Objective Analysis of the Strain Localization Based on Tangent Modulus

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ABSTRACT

In this paper, the philosophical concept of objectivity in scientific studies and analyses is described. Its meanings in damage analysis and especially "strain localization analyses" is presented. The most important solution for making an analysis objective i.e. using the nonlocal values of relevant variables, is discussed. Then the current idea on objective analysis based on secant modulus is represented and the authors' suggestion for objective analysis based on tangent modulus is proposed and finally, a sample problem is solved.

KEYWORDS

Objectivity, Nonlocal Analysis, Damage Mechanics, Secant Modulus, Tangent Modulus.

1. Introduction

It is true if we say, " the main aim of the philosophy is finding an objective analysis method". Marx said, "The philosophers have only interpreted the world in different ways, the point is to change it" [8]. In engineering, "changing the world" is so essential. In the other word "the main function of engineering is changing the world". For changing the world, we must know it correctly. In practice, we need a theory to simulate the world. Philosophers – at least from the age of the enlightenment –, scientists and engineers- from the age of industrial revolution- have been concentrated on the objectivity concept.

What is objectivity? And how is an objective analysis method? In practice, an objective analysis method is such a method that describes an event correctly. Results of analysis with an objective method must be the same as our observations. So, an objective method must not be affected by human assumptions. An objective method is only OBJECT dependent.

In the material engineering, objectivity has an essential role particularly in the INELASTIC analyses. In such analyses, by reaching the peak strength, material properties will change and then there will be at least two different materials, large strains are localized into a narrow zone. This condition is called "Strain Localization". In such a condition, difference between an objective and a non-objective analysis is a serious problem [7].

For making an analysis objective, several ways have been proposed. In this paper a new solution is proposed.

2. Objectivity in the Strain Localization Problem

For introducing the non-objectivity in the strain localization problem, a one-dimensional problem is considered. A straight bar with constant cross section area (A) and initial length (L), under tensile loading (lengthening) is shown in Figure 1.

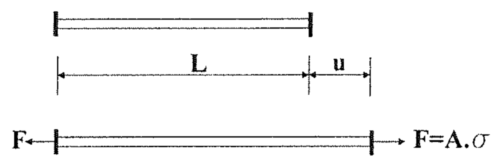

Figure 1: Straight bar under tensile loading.

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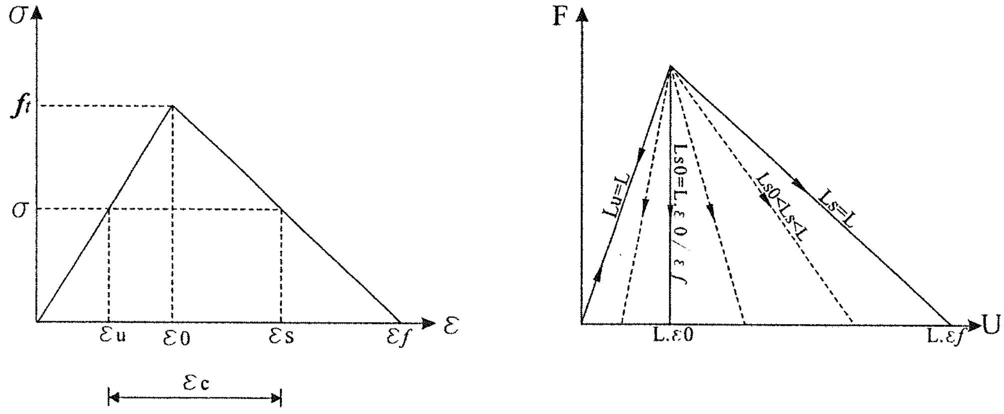
Material behavior is assumed to be bi-linear as shown in Figure 2(a). Before reaching the peak stress (strength), load-deformation (F-U) response is only affected by modulus of elasticity (E). After that, a section of the bar will be under loading and the rest of it will be under unloading. In fact, a fundamental question is that how much is the length of loaded section of the bar. It is assumed that the length of the loaded part is **Ls**. This assumption is so determinative in the softening branch of F-U diagram.

basis of nonlocal fields has been proposed. Eringen proposed this idea for the first time, in a microphysical context [3],[4]. Then, Bazant used this idea in the fracture mechanics context [1]. Hitherto, many of researchers have used this idea in different ways. It seems that using the nonlocal field instead of the local one, is an effective and reliable solution for the mesh sensitivity eliminating.



(a) (b)

Figure 2: (a) Material behavior diagram (b) Possible post peak responses of bar.

If the softening part of the bar (Ls) is infinitesimal, i.e. Ls→0, the whole bar is unloaded and the final deformation will be zero.

If the softening part is as large as possible, i.e. Ls=L, the whole bar is loaded and the final deformation will be $L.\varepsilon_f$.

Obviously, there are infinite cases between two above limits (as shown in Figure 2(b)). So, the most important assumption in the strain localization problem is Ls. This means that the problem has a serious sensitivity. This sensitivity is related to the dividing pattern of the bar. Ls is equal to the length of a segment of the fragmented bar. If we divide the bar length into n segments, Ls will be L/n. So, the problem will be mesh dependent. Such sensitivity is a kind of size effect that is an important part of the fracture mechanics studies.

In practice, there is only one response after the peak point of F-U diagram. This means that the softening part of the bar has a unique length. This length is a material property and is called Characteristic Length.

An objective analysis method must depend only on the characteristic length. Such a method must not affected by the mesh sensitivity.

3. Nonlocal Averaging

For eliminating the *mesh sensitivity* a solution on the

What is a nonlocal field? Assume that "y" is a function of "x" i.e. *y=f (x)*. This means that there is only a "y" against a specific "x". Such a field is a local one. Now we are going to define a field based on this local field: $\{y \mid \forall x \in D1, \ni y \in D2; y = f(x)\}$. If "y" at a point does not depend only on a specific "x", so, the field is nonlocal. We can define a nonlocal field by a specific averaging. This averaging is defined as follows:

$$\bar{f}(x) = \int_{v} \alpha(x,\xi).f(\xi)d\xi \qquad (1)$$

Integrating is performed over "v" i.e. the volume that is specified with the characteristic length. It can be a line with d_c (characteristic length) length in one-dimensional problem, a circle with d_c diameter in two-dimensional problem or a sphere with d_c diameter in three-dimensional problem. In all cases the center of integration domain is point "x".

α is a weight factor that is a function of the distance between the source point" ξ " and the receiver point "x".

Further details, see the recent paper by Jirasek and Bazant [2].

4. Objective Analysis by Damage Mechanics: Secant Modulus Method

One of the effective and practical smeared crack approaches is "crack band model". Mazars proposed a method based on crack band model by introducing a damage parameter" ω " [9],[10].

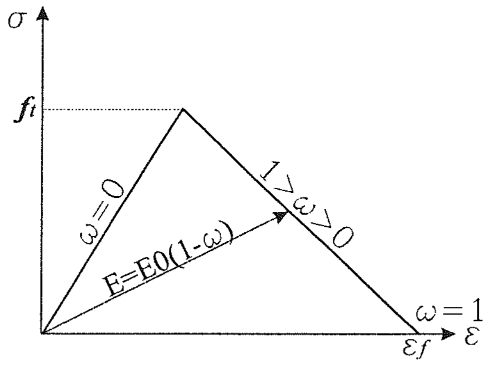


Figure 3: Mazars approach.

In the Mazars approach, modulus of elasticity decreases when the strain goes over the elastic limit as shown in Figure 3. This method of modeling is based on the secant modulus concept and can be written in the abstract form:

$$D = (1-\omega)D^e \tag{2}$$

In equation (2), "D^e" is the behavior matrix of the virgin material, and "D" is the behavior matrix of damaged or cracked material.

For making the method objective, we can use the nonlocal field of the damage parameter "ω": $\overline{\omega}$, or nonlocal field of the stiffness matrix: $\overline{K}$.

5. A New Approach: Tangent Modulus Method

A method based on tangent modulus has several advantages. For this reason, many of the researchers are after finding a practical Tangent Modulus Based Method (T.M.B.M). A newer one has been proposed by Jirasek and Patzak [6].

Jirasek and Patzak defined the nonlocal T.M.B.M stiffness matrix as follows:

$$K = \sum_{p} \omega_p (1-\omega_p) K_p^e - \sum_{p,q} \omega_p B_p^T f_p' k_p \omega_q \alpha_{pq} \sigma_p^e \eta_q^T B_q \tag{3}$$

Obviously, such a definition is so complex and is not practical. Of course, the tangent modulus concept has many complexities. But, we think by simplifying the concept, it is possible to form an objective T.M.B.M. Now we define our approach step by step:

1- Similar to (2) we define the tangent modulus for the cracked material:

$$D^t = \beta . D^e \tag{4}$$

In the equation (4), "D^e" is the tangent behavior matrix of the virgin (uncracked) material, "D^t" is the tangent matrix of cracked material, and "β" is a parameter that relates D^t to D^e. We called this parameter " Tangent Stiffness Factor (T.S.F.)".

2- β can be a function of the strain or damage energy release rate (Y) level:

$$\beta = f(\varepsilon) \tag{5}$$
$$\beta = f(Y) \tag{6}$$

3- β relates to the damage parameter ω:

$$\beta = \frac{d\sigma}{E.d\varepsilon} = 1 - \omega - \varepsilon \frac{d\omega}{d\varepsilon} \tag{7}$$

4-This is a successive method. In such methods an error correction procedure is necessary, while in direct methods such as the secant method, it is not so important.

Using only the first term of the Taylor expansion, the following modified formula is obtained:

$$K^t_{\text{modified}} = \sum_{p} \beta_p . K_p^e + \sum_{p} (\beta_Y')_p . B_p^T . D^e . B_p . U_p . \delta_p^T . K_p^e \tag{8}$$

The parameters in the above formula are shown in Figure 4. For further details see the Ph.D. thesis [11].

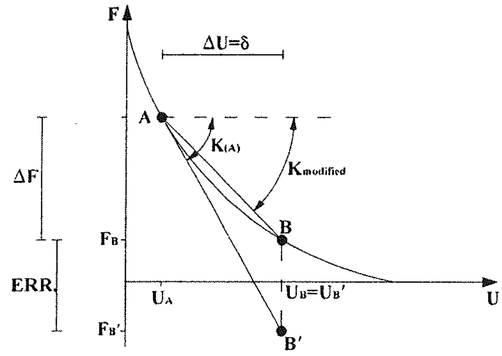


Figure 4: Error occurred in the tangent stiffness method.

Obviously, by making the loading steps (ΔU) small enough, the effect of error correction decreases. But, using the small loading steps increases the time consuming of analytical program. So, for optimizing the analysis, a mechanism for error correction is required.

5- Making the analysis objective: for this purpose, we used a nonlocal averaging, but with an especial weight factor. This weight factor has two parts: the first part is a usual weight function such as uniform or Gaussian function, and the second is n-th power of energy release rate (Y) [11].

So, the nonlocal value of the T.S.F. is defined as:

$$\tilde{\beta}_n = \frac{\iiint_D \alpha(r).[Y(x,y,z)]^n.\beta(x,y,z).dv}{\iiint_D \alpha(r).[Y(x,y,z)]^n.dv} \tag{9}$$

α is a usual weight function such as uniform or Gaussian function, Y is the energy release rate, β is the T.S.F. and finally, "$\tilde{\beta}_n$" is the n-th nonlocal value of T.S.F. over the domain D (defined with characteristic length). Several analyses showed us that there is an optimum value for "n". But for usual analyses, it seems that "n=2" is acceptable.

6. Solving a Sample Problem

Assume a straight bar divided into 3 segments, and is under one-dimensional tensile loading. This problem has 3 degrees of freedom as shown in Figure 5.

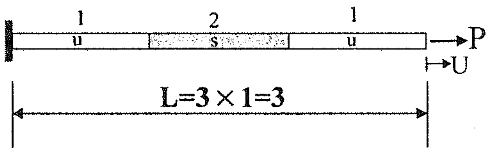


Figure 5: one-dimensional problem-Geometry and loading diagram.

The first and the third parts of the bar, that indicated with "u" are under loading and unloading condition. The middle part of the bar that indicated with "s" goes over the strain ε_0 (elastic limit as shown in Figure 2(a)), and so, is under loading. In this problem we assume that total length of the bar is: L=3*1=3, cross area of the bar is: A=1, peak stress is: f_t=2, peak strain is: $\varepsilon_0 = 1$ and fracture strain is: $\varepsilon_f = 10$ as shown in Figure 6. More over, we assume that the characteristic length is L_c=1.5.

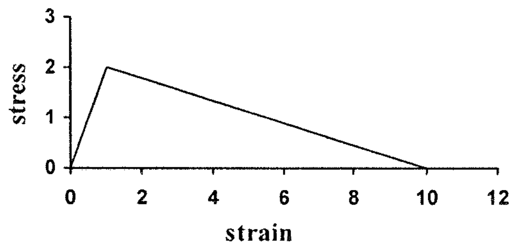


Figure 6: one-dimensional problem-Bi-Linear behavior of Material.

Wile the strain is less than 1 (stress is less than 2), segments 1 & 2 (u & s) have the same behavior. At the point ($\varepsilon = 1, \sigma = f_t = 2$) we have the peak point behavior of the bar that is defined with: P=2 & U=3.

A. Secant modulus scenario

After the peak point we have :

$$\omega = \frac{10}{9}.\frac{\varepsilon - 1}{\varepsilon} = \frac{10}{9}.\frac{\sqrt{Y} - 1}{\sqrt{Y}} \tag{10}$$

Reader must note that the damage energy release rate is defined in matrix form as: $Y = \frac{1}{2}\varepsilon^T.D^e.\varepsilon$, so for the one-dimensional problem, it is defined as: $Y = \frac{1}{2}.E.\varepsilon^2$. Using the nonlocal ω strategy, if ω in the first and the third segment is: "ω_1" and ω in the middle segment is: "ω_2", with uniform integration, nonlocal values will be obtained as:

$$\overline{\omega}_1 = \frac{5\omega_1 + \omega_2}{6} \quad , \overline{\omega}_2 = \frac{\omega_1 + 2\omega_2}{3} \tag{11}$$

The middle segment (now let we call it "element") is under loading and the others (1st and 3rd elements) are under unloading.

By exact (point to point or "local") solution we obtain:

U=3.0, P=2.0, $\varepsilon_1 = 1.0$, $\varepsilon_2 = 1.0$

U=6.0, P=1.14, $\varepsilon_1 = 0.57$, $\varepsilon_2 = 4.86$

U=9.0, P=0.29, $\varepsilon_1 = 0.143$, $\varepsilon_2 = 8.71$

U=10.0, P=0.0, $\varepsilon_1 = 0.0$, $\varepsilon_2 = 10.0$

In above data, "U" means the deformation of the bar end, and "P" is the point load acting on the bar end as shown in Figure 5. "ε_1" is the strain of the unloaded section of the bar, and "ε_2" is the strain of the loaded section of the bar.

In the first step after the peak we have:

U=6.0, $\varepsilon_1 = 0.57$, $\varepsilon_2 = 4.86$

=> $\overline{\varepsilon}_1 = 1.285, \overline{\varepsilon}_2 = 1.43$

=> $\overline{\omega}_1 = 0.246, \overline{\omega}_2 = 0.787$

=> $\overline{E}_1 = (1 - \overline{\omega}_1)E_0 = 1.508$, $\overline{E}_2 = (1 - \overline{\omega}_2)E_0 = 0.426$

=> $\varepsilon_1 = 1.083$, $\varepsilon_2 = 3.834$

=> P= $\overline{E}_1.\varepsilon_1 = 1.633$

For the other loading steps, using similar method, it is obtained:

U=9.0, P=1.12

U=12.0, P=0.50

U=15.0, P=0.0

B. Tangent modulus scenario

At first, two essential considerations must be reviewed:

Consideration 1: Secant modulus based approach, is a direct method. By this approach, process zone only one time are modified. But the modification in the T.M.B.M. is different: this approach is a successive one, and the problem is analyzed for several times. If process zone modification will be done in each analysis step, the cracked zone (process zone) will be expanded rapidly and the problem will be divergent. For preventing the huge expansion of the process zone, the modification (nonlocal averaging) must be done only in the first time that an integration point will go over the elastic limit ($\varepsilon \geq \varepsilon_0$). In the next steps, only the other integration points (with $\varepsilon \geq \varepsilon_0$) will be under nonlocal averaging.

Consideration 2: In proposed formula (9) an "n" has been appeared. The question is: "which value of "n" is optimum?" this is a very difficult question. Indeed, a specific value of "n" is the most effective. Authors are working on the problem, but a logical solution is existed for the problem:

$$Y_u^n.\beta_u + Y_s^n.\beta_s = Err. \to 0 \Rightarrow \mathrm{n} \cong \frac{\mathrm{Log}\left(-\frac{\beta_s}{\beta_u}\right)}{Log\left(\frac{Y_u}{Y_s}\right)} \qquad (12)$$

Subscripts "u" and "s" introduce the unloading and softening material, and are the same as "1" and "2" in the problem that is shown in Figure 5.

Solving the problem: In this problem, material behavior is defined with the following parameters (in the T.M.B.M.):

$$\beta_u = \beta_1 = 1.0 \quad \beta_s = \beta_2 = -\frac{1}{9}$$

By combining two formulas (9) and (11) we have:

$$\tilde{\beta}_u^n = \tilde{\beta}_1^n = \frac{5\beta_u.Y_u^n + \beta_s.Y_s^n}{5Y_u^n + Y_s^n}, \tilde{\beta}_s^n = \tilde{\beta}_2^n = \frac{\beta_u.Y_u^n + 2\beta_s.Y_s^n}{Y_u^n + 2Y_s^n} \qquad (13)$$

Those are the n-th nonlocal values of T.S.F. in each section (unloading and softening).

Now, the second step of previously analyzed problem, is considered:

U=6.0 , P= σ =1.14 , ε_1 =0.57 , ε_2 =4.86

Y= $\sigma.\varepsilon/2$ => Y_1=0.325 , Y_2=2.77

From equation (12) it is obtained: n= 1.025. Using (13) we have:

$$\tilde{\beta}_1^{1.025} = 0.286 \quad , \quad \tilde{\beta}_2^{1.025} = -0.0526$$

Using the above nonlocal parameters, for P=0, U= 15.02 is obtained.

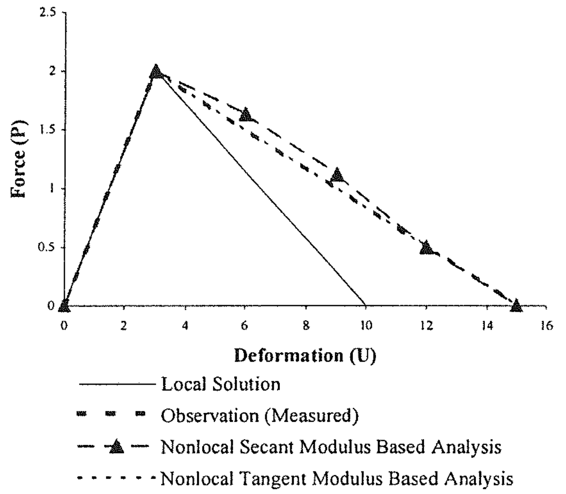


Figure 7: Results of Analyses in comparison with the Observation.

Results of analyses are shown in Figure 7. Obviously, proposed method is properly enough in conformity with the observation (objective results). So, the tangent modulus based approach can be an objective method.

7. CONCLUSION

In this paper, objectivity concept and its meaning in the engineering context has been described. Authors tried to find a solution for making the T.M.B.M., objective. This issue is opened, and has several non-solved problems. But authors believe that have found a practical method for using in engineering purposes.

8. REFERENCES

[1] Bazant, Z. P. (1976) "Instability, ductility, and size effect in strain-softening solids". J. of the Engineering Mechanics Division, ASCE, Vol. 102, p. 331-344.

[2] Bazant, Z. P. and Jirasek, M. (2002) "Nonlocal Integral Formulations of Plasticity and Damage". J. of Engineering Mechanics, ASCE, Vol. 128, No. 11, p. 1119-1149.

[3] Eringen, A. C. (1966) "A unified theory of thermomechanical materials". Int. J. of Engineering Science, Vol. 4, p. 179-202.

[4] Eringen, A. C. (1983) "Theories of nonlocal plasticity". Int. J. of engineering science, Vol. 21, p.741-751.

[5] Jirasek, M. (1999) "Computational Aspects of Nonlocal Models". Europian Conference on Computational Mechanics, ECCM 99.
[6] Jirasek, M. and Patzak, B. (1999) " Consistent tangent stiffness for nonlocal damage models". Technical report, EPFL, Switzerland.
[7] Jirasek, M. (2002) "Modeling of strain localization".Revue de genie civil, Vol.6, No.6, p.1119-1132.
[8] Marx, K. (1845) "Theses on Feuerbach". MEGA I/5, pp. 533-5.
[9] Mazars, J. (1981) "Mechanical damage and fracture of concrete structures". 5th Int. Conf. Fracture, Vol. 4, p. 1499-1506.
[10] Mazars, J. (1986) "A model for a unilateral elastic damageable material and its application to concrete". In fracture toughness and fracture energy of concrete, Elsevier, Amsterdam, p. 61-71.
[11] Shobeiri, H. (2005) " Behavioral model and numerical analysis of smeared cracks and softening behavior of rocks". Amirkabir University of Technology. Ph.D. thesis, that is going to submission.