70)$	$\widetilde{T}_{15}^2 = (410,420,425,430)$	$\widetilde{P}_1^2 = (690,700,705,710)$
$\widetilde{T}_{45}^2 = (110,120,125,130)$	$\widetilde{T}_{16}^1 = (150,160,165,170)$	$\widetilde{P}_2^1 = (490,500,505,510)$
$\widetilde{T}_{46}^1 = (30,40,45,50)$	$\widetilde{T}_{16}^2 = (270,280,285,290)$	$\widetilde{P}_2^2 = (390,400,405,410)$
$\widetilde{T}_{46}^2 = (70,80,85,90)$	$\widetilde{T}_{25}^1 = (290,300,305,310)$	$\widetilde{P}_3^1 = (490,500,505,510)$
$\widetilde{t}_{13} = \widetilde{t}_{24} = (5,6,6.5,7)$	$\widetilde{T}_{25}^2 = (150,160,165,170)$	$\widetilde{P}_3^2 = (90,100,105,110)$
$\widetilde{t}_{56} = (4.5,5.5,6,6.5)$	$\widetilde{T}_{23}^2 = (190,200,205,210)$	$\widetilde{P}_4^1 = (90,100,105,110)$
$\widetilde{t}_{12} = \widetilde{t}_{15} = \widetilde{t}_{34} = \widetilde{t}_{26} = \widetilde{t}_{45} = \widetilde{t}_{36} = (2,3,35,4)$	$\widetilde{T}_{26}^1 = (290,300,305,310)$	$\widetilde{P}_4^2 = (190,200,205,210)$
	$\widetilde{T}_{26}^2 = (230,240,245,250)$	$\widetilde{A}_5^1 = (690,700,705,710)$
$\widetilde{t}_{14} = \widetilde{t}_{23} = \widetilde{t}_{16} = \widetilde{t}_{25} = \widetilde{t}_{35} = \widetilde{t}_{46} = (4,5,55,6)$	$\widetilde{T}_{35}^1 = (190,200,20,210)$	$\widetilde{A}_5^2 = (490,500,505,510)$
	$\widetilde{T}_{35}^2 = (30,40,45,50)$	$\widetilde{A}_6^1 = (790,800,805,810)$
	$\widetilde{T}_{36}^1 = (290,300,305,310)$	$\widetilde{A}_6^2 = (890,900,905,910)$

If parameter values change in the future as follows (see table 2). Find trip distribution between the zones for two-commodity network in the future (assuming $L_{ij}^1 = L_{ij}^2 = 1$).

TABLE 2
PARAMETER VALUES IN THE FUTURE (GIVEN DATA)

$\tilde{t}_{13}' = \tilde{t}_{24}' = (4.5,5.5,6,6.5)$	$\tilde{P}_4^{1'} = (140,150,155,160)$	$\tilde{P}_1^{1'} = (590,600,605,610)$
$\tilde{t}_{56}' = (4,5,5.5,6)$	$\tilde{P}_4^{2'} = (240,250,255,260)$	$\tilde{P}_1^{2'} = (890,900,905,910)$
$\tilde{t}_{12}' = \tilde{t}_{15}' = \tilde{t}_{34}' = \tilde{t}_{26}' = \tilde{t}_{45}' =$ $\tilde{t}_{36}' = (2.4,2.5,2.55,2.6)$	$\tilde{A}_5^{1'} = (890,900,905,910)$	$\tilde{P}_2^{1'} = (590,600,605,610)$
	$\tilde{A}_5^{2'} = (690,700,705,710)$	$\tilde{P}_2^{2'} = (490,500,505,510)$
$\tilde{t}_{14}' = \tilde{t}_{23}' = \tilde{t}_{16}' = \tilde{t}_{25}' = \tilde{t}_{35}' =$ $\tilde{t}_{46}' = (3,4,4.5,5)$	$\tilde{A}_6^{1'} = (940,950,955,960)$	$\tilde{P}_3^{1'} = (490,500,505,510)$
	$\tilde{A}_6^{2'} = (1090,1100,1105,1110)$	$\tilde{P}_3^{2'} = (140,150,155,160)$

Solution:
In this example, we consider $M=N$, then, the construction and solution of (model F) will be required. Here is the model F.

Min λ (Model F)

$$-5(1-\lambda) \le 600 - (\hat{T}_{15}^{1'}(I) + \hat{T}_{16}^{1'}(I)) \le 5(1-\lambda)$$

$$-5(1-\lambda) \le 900 - (\hat{T}_{15}^{2'}(I) + \hat{T}_{16}^{2'}(I)) \le 5(1-\lambda)$$

$$-5(1-\lambda) \le 600 - (\hat{T}_{25}^{1'}(I) + \hat{T}_{26}^{1'}(I)) \le 5(1-\lambda)$$

$$-5(1-\lambda) \le 500 - (\hat{T}_{25}^{2'}(I) + \hat{T}_{26}^{2'}(I)) \le 5(1-\lambda)$$

$$-5(1-\lambda) \le 500 - (\hat{T}_{35}^{1'}(I) + \hat{T}_{36}^{1'}(I)) \le 5(1-\lambda)$$

$$-5(1-\lambda) \le 150 - (\hat{T}_{35}^{2'}(I) + \hat{T}_{36}^{2'}(I)) \le 5(1-\lambda)$$

$$-5(1-\lambda) \le 150 - (\hat{T}_{45}^{1'}(I) + \hat{T}_{46}^{1'}(I)) \le 5(1-\lambda)$$

$$-5(1-\lambda) \le 250 - (\hat{T}_{45}^{2'}(I) + \hat{T}_{46}^{2'}(I)) \le 5(1-\lambda)$$

$$-5(1-\lambda) \le 900 - (\hat{T}_{15}^{1'}(I) + \hat{T}_{45}^{1'}(I) + \hat{T}_{25}^{1'}(I) + \hat{T}_{35}^{1'}(I)) \le 5(1-\lambda)$$

$$-5(1-\lambda) \le 700 - (\hat{T}_{15}^{2'}(I) + \hat{T}_{45}^{2'}(I) + \hat{T}_{25}^{2'}(I) + \hat{T}_{35}^{2'}(I)) \le 5(1-\lambda)$$

$$-5(1-\lambda) \le 950 - (\hat{T}_{26}^{1'}(I) + \hat{T}_{36}^{1'}(I) + \hat{T}_{16}^{1'}(I) + \hat{T}_{46}^{1'}(I)) \le 5(1-\lambda)$$

St:

$$-5(1-\lambda) \le 1100 - (\hat{T}_{26}^{2'}(I) + \hat{T}_{36}^{2'}(I) + \hat{T}_{16}^{2'}(I) + \hat{T}_{46}^{2}(I)) \le 5(1-\lambda)$$

$$\hat{T}_{15}^{1'}(I) = 318.709 x_1^{1'} . x_5^{1'}$$
$$\hat{T}_{16}^{1'}(I) = 263.204 x_1^{1'} . x_6^{1'}$$
$$\hat{T}_{25}^{1'}(I) = 281.775 x_2^{1'} . x_5^{1'}$$
$$\hat{T}_{26}^{1'}(I) = 300.404 x_2^{1'} . x_6^{1'}$$
$$\hat{T}_{35}^{1'}(I) = 235.419 x_3^{1'} . x_5^{1'}$$
$$\hat{T}_{36}^{1'}(I) = 256.766 x_3^{1'} . x_6^{1'}$$
$$\hat{T}_{45}^{1'}(I) = 69.095 x_4^{1'} . x_5^{1'}$$
$$\hat{T}_{46}^{1'}(I) = 53.881 x_4^{1'} . x_6^{1'}$$
$$\hat{T}_{15}^{2'}(I) = 424.804 x_1^{2'} . x_5^{2'}$$
$$\hat{T}_{16}^{2'}(I) = 289.287 x_1^{2'} . x_6^{2'}$$
$$\hat{T}_{25}^{2'}(I) = 167.019 x_2^{2'} . x_5^{2'}$$
$$\hat{T}_{26}^{2'}(I) = 232.302 x_2^{2'} . x_6^{2'}$$
$$\hat{T}_{35}^{2'}(I) = 48.816 x_3^{2'} . x_5^{2'}$$
$$\hat{T}_{36}^{2'}(I) = 68.080 x_3^{2'} . x_6^{2'}$$
$$\hat{T}_{45}^{2'}(I) = 118.736 x_4^{2'} . x_5^{2'}$$
$$\hat{T}_{46}^{2'}(I) = 80.398 x_4^{2'} . x_6^{2'}$$
$$x_1^{1'}, x_1^{2'}, x_2^{1'}, x_2^{2'}, x_3^{1'}, x_3^{2'}, x_4^{1'}, x_4^{2'}, x_5^{1'}, x_5^{2'}, x_6^{1'}, x_6^{2'}, \lambda \geq 0$$
$$\hat{T}_{15}^{1'}(I), \hat{T}_{16}^{1'}(I), \hat{T}_{25}^{1'}(I), \hat{T}_{26}^{1'}(I), \hat{T}_{35}^{1'}(I), \hat{T}_{36}^{1'}(I), \hat{T}_{45}^{1'}(I), \hat{T}_{46}^{1'}(I),$$
$$\hat{T}_{15}^{2'}(I), \hat{T}_{16}^{2'}(I), \hat{T}_{25}^{2'}(I), \hat{T}_{26}^{2'}(I), \hat{T}_{35}^{2'}(I), \hat{T}_{36}^{2'}(I), \hat{T}_{45}^{2'}(I), \hat{T}_{46}^{2'}(I) \geq 0$$

The optimal solution of model (F) is given in Table 3(solved by Lingo software):

TABLE 3
THE OPTIMAL SOLUTION

New Model				Survey model		Trip distribution
Forecasting		present		present		
Commodity 2	Commodity 1	Commodity 2	Commodity 1	Commodity 2	Commodity 1	
404.3265	310.5853	420	240	420	240	T15
490.6735	284.4147	280	160	280	160	T16
145.1731	277.2482	160	200	160	200	T25
359.8269	327.7518	240	300	240	300	T26
44.47258	224.0430	40	200	40	200	T35
110.5274	270.9570	60	300	60	300	T36
111.0278	83.12359	120	60	120	60	T45
133.9722	71.87641	80	40	80	40	T46

By referring to Table 3, we could see the forecasted trip distribution between the zones for some commodity gets higher (i.e. T16 for commodity 2 which is 280 at present gets 490.67 in the future). If we consider the Table 3 in more details, we could find more similar cases. This trip distribution forecasting identifies that the volume of traffic will exceed the capacity of paths. Thus, the capacity planning and traffic engineering are required.

3. CONCLUSION

In this research, the trip distribution forecasting has been formulated in multi-commodity transportation networks by a fuzzy multi-objective optimization model. Then, this multi-objective model has been converted to single objective model by Zimmermann's approach. The special structures of these two models are very close to quadratic programming, where its convexity is confirmed. Therefore, there was no difficulty applying Lingo software to solve these models. In both models, the amounts of tolerances are given by priori-information.

4. ACKNOWLEDGEMENT

The authors gratefully acknowledge the anonymous referees for their useful comments on the original version of this paper.

5. REFERENCES

[1] S. Bazaraa,D. Sherali, and J. Jarvis, Linear *Programming and Network Flows. 3rd Edition.* John Wiley & Sons, 2004.

[2] M. Kalic, D. Teodorović, "Trip Distribution Using Fuzzy Logic and Genetic Algorithm," Transportation Planning and Technology, Vol. 26, pp. 213-238, 2003.

[3] C. Khisty, B. Lall., *Transportation Engineering*: An Introduction, 3rd Edition, Prentice-Hall International, August 2002.

[4] M. Mozolin., J.-C. Thill, and E.L. Usery, "Trip Distribution Forecasting with Multi-layer Perceptron Neural Networks: A Critical Evaluation," *Transportation Research B*, Vol. 34, pp. 53-73, 2000.

[5] Federal Highway Administration, *Guidebook on Statewide Travel Forecasting. Center for Urban Transportation Studies.* University of Wisconsin,1999.[Online].Available:http://www.fhwa.dot.gov/hep10/state /swtravel.pdf

[6] M. Kalic,and D. Teodorović, "Trip Distribution Modeling Using Soft Computing Techniques," *EURO XV / INFORMS XXXIV*, pp. 74, Barcelona, Spain, 1997.

[7] S. Haykin, *Neural Networks: a Comprehensive Foundation*, 2nd Edition, New Jersey, Prentice Hall, 1998.

[8] Zimmerman, *Fuzzy Set Theory and Its Application*, Kluwer Acedemic Publisher, 1993.

[9] S. Bazaraa, H.D. Sherali, and C.M. Shetty, *Nonlinear Programming: Theory and Algorithms*, 2nd Edition, John Wiley Sons, 1993

[10] D. P. Walting and M.J. Maher ,"A Statistical Procedure for Estimating a Mean Origin-Destination Matrix from a Partial Registration Plate Survey," *Transportation Research, 26B*, pp. 171-193, 1992

[11] D. P. Walting and D. R. Grey , "Analysis of Partial Registration Plate Data Using a Model with Poisson Input and Output," in Proc. 1991 Mathematics in Transport Planning and Control Conf., Oxford University Press

[12] T. Poggio and F. Girosi, "Networks for Approximation and Learning," in Proc. 1990 IEEE, Vol.78, pp.1481-1497.

[13] C.L., Hwang, Fuzzy Mathematical Programming Methods and Application, Spinger-Verlag, 1989.

[14] H Masud, Multi Objective Decision Making, Methods and Application, Springer-Verlag, 1989.

[15] H. Spiess," A Maximum Likelihood Model for Estimating Origin-Destination Matrices," Transpn.Res, B21, pp. 395-412, 1987.

[16] K. G. Murty, Linear programming, John Wiley Sons. 1983.

[17] S. McNeiland, C. Hendrickson, "A regression Formulation on Matrix Estimation ," Transportation Science, 19(3), pp. 278-292, 1985.

[18] M. J. Maher, "Inferences on Trip Matrices From Observations on Link Volumes: A Bayesian Statistical Approach," Transportation Research, vol. 17B(6), pp. 435-447, 1983.

[19] I. Geva, E. Hauerlandau, "Maximum-Likelihood and Bayesian methods for the estimation of origin-destination flows," Transportation Research Record, n. 944, pp. 101-105, 1983.

[20] H. J. Van Zuylen, "The Information Minimization Method: Validity and Applicability to Transport Planning," 1978.

[21] L. G. Willumsen, "Estimation of an O-D Matrix from Traffic Counts – A Review, " Institute of Transportation Studies, University of Leeds, England , 1978.

[22] M. Gaudryand, L. LaMarre, "Estimating Origin-Destination Matrices from Traffic Counts: A Simple Linear Intercity Model for Quebec." Publication No. 105, Ctr. for Transp. Res., Univ. of Montreal, Montreal, Quebec, 1978.

[23] P. Hogberg, "Estimation of Parameters in Models for Traffic Prediction: A Non-Linear Regression Approach," Transportation Research, Vol. 10, pp. 263-265, 1976

[24] Holm, T. Jensen, S. K. Nielsen, A. Christensen, B. Johnsen and G. Ronby, "Calibrating Traffic Models on Traffic Census Results Only," Traffic Engineering and Control, pp. 137-140, 1976.

[25] D. A. Low, "A new approach to transportation system modeling." Traffic Quarterly, Vol. 26, No. 3, 1972.

[26] P. Robillard, "Estimating the O-D Matrix From Observed Link Volumes", Transportation Research, vol. 9, pp. 123-128, 1975.