Dynamic Response and Stability Analysis of A Double Race Automatic Dynamic Ball-Balancer

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Abstract

In this paper Dynamic stability and behavior of an automatic dynamic ball-balancer (ADB) with double race are analyzed.

Using the nonlinear equations of motion the linear variational equations are obtained by the perturbation method. Based on the linearized equations, the stability of the ball-balancer is analyzed around the balanced and unbalanced equilibrium positions. In addition, the time responses for the nonlinear equations of motion are computed by a numerical approach. However, the effects of fluid damping and external damping on the stability of the ADB are considered, also the effect of number of races on the operation of the ADB are investigated by the stability analysis and the time responses.

Keywords

Automatic Ball Balancer - Double Race - Dynamic Stability

Introduction

Unbalance in rotating machines is a common source of vibration excitation. For a rotor with a constant unbalanced mass, only one time of balancing is sufficient .However, if a rotor has variable unbalanced mass depending on running conditions, balancing of the rotor can not be achieved by only one time of balancing. For this purpose automatic dynamic ball-balancer is used, to reduce the imbalance in rotating mechanisms, such as washing machine, turning lathes, etc. ADB is a device to automatically eliminate the variable imbalance of rotating mechanisms. ADB, is usually composed of a circular disk with a groove, or race, containing spherical or cylindrical weights and a low viscosity damping fluid, although early attempts used other approaches. Many inventors have suggested various kinds of ADB's through U.S. patents, but they left it for others to explain why this system will or will not work. Basic research was initiated by Thearle[1,2], Alexander[3], and Cade[4]. Dynamic analysis for various ball-balancers can be found in references [1-5]. Recently, Lee[6] and Lee and Vanmoorhem[7] presented theoretical and experimental analyses of an ADB, but their presentation did not provide explicit requirements for the ADB, to balance the system. In addition, they used rectangular coordinate system instead of polar coordinates. Consequently, they derived the equations of motion for a non-autonomous system that requires the application of the floquet theory to stability analysis, however, this method has limitations on the complete stability analysis and has inaccurate stability results. In order to overcome to this problems Chung and Ro [8] derived the non-linear equations of motion using the polar co-ordinate system, for an autonomous system. Based on these equations, they obtained equilibrium positions and the linear variational equations, by the perturbation method and then by using Routh-Hurwitz criteria, they analyzed the stability of the system. In addition they computed the time responses of the system by the generalized-α method [9]. Hwang and

Chung [10] applied this approach to the analysis of an ADB with double race, but they didn't consider the effect of fluid damping and external damping on the operation of the ADB. Also they did not theoretically show what benefits, the ADB with double race has with respect to the ADB with one race, and also they did not consider the effect of inner race radius on the stability analysis. Also, in another paper[11] Chung and Jang considered the Stodola–Green rotor model, of which the shaft is flexible, and their model is able to include the influence of rigid-body rotations due to the shaft flexibility on dynamic responses

In this paper, the stability and dynamic behavior of an ADB with double race are analyzed, and the effects of fluid damping, external damping, number of races and other parameters on the stability of the ADB are considered. Also For the stability analysis the roots of characteristic equation are considered instead of Routh-Hurwitz criteria, because of simplicity in programming. The advantages of the ADB with double race compared to the ADB with single race are: (1) the impact between the balls can be avoided so the fracture of the balls is prevented, and (2) fine balancing is possible because there is no interference between the balls.

1-Non-Linear Equations of Motion

In order to obtain the non-linear equations of motion, the Lagrange's equation can be used. For this purpose, at first we evaluate the kinematics and potential energy and Rayleigh's dissipation function, and then we apply the Lagrange's equation to them [5-8, 10, 11, 12].

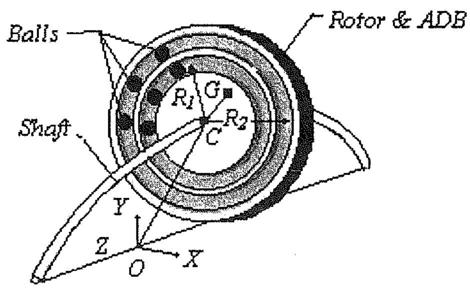

Figure (1) Automatic Dynamic Balancer(ADB) with double race.

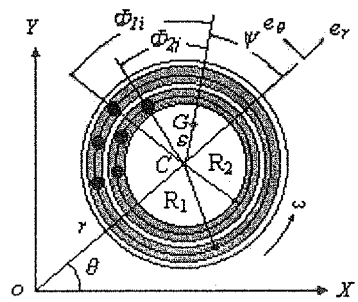

Figure (2) Configuration of the ADB with double race.

However by considering Figures (1)-(2), the non-linear equations of motion are derived in the polar coordinate system as follows [12]:

$$
\begin{aligned}
&(M+nm)\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)+cr\dot{\theta} \\
&+mR_1\sum_{i=1}^{n1}\ddot{\phi}_{1i}Cos(\phi_{1i}+\omega t-\theta)+(\dot{\phi}_{1i}+\omega)^2 Sin(\phi_{1i}+\omega t-\theta) \\
&+mR_2\sum_{i=1}^{n2}\ddot{\phi}_{2i}Cos(\phi_{2i}+\omega t-\theta)+(\dot{\phi}_{2i}+\omega)^2 Sin(\phi_{2i}+\omega t-\theta)= \\
&M\varepsilon\omega^2 Sin(\omega t-\theta)
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
&mR_1^{\,2}\ddot{\phi}_{1i}+D\dot{\phi}_{1i}-mR_1\left(\ddot{r}-r\dot{\theta}^2\right)Sin(\phi_{1i}+\omega t-\theta) \\
&\qquad +mR_1\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)Cos(\phi_{1i}+\omega t-\theta)=0, i=1,2,\ldots,n_1
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
&mR_2^{\,2}\ddot{\phi}_{2i}+D\dot{\phi}_{2i}-mR_2\left(\ddot{r}-r\dot{\theta}^2\right)Sin(\phi_{2i}+\omega t-\theta) \\
&\qquad +mR_2\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)Cos(\phi_{2i}+\omega t-\theta)=0, i=1,2,\ldots,n_2
\end{aligned} \tag{3}
$$

The angular velocity (ω) is assumed constant ,during the derivation. If the mass of balls equals to zero, equations (1)-(3) reduce to the equations for the Jeffcott rotor which are expressed as[8,10,12]:

$$M\left(\ddot{r}-r\dot{\theta}^2\right)+c\dot{r}+kr=M\varepsilon\omega^2 Cos(\omega t-\theta) \tag{4}$$

$$M\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)+cr\dot{\theta}=M\varepsilon\omega^2 Sin(\omega t-\theta) \tag{5}$$

2-Balanced Position and Linearized Equations

As seen in equations (1)-(3), the equations of motion correspond to the non-autonomous system, and generally it is very cumbersome to analyze the stability for non-autonomous systems. Therefore, the non-linear equations of motion should be transformed into those for an autonomous system: For simplicity, in order to overcome the difficulties, this study uses a generalized coordinate instead, which is defined by

$$\psi=\omega t-\theta \tag{6}$$

This new parameter represents the angle from the r direction to the center of mass G, as shown in Fig.2. In order to analyze the stability of the system the state equations can be easily employed, using equation (6), let us rewrite the equations of motion (1)-(3) as the state equations, to do this, we consider new symbols for r, ψ, φ_{1i}, φ_{2i} as follows:

$$\dot{r}\equiv\hat{r} \quad , \quad \dot{\psi}\equiv\hat{\psi} \quad , \quad \dot{\phi}_{1i}\equiv\hat{\phi}_{1i} \quad , \quad \dot{\phi}_{2i}\equiv\hat{\phi}_{2i} \tag{7}$$

Substituting equation(6) into equation (1)-(3) and using the notations given by equations (7), the equations of motion can be expressed as the state equations, which are 2n+4 first-order differential equations.

The state equations may be written by a matrix-vector equation:

$$\mathrm{A}(x)\dot{x}=\mathrm{N}(x) \tag{8}$$

where,

$$A = \begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix} \tag{9}$$

which M can be found in [12].

and

$$x = \left\{r, \psi, \phi_{11}, \ldots, \phi_{1n1}, \phi_{21}, \ldots, \phi_{2n2}, \hat{r}, \hat{\psi}, \hat{\phi}_{11}, \ldots, \hat{\phi}_{1n1}, \hat{\phi}_{21}, \ldots, \hat{\phi}_{2n2}\right\}^T \tag{10}$$

$$\begin{aligned} N = \{ & N_r, N_\psi, N_{\varphi_{11}}, \ldots, N_{\varphi_{1n1}}, N_{\varphi_{21}}, \ldots, N_{\varphi_{2n2}}, \hat{N}_r, \hat{N}_\psi, \\ & \hat{N}_{\varphi_{11}}, \ldots, \hat{N}_{\varphi_{1n1}}, \hat{N}_{\varphi_{21}}, \ldots, \hat{N}_{\varphi_{2n2}} \}^T \end{aligned} \tag{11}$$

in which I is the (n+2)*(n+2) identity matrix, M and X are the mass matrix and state variable vector respectively and N is obtained in terms of state variables. The equilibrium position may be classified into two cases, the balanced and unbalanced cases, which corresponds to r*=0 & r*≠0 respectively. Since the balanced equilibrium position of r*=0 is important, practically, only the case that r*=0 is considered. The equilibrium positions in this case are as follows [10, 12]:

$$R_1 \sum_{i=1}^{n1} Cos\phi_{1i}^* + R_2 \sum_{i=1}^{n2} Cos\phi_{2i}^* + \frac{M\varepsilon}{m} = 0 \tag{12}$$

$$R_1 \sum_{i=1}^{n1} Sin\phi_{1i}^* + R_2 \sum_{i=1}^{n2} Sin\phi_{2i}^* = 0 \tag{13}$$

and the linear variational equations (linearization of equations of motion about the equilibrium points) can be expressed as[8]:

$$A^* \Delta\dot{x} = B^* \Delta x + O(\Delta x) \tag{14}$$

where A* and B* are constant and O is a function of x with a second and higher order of magnitude. The A* and B* matrices are defined by:

$$A^* = A(x^*) \tag{15}$$

$$B^* = \begin{bmatrix} 0 & I \\ -k^* & -c^* \end{bmatrix} \tag{16}$$

where, k*, c* cn be found in [12]. Assuming x is sufficiently small to permit O to be ignored, equation (14) can be approximated by:

$$A^* \Delta\dot{x} = B^* \Delta x \tag{17}$$

which represents the linear variational equation.

3-Stability Analysis

For analyzing the stability of the system around the equilibrium position, the linear variational equation given by equation (17) is used.

for the simplicity of the analysis, consider the case that ADB has two races and one ball in each race, i.e. (n1=n2=1 & n=n1+n2=2)

Since the case that corresponds to balanced equilibrium position is important, practically, we consider it, in stability analysis.

By considering the solution in the form of

$$\Delta x = \Delta X\, e^{\lambda t} \tag{18}$$

where λ is an eigen value and X is an eigen vector corresponding to,

and Substituting it into equation (17) and eliminating the ψ^*, we apply the non-trivial solution condition. (As it is seen in equations (12-13) the value of ψ^* is not defined when $r^*=0$ i.e. the balanced equilibrium position. As a result, the characteristic equation can not be determined directly).

In this way the characteristic equation, is obtained which can be expressed as a polynomial of λ [12]:

$$\sum_{k=0}^{8} C_k \lambda^k = 0$$

where the coefficient of C_k (k=0,1,...,8) are functions of, m, M, R1, R2,C, D, k.The roots of characteristic equation are used to investigate the stability of the system, i.e. if the maximum of real part of the eigenvalues is positive, the system is unstable and if it is negative, the system is stable. The stability of the system is investigated for variations of system parameters such as M, m, c, D. The system parameters for all the stability analyses, are given by R_1=0.1, R_2=0.08, m=0.01Kg, M=1Kg, k=10000 N/m, c=5 N.m .Sec, D=0.01 (N/m).Sec, ε =0.001m.

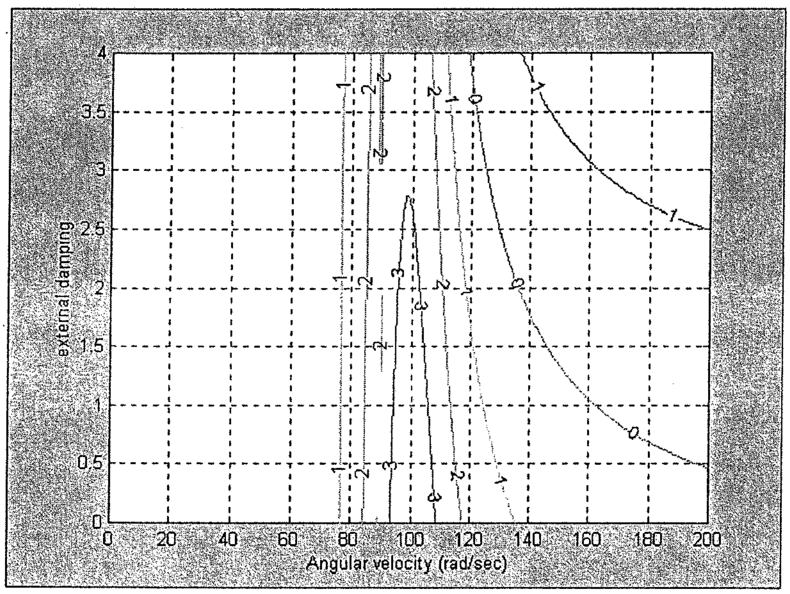

Figure (3-a) Contour plot of maximum real part of eigenvalues as function of angular velocity and external damping (c).

First, consider the stability of the two races ADB, in the neighborhood of the balanced equilibrium position for the variation of the rotating speed (ω) and external damping(c). Fig.(3a) shows that by increasing the value of external damping (c), the operating domain of the ADB increases.

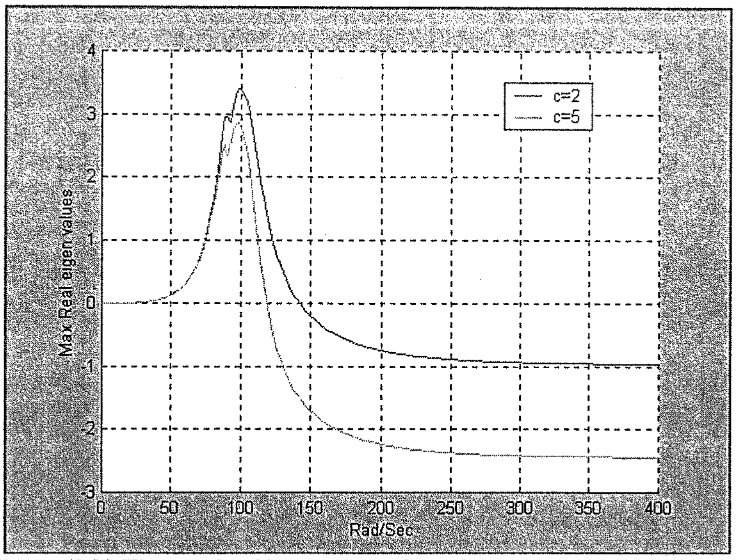

Figure (3-b) The result of increasing c from 2 to 5 in operating domain.

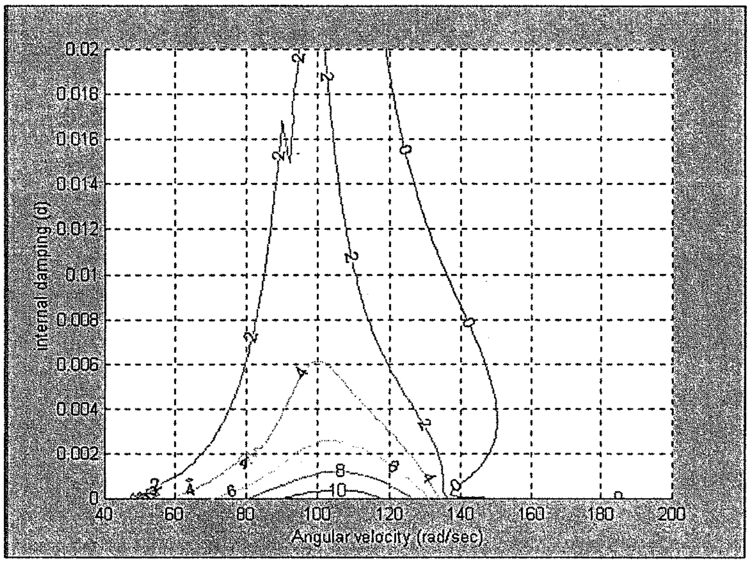

Figure (4) Contour plot of maximum real part of eigenvalues as function of angular velocity and internal damping (D).

Fig.(3b) indicates the stability versus variation of in to cases, i.e. c=2 & c=5. As you see, by increasing c from 2 to 5, the time that the system is balanced decreases. Because the maximums of real part of the eigen values, in this case are smaller than the case, c=2.

Fig. (4) shows the contour plot of maximum real part of eigenvalues versus angular velocity (ω) and internal damping(D). As it is seen, by varying in the value of (D) the operating domain of the ADB decreases until d=0.003, after this, the operating domain begins to increase. Fig.(4) also shows that that the ADB doesn't work near d=0, i.e. when there is not any fluid damping ADB doesn't work properly.

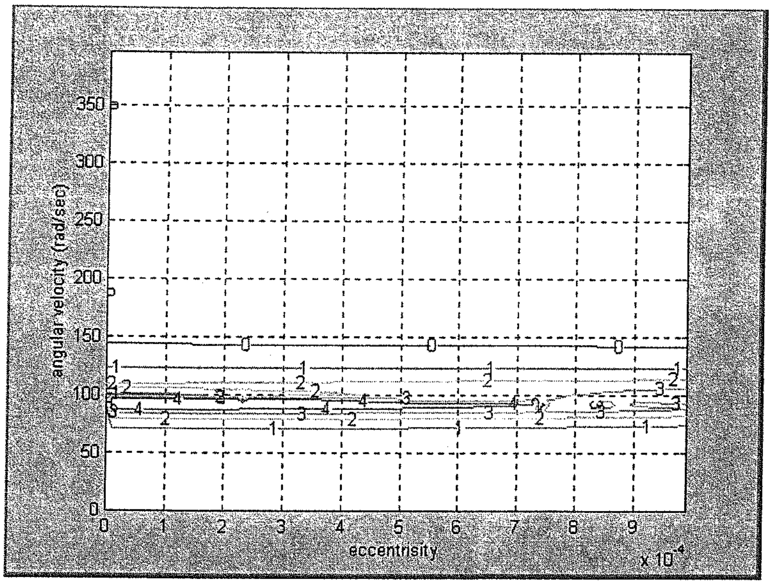

Figure (5) Contour plot of maximum real part of eigenvalues as function of angular velocity and imbalanced load (ε).

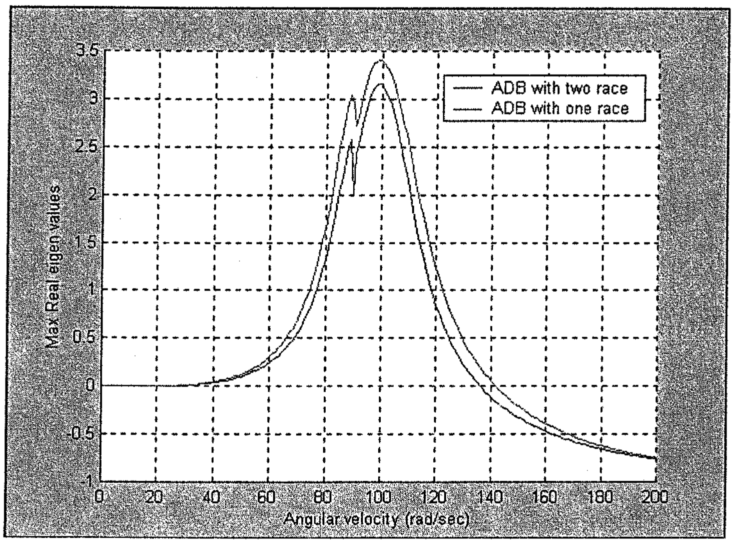

Figure (6) The effect of number of races in operating domain.

Fig.(5) shows the contour plot of maximum real part of eigenvalues versus angular velocity (ω) and imbalanced load (ε). By varying ε in the allowable domain, the operating domain of the ADB doesn't change very much, i.e. the ADB works properly by varying the imbalance load.

Fig.(6) shows the stability versus (ω) in two cases, i.e. two races ADB & single race ADB. In the case of two races ADB we assume that R1=0.1 and R2=0.08 and in the case of single race ADB we have R=0.1. As you see, in the case of two races ADB the operating domain is greater than a single race ADB.

4-Time Responses

Time responses of an ADB are investigated to verify the stability of the ball balancer and to analyze the dynamic behavior. From the non-linear equations of motion given by equations (1)-(3), the time responses are computed by the generalized-α time integration method [9], when the ball balancer has two races with one ball, in each race, the non-linear equations (1)-(3) may be expressed by the following matrix-vector equation

$$M(x)\dot{x} + N(x,\dot{x}) = 0 \tag{19}$$

where M is the mass matrix, N is the non-linear internal force vector, and x is the displacement vector. x, M, N are respectively:

$$x = \{r, \psi, \phi_{11}, \phi_{21}\}^T \tag{20}$$

and

$$M = \begin{bmatrix} M+2m & 0 & -mR_1\sin(\phi_{11}+\psi) & -mR_2\sin(\phi_{21}+\psi) \\ 0 & (M+2m)r & -mR_1\cos(\phi_{11}+\psi) & -mR_2\cos(\phi_{21}+\psi) \\ -mR_1\sin(\phi_{11}+\psi) & -mR_1 r\cos(\phi_{11}+\psi) & mR_1^{\ 2} & 0 \\ -mR_2\sin(\phi_{21}+\psi) & -mR_2 r\cos(\phi_{21}+\psi) & 0 & mR_2^{\ 2} \end{bmatrix} \tag{21}$$

and

$$N = \begin{Bmatrix} \begin{bmatrix} -(M+2m)r(\omega-\hat{\psi})^2 + c\hat{r} + kr - M\varepsilon\omega^2\cos\psi - \\ mR_1(\hat{\phi}_{11}+\omega)^2\cos(\phi_{11}+\psi) - mR_2(\hat{\phi}_{21}+\omega)^2\cos(\phi_{21}+\psi) \end{bmatrix} \\ \begin{bmatrix} -2(M+2m)\hat{r}(\omega-\hat{\psi}) - cr(\omega-\hat{\psi}) + M\varepsilon\omega^2\sin\psi + \\ mR_1(\hat{\phi}_{11}+\omega)^2\sin(\phi_{11}+\psi) + mR_2(\hat{\phi}_{21}+\omega)^2\cos(\phi_{21}+\psi) \end{bmatrix} \\ D\hat{\phi}_{11} + mR_1 r(\omega-\hat{\psi})^2\sin(\phi_{11}+\psi) + 2mR_1\hat{r}(\omega-\hat{\psi})\cos(\phi_{11}+\psi) \\ D\hat{\phi}_{21} + mR_2 r(\omega-\hat{\psi})^2\sin(\phi_{21}+\psi) + 2mR_2\hat{r}(\omega-\hat{\psi})\cos(\phi_{21}+\psi) \end{Bmatrix} \tag{22}$$

The system parameters for computation time responses are given by R1=0.1m, R2=0.08m, m=0.01Kg, M=1Kg,ε=0.001m, k=10000N/m, c=5N.m.Sec, D=0.01(N/m).Sec

The procedure to obtain the time responses by using the generalized-α method can be found in reference [9].Time responses are computed for $\omega/\omega_n = 0.6, 1, 2$.

The initial condition are given as r(0)=0.001m, $\psi(0)=0$, $\phi_{11}(0)=\pi/3$, $\phi_{21}(0)=\pi/6$.

Figures7 and 8 show that the rotating speed ω/ω_n =0.6 and 1 are in the unstable region for the balanced equilibrium position. If the balancer has the system parameters given above, on the other hand it is observed that the speed of ω/ω_n =2 is in the stable region. Time responses of an ADB, when ω/ω_n =0.6 are presented in Fig. (7) as you see, it demonstrates that the ADB does not work properly in this case, in addition, the non-pleasant vibrations increase. Anyhow, we ignore from the ball positions in this case and the case, ω/ω_n =1, because it is not important, practically.

Fig.(8) demonstrate that when ω/ω_n =1 the ball balancer is also unstable around the balanced equilibrium point. However, when the ball balancer is in the stable region for the balanced equilibrium position, e.g., when ω/ω_n =2, the balancer achieves the balancing of the rotor, in this case the radial displacement converges to zero. The converged values for the ball positions, are ϕ_{11}=-132.6° and ϕ_{21}=112.13°, which can be obtained from equations (12)-(13). This means that the balancer is not only in static equilibrium but also in dynamic equilibrium, Fig.(9). Fig. (10) & Fig. (11) Show the time responses for the position of ball in the first and the second race, respectively.

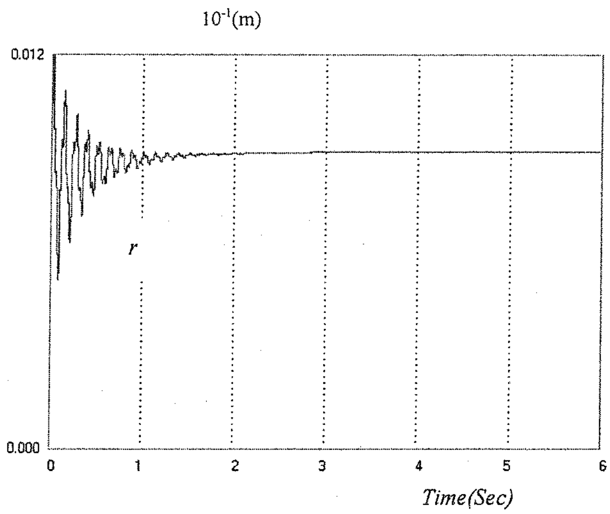

Figure (7) Time response of the radial displacement when ω/ω_n=0.6.

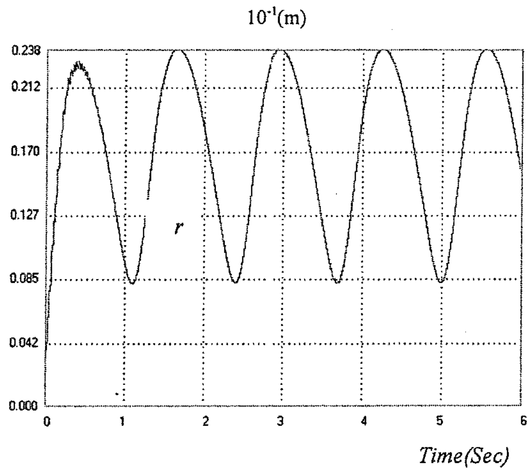

Figure (8) Time response of the radial displacement when ω/ω_n =1.

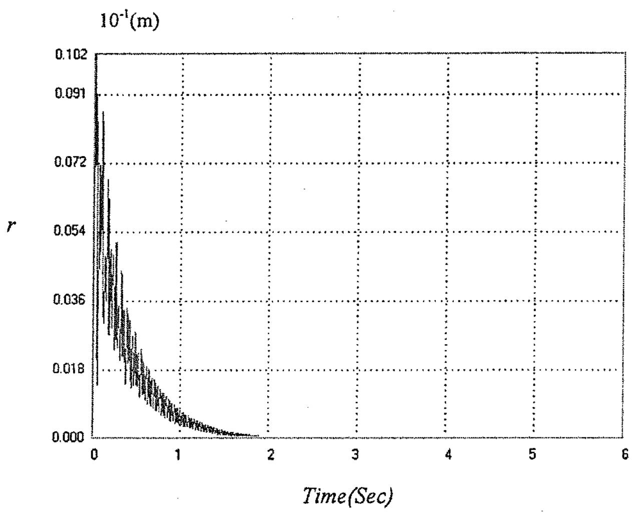

Figure (9) Time response of the radial displacement when ω/ω_n =2.

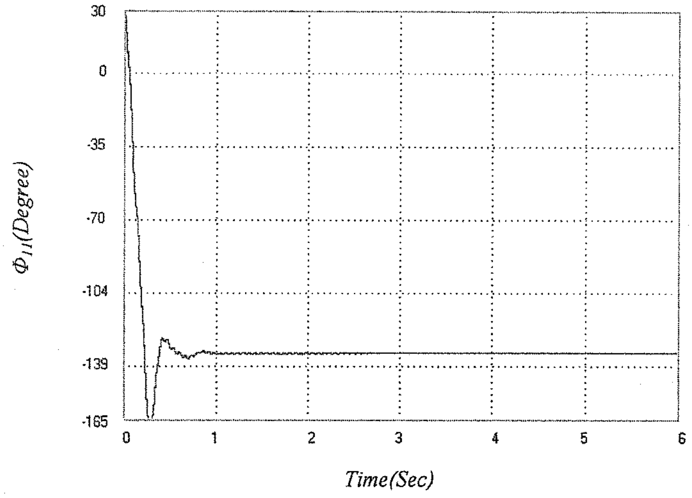

Figure (10) Time response of position of the ball which is in the first race, $\omega/\omega_n = 2$.

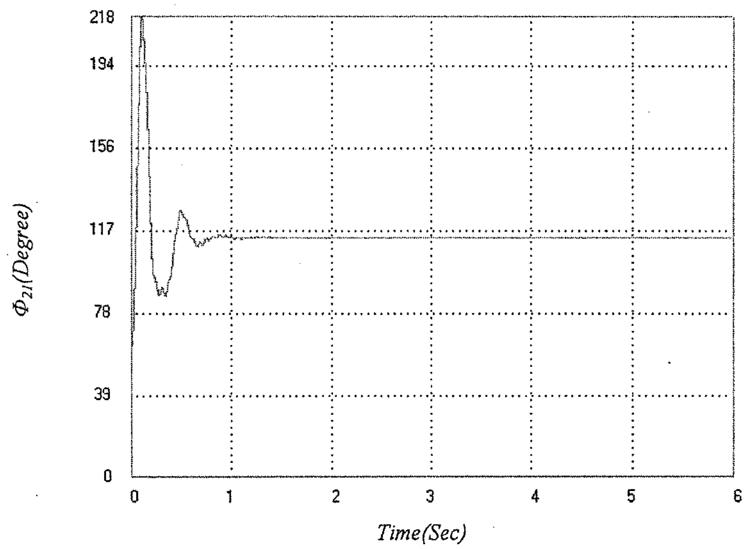

Figure (11) Time response of position of the ball which is in the second race, $\omega/\omega_n = 2$.

5-Conclusions

In this paper, dynamic stability and time responses are analyzed for a double race automatic ball balancer. By using the nonlinear equations of motion and applying the perturbation method to these equations, balanced equilibrium position and linearized equations in the neighborhood of the equilibrium position are obtained. Based on the linearized equations, around the balanced equilibrium position, the stability analysis is performed by using the roots of characteristic equation in this position. On the other hand, time responses are computed from the nonlinear equations of motion and they are investigated. The results of this study may be summarized as follows.

1- The two races automatic ball balancer can achieve the balancing of the jeffcott rotor as well as a single race ADB.

2- By increasing the external damping (c) the operating domain of the ADB increases, also the time which system is balanced, decreases.

3- By increasing the number of races, interference between the balls decreases, and the operating domain increases

References

[1] E.L. THEARLE, "Automatic dynamic balancers (Part 1. Leblanc balancer)." Machine Design Vol.22,1950, pp. 119-124.

[2] E.L. THEARLE, "Automatic dynamic balancers (Part 2. ring, pendulum, ball balancers)" Machine Design Vol. 22, pp. 103-106.

[3] J.D.Alexander, "An automatic dynamic balancer", Proceeding of 2 nd Southeastern Conference vol. 2,1964, pp. 415-426.

[4] J.W.CADE, "Self-compensating balancing in rotating mechanisms", Design News, 1965, pp. 234-239.

[5] J.Adolfsson, "A Study of Stability in Auto Balancing Systems using Multiple Correction Masses", Department of Mechanics, Royal Institute of Technology S-100 44 STOCKHOLM, Sweden, Licentiate Thesis

[6] J.Lee. "An analytical study of self-compensating dynamic balancer with damping fluid and ball", Shock and Vibration Vol. 2, 1995, pp. 59-67.

[7] J. Lee and W.K. Van Moorhem "Analytical and experimental analysis of a self-compensating dynamic balancer in a rotating mechanism", ASME Journal of Dynamic Systems, Measurement, and Control Vol.118, 1996, pp. 468-475.

[8] J. Chung and D.S. Ro "Dynamic analysis of an automatic dynamic balancer for rotating mechanisms", Journal of Sound and Vibration Vol.5, 1999, pp. 1035-1056

[9] J. Chung and G.M. Hulbert, "A time integration algorithm for structural dynamics with improved numerical dissipation: the generalized- α method", ASME Journal of Applied Mechanics Vol. 60, 1993, pp. 371-375.

[10] C.H. Hwang and J. Chung, "Dynamic analysis of an automatic ball-balancer with double race", JSME International Journal Series C vol.42, 1999, pp.265-272

[11] Chung J. and Jang I "Dynamic response and stability analysis of an automatic ball balancer for a flexible rotor", Journal of sound and vibration vol. 1, 2003, pp 31-43

[12] J.Ehyaee, "An Analytical Development of Automatic Dynamic Balancers of Rotating Shafts", Department of Mechanical Engineering, Tarbiat Modarres University, M.Sc Thesis, 2002