Optimal Trajectory of Mobile Manipulators with Maximum Load Carrying Capacity

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Abstract

A new technique is developed for determining optimal trajectory of mobile manipulators to maximize their load carrying capacity between two points of their workspace. This problem in mobile manipulators is more complicated, since, wheeled mobile bases are usually subjected to non integrable kinematic constraints besides to have extra degrees of freedom due to combined motion of base and manipulator. In this paper, to solve the non holonomic constraint problem and redundancy resolution, the extended Jacobian matrix concept is used. Then, the problem of dynamic load carrying capacity on mobile manipulators is converted into a trajectory optimization problem. Dynamic equations are formulated in the state space form and then are linearized as well as constraint equations. Then, Iterative Linear Programming (ILP) method is used to determine maximum load of mobile manipulators. Finally, by a numerical example involving a PUMA robot using the method is presented.

Keywords

optimal trajectory, load, optimization, mobile manipulators

Introduction

Some common types of mobile manipulators are wheeled mobile manipulators, tracked robotic manipulators and gantry robots. One of the main applications of mobile manipulators is handling heavy loads from one place to another. Finding the full load motion for a given point-to-point task can maximize the productivity and economic usage of the mobile manipulators. In the classical fixed base manipulators, dynamic load carrying capacity is defined as maximum load, which a manipulator can carry repeatedly on its fully extended configuration, while the dynamics of the load and the robot manipulator itself must be taken into account [1]. For the fixed base manipulators, the major limiting factor in determining the maximum load is the joint actuator capacity. In mobile manipulators, end effector motion is a superposition of the base and the manipulator motions, thus the overall system has extra D.O.F in its motion. Therefore, redundancy resolution should be considered in the trajectory synthesis of mobile manipulators. Also, considering tipping over stability increases the analysis complexity.

Wang and Ravani [2] are presented that in fixed base robots, determining the load carrying capacity can be formulated as a trajectory optimization problem. On their analysis actuators torque capacity is stressed as the main constraint. Korayem and Basu [3-4] by relaxing the rigid body assumption on robot joints and links imposed an additional constraint to the allowable deformation at end effector besides the joint actuators capacity. S. Yue et. al. [5] used the finite element method for describing the dynamics of the system and are computed the maximum payload of kinematically redundant flexible manipulators. Korayem and Ghariblu [6] introduced the load workspace concept and considered the effect of base initial location on dynamic load carrying capacity of robotic manipulators on a given trajectory. Due to increasing applications of mobile manipulators especially on carrying heavy loads on large workspaces, there have been some researches on tipping over stability on mobile manipulators [7-9]. Also, some works done involves mathematical and kinematic modeling of mobile manipulators [11-13].

A new computational technique is developed for determining load carrying capacity of mobile manipulators between two points of their workspace in this paper. By considering non-integrable kinematic constraints and extra degrees of freedom, which both arise from base mobility, solving the dynamic equation of system is more difficult with respect to classical robotic manipulators. Thus, to overcome these difficulties the extended Jacobian matrix concept is used which simplifies the system coordination and control during the motion. The problem of increasing load carrying capacity of mobile manipulators by proper formulation is converted to a trajectory optimization problem. Suitable objective function is defined and dynamic equations are linearized as well as other constraints on its state space form. Then, Iterative Linear programming (ILP) method is used to numerical solution of the linearized trajectory optimization problem. Finally, by simulation study including a PUMA arm, application of the method is investigated.

1-Kinematic Modeling and Motin Planning of Mobile Manipulators

The position of the end effector in the task space of mobile manipulators can be defined as follows:

$$X = X_b(q_b) + X_{m/b}(q_m) \tag{1}$$

where $X = [x \quad y \quad z]^T$ and $X_b = [x_b \quad y_b \quad z_b]^T$ are the position of the end effector and the base in the inertial reference frame. $X_{m/b} = [x_{m/b} \quad y_{m/b} \quad z_{m/b}]^T$ is the position vector of manipulator with respect to the base, also q_b and q_m are base configuration space and joint space vectors. The Jacobian equation of the mobile manipulator can be determined as:

$$\dot{X} = J\dot{q} \tag{2}$$

where $J = (J_b \quad J_m)$ and $\dot{q} = (\dot{q}_b \quad \dot{q}_m)^T$. $\dot{X} \in R^m$ denotes the end effector velocity with respect to the fixed coordinate frame and $\dot{q} \in R^n$ is the joints velocity space. We denote the mobility plus manipulation D.O.F of the system by n and working cartesian space dimension by m. If $n > m$, then the degree of redundancy $n - m$ is denoted by r. By the other means for solvability generally r additional function must be applied to relate joint vectors to each other. If $n < m$, then there exists $c = m - n$ constraint equation in the system dynamics.

In mobile manipulators, the mobile bases are subject to nonintegrable kinematic constraints. Rolling contact between the wheels and ground generally causes such constraints. As a result the base must be move in the direction of its main axis of symmetry. The general form of the nonholonomic constraint equation can be written as:

$$J_c\dot{q} = 0 \tag{3}$$

where $J_c \in R^{c\times n}$. The combined system of mobile manipulator has extra degrees of freedom $(n > m)$ on its motion. Therefore to resolve the redundancy, we can apply r additional constraint equations, which can be given as:

$$\dot{X}_Z = J_z \dot{q} \tag{4}$$

where $J_z \in R^{r\times n}$. Hence, the kinematic equation of mobile manipulators by combining the equations (2), (3), and (4) one obtains:

$$\begin{pmatrix} \dot{X} & \dot{X}_z & 0 \end{pmatrix}^T = (J \quad J_z \quad J_c)^T \dot{q} \tag{5}$$

Here $J_a = (J \quad J_z \quad J_c)^T$ is named as augmented Jacobian matrix. As explained by Seraji [11] to resolving the system's redundancy the simple method is to choose user specified constraint equations in general form as follows:

$$X_z = g(q) \tag{6}$$

By differentiating of Eq. (6) with respect to time, we have $\dot{X}_z = J_z \dot{q}$ similar to Eq. (4). Note that the augmented Jacobian equation enables us to consider the differential kinematics of mobile manipulators as if they were non-redundant robots.

The augmented Jacobian matrix J_a, regardless of the configuration q of the mobile manipulator must be non-singular, or the determinant of J_a must be non-zero:

$$\mathrm{Det}(J_a) \neq 0. \tag{7}$$

If the resultant J_a to be non-singular then joints velocity acceleration vectors are found:

$$\dot{q} = J_a^{-1} \begin{pmatrix} \dot{X} & \dot{X}_z & 0 \end{pmatrix}^T \tag{8}$$

$$\ddot{q} = J_a^{-1} \left(\begin{pmatrix} \ddot{X} & \ddot{X}_z & 0 \end{pmatrix}^T - \dot{J}_a \dot{q} \right) \tag{9}$$

2-Linearized State Space Representation of Dynamic Equations

To determine an optimal trajectory for a mobile manipulator, proper modeling of the system and load dynamic is a prerequisite. The Lagrangian method is used to model the mobile manipulator and load m_L, which is presented on its closed form as:

$$\vec{\tau} = [M(\vec{q}, m_L)]\ddot{\vec{q}} + \vec{C}(\vec{q}, \dot{\vec{q}}, m_L) \tag{10}$$

where $\vec{\tau} \in R^n$ is the joints actuator torque, $M(\vec{q}, m_L)$ is the inertia matrix (mass and inertia of actuators are seen in main diagonal elements of the M matrix). Also $\vec{C}(\vec{q}, \dot{\vec{q}}, m_L)$ includes gravity, Coriolis and centrifugal force components.

To compute the numerical solution of nonlinear constrained trajectory optimization problem to increase load carrying capacity, the dynamic equation (10) are rearranged as bellow:

$$\ddot{\vec{q}} = [M(\vec{q}, m_L)]^{-1}(\vec{\tau} - \vec{C}(\vec{q}, \dot{\vec{q}}, m_L) = \vec{f}(\vec{q}, \dot{\vec{q}}, \vec{\tau}, m_L) \tag{11}$$

By defining state vector as $\vec{X} = [x_1\ x_2]^T$ which $\vec{x}_1 = (q_1, q_2, ..., q_n)^T$ and $\vec{x}_2 = (\dot{q}_1, \dot{q}_2, ..., \dot{q}_n)^T$, the equation (11) is written as:

$$\dot{\vec{X}} = \begin{bmatrix} \dot{\vec{x}}_1 \\ \hline \dot{\vec{x}}_2 \end{bmatrix} = \begin{bmatrix} \vec{x}_2 \\ \hline \vec{f}(\vec{X}(j), \vec{\tau}(j), m_L) \end{bmatrix} = \vec{F}(\vec{X}(j), \vec{\tau}(j), m_L) \tag{12}$$

The equation (12) is the state space representation of dynamic equation (11), where $\vec{X} \in R^{2n}$ and $\vec{f}$ consist of n nonlinear functions. Discretized form of the equation (12) results:

$$\frac{\vec{X}(j+1) - \vec{X}(j)}{h} = \vec{F}(\vec{X}(j), \vec{\tau}(j), m_L) \tag{13}$$

where, $h = \frac{\Delta T}{m}$ and ΔT is the overall motion time and m is the number of points are used for discretizing trajectory. The nonlinear function f at the *(k+1)*th trajectory is expanded in Taylor series about the *k*-th trajectory. After neglecting the higher order (non linear) terms and simplifying the expressions the following equation is obtained:

$$\vec{X}(j+1) = [G_j]\vec{X}(j) + [H_j]\vec{\tau}(j) + \vec{B}_j m_L + \vec{D}_j \tag{14}$$

where, the matrices $[G_j], [H_j]$ and $\vec{B}_j, \vec{D}_j$ are as bellow

$$[G_j] = \left| \begin{array}{c|c} [I] & h[I] \\ \hline \left[h\frac{\partial f^k}{\partial x_1}(j)\right] & \left[h\frac{\partial f^k}{\partial x_2}(j) + [I]\right] \end{array} \right| \in R^{2n\times 2n} \tag{15}$$

$$[H_j] = \left| \begin{array}{c} [O] \\ \hline \left[h\frac{\partial f^k}{\partial \vec{\tau}}(j)\right] \end{array} \right| \in R^{2n\times n} \tag{16}$$

$$\vec{B}_j = \begin{bmatrix} \vec{O} \\ \left[h \frac{\partial f^k}{\partial m_L}(j) \right] \end{bmatrix} \in R^{2n} \tag{17}$$

$$\vec{D}_j = \begin{bmatrix} \vec{O} \\ \left[h\vec{c}^k(j) \right] \end{bmatrix} \in R^{2n} \tag{18}$$

In the above expressions $\vec{c}^k(j)$ is

$$\vec{c}^k(j) = \vec{f}^k(j) - \frac{\partial f^k}{\partial x_1}(j) \cdot \vec{x}_1^k(j) - \frac{\partial f^k}{\partial x_2}(j) \cdot \vec{x}_2^k(j) - \frac{\partial f^k}{\partial \tau}(j) \cdot \vec{\tau}^k(j) - \frac{\partial f^k}{\partial m_L}(j) \cdot m_L^k \tag{19}$$

In these equations, *[I]* is an $n \times n$ identity matrix, *[0]* is an $n \times n$ null matrix, and $\vec{O}$ is an $n \times 1$ null vector.

$\vec{X}(j+1)$ can be written as a linear combination of the payload m_L and the torque control $\vec{\tau}(i)$, $i = 1,2,......,j$. Equation (14) then becomes:

$$\vec{X}(j+1) = \vec{X}_h(j+1) + \vec{\beta}_j m_L + \sum_{i=1}^{j} [\alpha_{ji}] \vec{\tau}(i) \quad \text{and } j = 1,2,...,m \tag{20}$$

This equation is the basic linearized equation, where:

$$\vec{X}_h(1) = \vec{X}(t_i) \tag{21}$$

$$\vec{X}_h(j+1) = [G_j] \vec{X}_h(j) + \vec{D}_j \tag{22}$$

$$\vec{\beta}_1 = \vec{B}_1 \tag{23}$$

$$\vec{\beta}_j = [G_j] \vec{\beta}_{j-1} + \vec{B}_j \tag{24}$$

$$[a_{ji}] = [G_j][a_{j-1,i}] \qquad \text{for } i \langle j \tag{25}$$

$$[a_{ji}] = [H_j] \qquad \text{for } i = j \tag{26}$$

4-The Problem Formulation

The problem of synthesizing dynamic mobile manipulator trajectories with maximum load carrying capacity can be formulated as a trajectory optimization problem. By considering point-to-point motions with actuator, joint variables, and redundancy constraints throughout the trajectory, the complete formulation can be written to maximize:

The m_{Load}

While ensuring that the state space Equation 12 is satisfied, where the individual joint torques are bounded by:

$$\vec{\tau}_{min}(\vec{X}(t)) \le \vec{\tau}(t) \le \vec{\tau}_{max}(\vec{X}(t)) \tag{27}$$

The bounds $\vec{\tau}_{\min}(\vec{X}(t))$ and $\vec{\tau}_{\max}(\vec{X}(t))$ are arbitrary known functions of the actuator joint angles and velocities. In addition to constraints of the joint torques, the initial and final states must be reached. Thus, the following conditions must be satisfied:

$$\vec{x}_1(t_i)=\vec{q}(t_i)=\vec{x}_{1i} \quad , \quad \vec{x}_2(t_i)=0$$
$$\vec{x}_2(t_f)=\vec{q}(t_f)=\vec{x}_{1f} \quad , \quad \vec{x}_2(t_f)=0 \tag{28}$$

During the motion, the joint displacements are also usually bounded by

$$\vec{x}_1^- \leq \vec{x}_1(t) \leq \vec{x}_1^+ \tag{29}$$

where $\vec{x}_1^+, \vec{x}_1^-$ are the upper and lower bounds of the joint variables, respectively. Redundancy constraint equations in general form of equation (6) must be satisfied.

The final constraint is that of the payload m_L upper bound must be smaller than the static load carrying capacity at the two end positions. The trajectory synthesis problem formulated above is a constrained nonlinear optimization problem. In the above formulation, the objective function consists of a single variable m_L, which is time independent and is a single valued quantity for the entire trajectory.

5- The Problem Solving

The Iterative Linear Programming (ILP) method is used to solve the trajectory synthesis problem described above. At first, the final state reaching condition from equation (15) can be obtained as:

$$\vec{X}(m)=\vec{X}_h(m)+\vec{\beta}_m m_L+\sum_{i=1}^{m}[a_{mi}]\vec{\tau}(i)=\vec{X}(t_f) \tag{30}$$

Equation (30) can be written as:

$$\vec{\beta}_m m_L+[E]\vec{\tau}=\vec{X}(t_f)-\vec{X}_h(m) \tag{31}$$

where, $[E]=[\ [a_{m1}],[a_{m2}],\ldots,[a_{mm}]\ ]^T \in R^{2n\times mn}$ and $\vec{\tau}=[\vec{\tau}(1),\vec{\tau}(2),\ldots,\vec{\tau}(m)]^T \in R^{mn}$.

It should be noted that $[E],\beta_m, X_h(m)$ are computed based on the values of the state and control variables of the previous iteration. Therefore, the only unknowns in the equation (26) are m_L and $\vec{\tau}_m$. In order to facilitate the LP solution, equation (31) can be written with two sets of inequalities:

$$\vec{\beta}_m m_L+[E]\vec{\tau}_m-\vec{e}\leq\vec{X}(t_f)-\vec{X}_h(m) \tag{32}$$

$$\vec{\beta}_m m_L+[E]\vec{\tau}_m+\vec{e}\geq\vec{X}(t_f)-\vec{X}_h(m) \tag{33}$$

where, $\vec{e}=[e_{pos1},e_{pos2},\ldots,e_{vel1},e_{vel2},\ldots]\in R^{2n}$ represents the final position and velocity error tolerances. This modification introduces two more variables (e_{pos}, e_{vel}) and $2n$ inequality constraints. The actuator constraints can be written separately as follows:

$$\vec{\tau}(j)\geq\vec{\tau}_{\min}(\vec{X}(j)) \qquad \text{for} \qquad j=1,2,\ldots\ldots,m \tag{34}$$

$$\vec{\tau}(j) \le \vec{\tau}_{max}(\vec{X}(j)) \qquad \text{for} \qquad j=1, 2, \ldots, m \tag{35}$$

Assuming a typical torque-speed characteristics for DC motors the $\vec{\tau}_{min}(\vec{X}(j))$ and $\vec{\tau}_{max}(\vec{X}(j))$ can be approximated as

$$\vec{\tau}(j) \ge \vec{\tau}_{min}(\vec{X}(j)) = -\vec{K}_1 - [K_2]\vec{x}_2(j) \tag{36}$$

$$\vec{\tau}(j) \le \vec{\tau}_{max}(\vec{X}(j)) = \vec{K}_1 - [K_2]\vec{x}_2(j) \tag{37}$$

where $\vec{K}_1$ is an $n \times 1$ constant vector and $[K_2]$ is an $n \times n$ diagonal constant matrix obtained from the equivalent motor constants. Writing these constraints in matrix form leads to:

$$\vec{\tau} \le \vec{b}_u = \begin{vmatrix} \vec{K}_1 - [K_2]\vec{x}_2^k(1) \\ \vec{K}_1 - [K_2]\vec{x}_2^k(2) \\ \vdots \\ \vec{K}_1 - [K_2]\vec{x}_2^k(m) \end{vmatrix} \tag{38}$$

$$-\vec{\tau} \le -\vec{b}_l = \begin{vmatrix} -\vec{K}_1 - [K_2]\vec{x}_2^k(1) \\ -\vec{K}_1 - [K_2]\vec{x}_2^k(2) \\ \vdots \\ -\vec{K}_1 - [K_2]\vec{x}_2^k(m) \end{vmatrix} \tag{39}$$

where $\vec{b}_l$ *and* $\vec{b}_u$ are the lower and upper bound vectors of joint actuator torques, respectively. The problem can be converted to a standard linear programming problem by a change of variable as below

$$\vec{Y} = \vec{b}_u - \vec{\tau} \qquad \text{or} \qquad \vec{\tau} = \vec{b}_u - \vec{Y} \;, \vec{Y} \ge 0 \tag{40}$$

Substituting equation (35) into (33) leads to:

$$\vec{Y} \le \vec{b}_u - \vec{b}_l \tag{41}$$

Using equation (20), the joint variable constraints outlined above can be written as:

$$\vec{x}_I^- - \vec{X}_{1h}(j) \le \vec{\beta}_{1j} m_L + \sum_{i=1}^{j} [a_{1ji}]\vec{\tau}(i) \le \vec{x}_I^+ - \vec{X}_{1h}(j) \qquad \text{for} \qquad j=1, 2, \ldots, m \tag{42}$$

where $\vec{X}_{1h}(j+1)$ and $\vec{\beta}_{1j}$ are the upper $n \times 1$ vectors of $\vec{X}_h(j+1)$ and $\vec{\beta}_j$, respectively and $\lfloor a_{1ji} \rfloor$ is the upper $n \times n$ submatrix of $\lfloor a_{ji} \rfloor$. Equation (42) can be written in the following form by letting $[A_j] = [\alpha_{1j1} \quad \alpha_{1j2}, \ldots, \alpha_{1jj}, 0, 0, \ldots, 0] \in R^{n \times nm}$

$$\vec{\beta}_{1j} m_L - [A_j]\vec{Y} \le \lfloor \vec{X}_I^+ - \vec{X}_{1h}(j+1) \rfloor - [A_j]\vec{b}_u \quad \text{for} \;\; j = 1, 2, \ldots, m \tag{43}$$

$$-\vec{\beta}_{1j} m_L + [A_j]\vec{Y} \le \lfloor \vec{X}_{1h}(j+1) - \vec{X}_I^- \rfloor + [A_j]\vec{b}_u \quad \text{for} \;\; j = 1, 2, \ldots, m \tag{44}$$

Combining all the constraints and expressing the result in matrix form gives:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \vec{\beta}_{1j} & -[A_j] & 0 \\ -\vec{\beta}_{1j} & [A_j] & 0 \\ \vec{\beta}_j & -[E] & -1 \\ -\vec{\beta}_j & [E] & -1 \end{bmatrix} \begin{bmatrix} m_L \\ \vec{Y} \\ \vec{e} \end{bmatrix} \le \begin{bmatrix} m_{max} \\ \vec{b}_u - \vec{b}_l \\ \vec{x}_l^+ - \vec{X}_{lh}(j) - [A_j]\vec{b}_u \\ \vec{X}_{lh}(j) - x_l^- + [A_j]\vec{b}_u \\ \vec{X}(t_f) - \vec{X}_h(m) - [E]\vec{b}_u \\ \vec{X}_h(m) - \vec{X}(t_f) + [E]\vec{b}_u \end{bmatrix} \tag{45}$$

The objective function of this problem is defined as:

$$Z = m_L + W\vec{e} \tag{46}$$

where $W = (w_1, w_2, \ldots, w_{2n}) > 0$ are weighting factors of final state error tolerances. With this objective function we can maximize the load carrying capacity m_L and simultaneously minimize the position and velocity errors at the end points of the trajectory. Since the objective function (46) and the constraints (45) are all linear, we have a standard linear programming problem.

6- The Computing Method

The computing method for the optimal trajectory problem is formulated as shown in Figure 1. At first, an initial control and state variable trajectory is suggested such that non of constraints are violated. By discretizing the initial trajectory into m points the corresponding linearized constraint coefficients are computed and then iterative linear programming subroutine is invoked to update $m_L, \vec{Y}$ and $\vec{e}$. Using these updated variables, the new trajectory $X^{k+1}(j)$ is synthesized. Then, the termination conditions are checked:

$$\max\{e_{pos}, e_{vel}\} \le \varepsilon_1 \tag{47}$$

$$\max\{\left|\vec{X}^{k+1}(j) - \vec{X}^k(j)\right|, j = 2, \ldots, m\} \le \varepsilon_2 \tag{48}$$

$$\left|m_L^{k+1} - m_L^k\right| \le \varepsilon_3 \tag{49}$$

where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are predefined small positive constants. If the termination conditions are satisfied then, the updated trajectory is the optimal and corresponding value of m_L is the maximum allowable load, which can be carried by the mobile manipulator. Otherwise the program jumps to Step 2. Also, satisfying termination criterion means that linearization errors eliminated or significantly reduced.

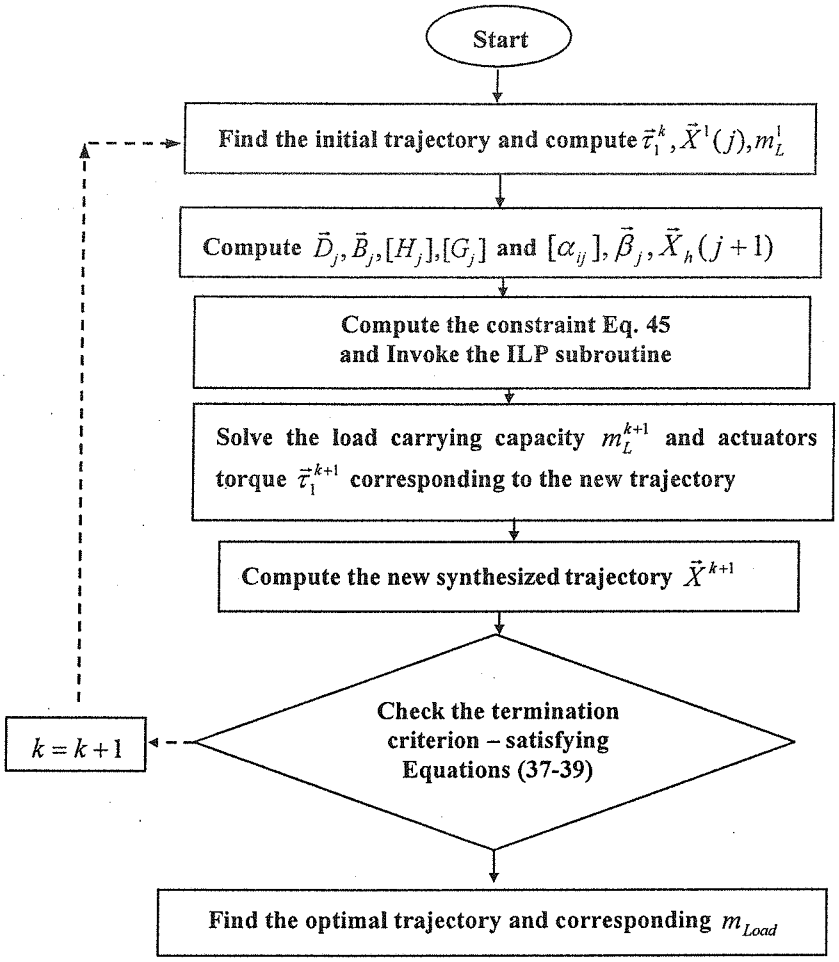

Figure 1: Flowchart of computing procedure for mobile manipulator

7- Simulation Results

A simulation study was carried out to investigate the validity and effectiveness of the proposed algorithm. A spatial three-jointed PUMA robot (without the wrist) mounted on a linear tracked base is considered as shown in Figure 2. The combined system include manipulator and its base have degrees of freedom of order $n=n_m+n_b=4$. Conversely, the end effector motion in cartesian space is of order $m=3$. Therefore, the system has extra degrees of freedom equal to $r=n-m=1$, and it is necessary adding a user specified kinematic constraint into system for its proper coordination. In this case the redundancy is resolved by the controlling the "elbow angle" β between the upper-arm and fore-arm and the forward kinematic model of the robot is extended by

$$X_z = \beta = \pi + \theta_2 \qquad (50)$$

In this simulation the angle β variations are considered such that during the system motion is changed from $\beta_i = \frac{\pi}{4}$ *to* $\beta_f = \frac{2\pi}{5}$. The extended differential kinematic model relating the rate of change of task variables to the joint velocities is found to be [11]:

$$\begin{bmatrix} -l\sin(\theta_0)(\sin(\theta_1)+\sin(\theta_1+\theta_2)) & l\cos(\theta_0)(\cos(\theta_1)+\cos(\theta_1+\theta_2)) & l\cos(\theta_0)\cos(\theta_1+\theta_2)) & 1 \\ l\cos(\theta_0)(\sin(\theta_1)+(\sin(\theta_1+\theta_2)) & l\sin(\theta_0)(\cos(\theta_1)+\cos(\theta_1+\theta_2)) & l\sin(\theta_0)\cos(\theta_1+\theta_2)) & 0 \\ 0 & -l(\sin(\theta_1)+\sin(\theta_1+\theta_2)) & -l\sin(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_0 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{x}_b \end{bmatrix} = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{z}_e \\ \dot{z} \end{bmatrix} \tag{51}$$

Singularities of J_a are found from

$$\mathrm{Det}(J_a) = l\times\cos(\theta_0)(\sin(\theta_1)+\sin(\theta_{1+}\theta_2))^2 \tag{52}$$

It is seen that J_a is singular when:

1: Base singularity occurs $\cos(\theta_0)=0 \Rightarrow \theta_0 = 270^\circ$ or $\theta_0 = 90^\circ$

2: Manipulator singularity occurs $\sin(\theta_1)+\sin(\theta_1+\theta_2)=0$

At the singular condition 1, the first and forth columns of J_a are multiplies, at singular condition 2 the first columns of J_a is zero and in both conditions J_a is not full rank. In comparison with the classical fixed base PUMA singularities, it is seen that the base mobility eliminates the elbow singularity $\sin(\theta_2)=0$. It means that, the arm is no longer singular when it is fully extended or fully folded. However, a new singularity has been introduced $\cos(\theta_0)=0$ when the end effector and the base have the same x-direction and is due to additional task variable β.

Table (1) Link parameters and inertia properties of the PUMA arm.

LINKS NUMBER	MASS (KG)	LENGTH (M)	MOMENT OF INERTIA (kg.m^2)	LINKS CENTER OF MASS REL.TO DISTAL JOINT (M)
1	12.0	a1=0.40	0.2 0 0 0 0.2 0 0 0 0	0.00 0.00 -0 20
2	10.0	a2=0.50	0 0 0 0 0.2 0 0 0 0.2	-0.25 0.00 0.1
3	5.0	a3=0.50	0 0 0 0 0.105 0 0 0 0.105	-0.25 0.00 0.00

Suppose that initially the load is at a point with coordinates $\{x_e = 0.50\ m, y_e = 0.20\ m, z_e = 0.30\ m)_i$ and it must reaches to final point with coordinates $\{x_e = 1.30\ m, y_e = 0.40\ m, z_e = 0.30\ m)_f$ at *T=2.4* sec. The base motion coordinate x_b is limited between *0 m* to *2 m* and joint limits of the manipulator are such that the first joint is free, second joint $-2\pi/3 \le \theta_1 \le 2\pi/3$, also θ_2 is controlled by the user specified additional task variable. The base mass is assumed to be 21 kg and manipulator link and inertia characteristics are shown in Table 1. The joint actuator constants are:

$$\vec{K}_1 = [40.62 \quad 10.02 \quad 30.0 \quad 6.67]\ \text{N or N.m}$$

$$[K_2] = \begin{bmatrix} 6.15 & 0 & 0 & 0 \\ 0 & 1.75 & 0 & 0 \\ 0 & 0 & 4.68 & 0 \\ 0 & 0 & 0 & 1.45 \end{bmatrix} \ \text{N.s/rad or N.m.s/rad}$$

Since the angle θ_2 is known priory as the additional task variable, therefore its joint and control variables are not considered on trajectory optimization analysis. Selecting m=50, the procedure for synthesizing the optimum trajectory converged after 8 iterations and m_{Load} $=4.38$ *kg* is found. Figure 3 gives the linear programming solution of the m_{Load} at iterations.

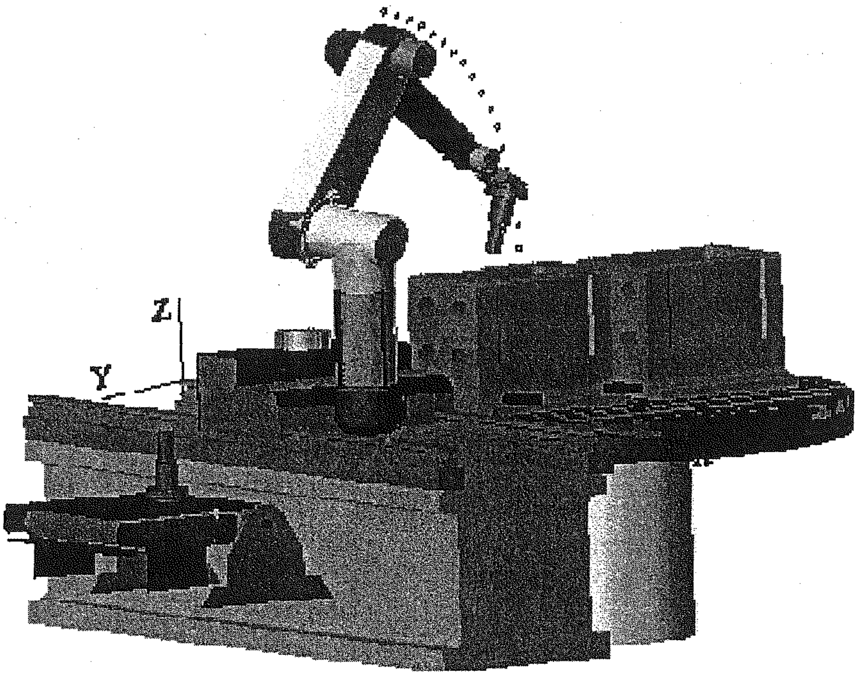

Figure (2) A view of linear tracked mobile PUMA manipulator.

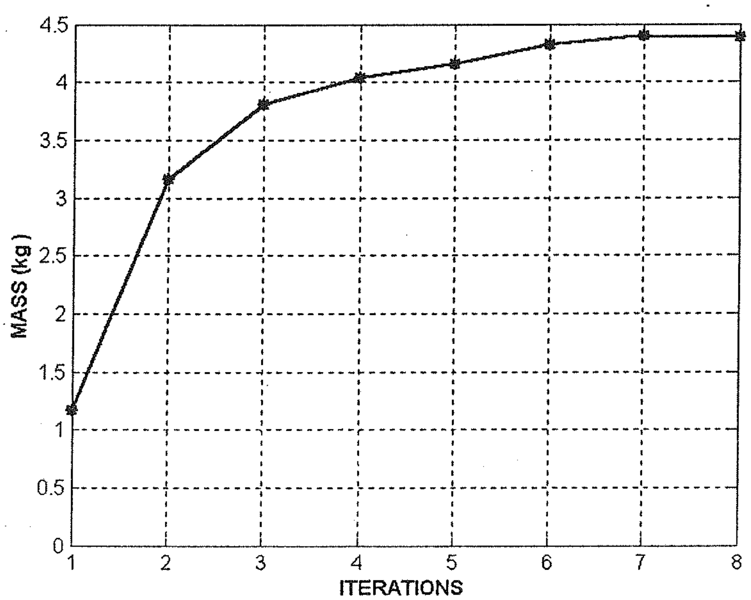

Figure (3) Optimal load carrying capacity at each iteration.

Figure 4 shows the initial and final optimal position and velocity of the base and manipulator's joints. Also Figure 5 shows the initial and final optimal path of the load on cartesian space, and Figures 6-8 are the optimal control trajectories.

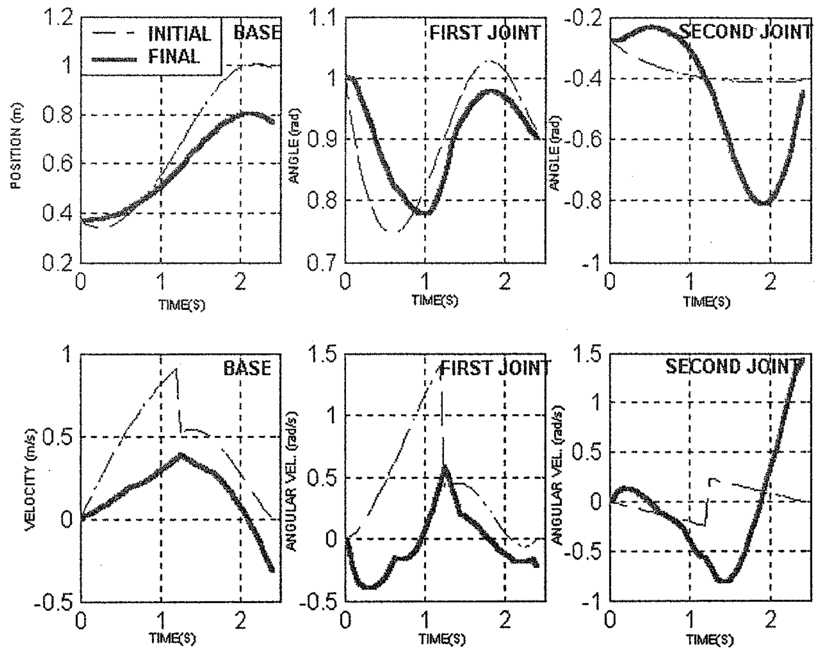

Figure (4) Joint positions and velocities at the initial and final optimal trajectories.

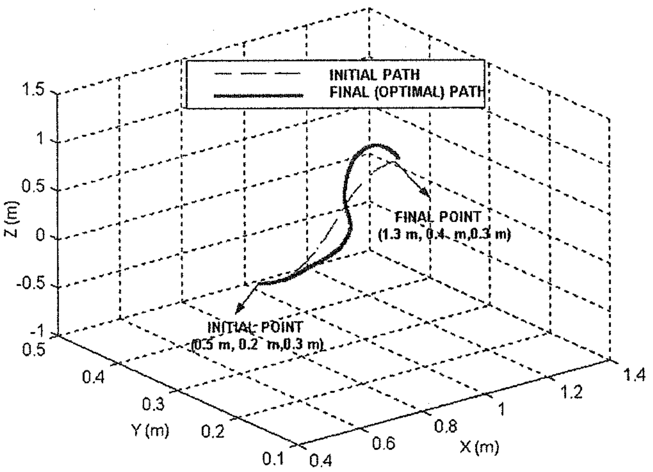

Figure (5) Load initial and optimal final paths and corresponding base path.

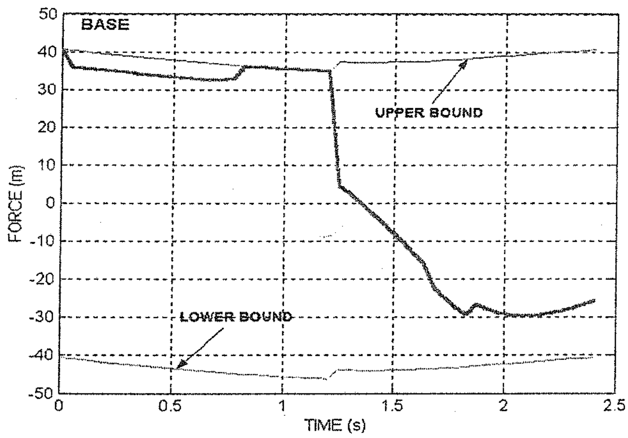

Figure (6) Actuator force at the base.

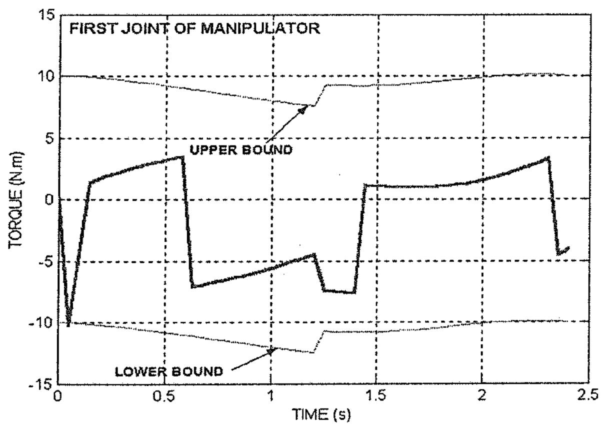

Figure (7) Actuator torque at the first joint of the manipulator.

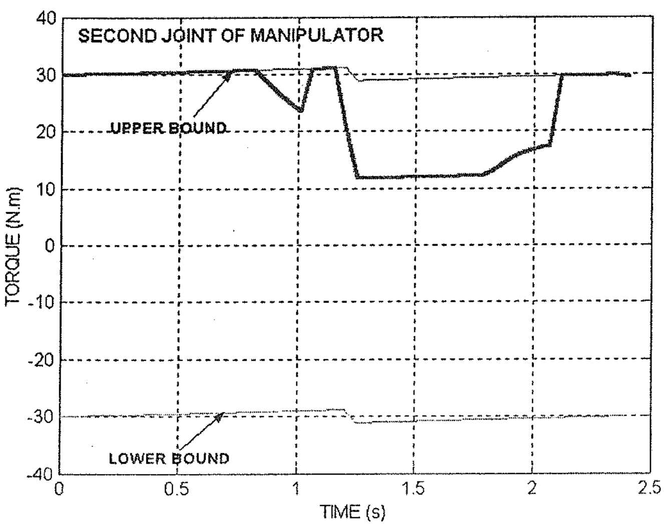

Figure (8) Actuator torque at the second joint of the manipulator.

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8-Conclusion

This paper presented a computational algorithm to find an optimal trajectory to increasing the load carrying capacity on mobile manipulators, for a given two end positions. This was achieved by considering additional kinematic constraints to redundancy resolution. A simulation study is presented to investigate the application and validity of the algorithm. It is seen, during the motion actuators are working with full or near to full capacity. A linear tracked base PUMA robot is used for simulation study. It is seen that, in the optimal trajectory the load carrying capacity is increased to $m_{Load} = 4.38\ kg$ from its initial value 1.17 kg.

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