Second Order Analysis of Steel Frames with Semirigid Connections Using Stability Functions

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Abstract

Prismatic steel beam-column of double symmetrical cross section with semirigid end connections was analyzed using second order analysis theory. For this purpose, the stability functions are utilized to form the stiffness matrix in which the effect of axial force on the flexural stiffness as well as the effect of flexural deformations on axial stiffness are involved. Since, in forming the stiffness matrix, the interaction between the axial and flexural forces on the corresponding stiffnesses are taken into account, performance of slender members can be properly evaluated. The effect of semirigid connections is included in the stiffness matrix by a power model. Finally, three examples are included in order to: (1) verify the validity of formulations, (2) compare the results obtained from the proposed second order analysis with those resulted from AISC/LRFD method, and (3) investigate the effect of semirigid connections on frame behaviour. The results are in an acceptable accordance with those obtained by others.

Keywords

Steel frames, Second order analysis, Semi-rigid connections, Stability functions

Introduction

In framed structures, the lateral deformation along each element ($P-\delta$ effect), the reletive displacement between the element ends ($P-\Delta$ effect), and the effect of flexural moments on axial stiffness result in geometric nonlinear behaviour. These three nonlinear sources alter not only the stiffness matrix, but the stability of each element and the whole structure. The geometric nonlinearity of framed structures can be evaluated by three means: (1) the second order finite element analysis, (2) using the stability functions method, and (3) the approximate methods based on moment amplification (such as the method proposed by AISC/LRFD 1994).

In the second order finite element analysis, iterative procedures are used to determine the induced axial load in each element. This method initially developed by Turner et al. (1960) and improved by other researchers (Martin and carey 1973). The most complete second order finite element formulation belongs to Yang and McMuire (1986a,b) whose papers comprise the formulation of stiffness matrices for a three dimensional thin-walled beam-column as well as a discusion about the lack of static equilibrium caused by the finite joint rotations. Even though various advanced finite element softwares are available, the stability and second order analyses of framed structures are still cumbersome and unpractical. This is due to the complexity of the structural modeling and the large number of degree of freedoms (DOFs) required to achieve a desired level of accuracy.

Second order analysis based on the classical stability functions (Timoshenko and Gere 1961) results in more exact solution in less time. In fact, in this method, less iteratives are needed, since the effect of axial force on the flexural stiffness is considered in the stiffness matrix. In the research conducted by Ekhande et al. (1989), the stiffness matrix for a 12 DOF beam-column with rigid connections was developed by the classical stability functions. The formulations contain the effect of shear deformations and the effect of interaction between the axial loads and flexural moments on the corresponding stiffnesses. Since then, the semirigid connections were approximately included in the above-mentioned stiffness matrix by Aristizabal-Ochoa (1997).

In the approximate methods, the nonlinear behaviour of frames is evaluated by applying amplification factors on the flexural moments obtained from the linear analysis (AISC/LRFD 1994, Chen and Lui 1987). However, this method can be merely used for rectangular frames with rigid connections.

The main objective of this study is to present a second order analysis method for steel structures using stability functions. The effect of semirigid connections is included in the stiffness matrix by a power model. The connection flexibility is a major issue in the nonlinear behavior of steel frames (Barakat and Chen 1991, Dhillon and O'Malley 1999). As a result of using the power model, semirigid connections are modeled directly and more accurately than the method proposed by Aristizabal-Ochoa (1997). Three examples are mentioned in order to: verify the validity of formulations, compare the results obtained from the proposed second order analysis with those resulted from AISC/LRFD method, and investigate the effect of semirigid connections on frame behaviour. The results are in an acceptable accordance with those obtained by others.

Modeling of semirigid connections

The typical moment-rotation curves of a number of conventional semirigid connections are shown in Fig. 1 (Xu and Grierson 1993). All types of connections exhibit a nonlinear moment-rotation behavior that lies between the two extreme cases of ideally simple and fully rigid connections.

The moment-rotation relationship of all types of semirigid connections is nonlinear, and varies depending on the connection flexibility. The stiffness or flexibility of a semirigid connection depends on the connection geometric parameters.

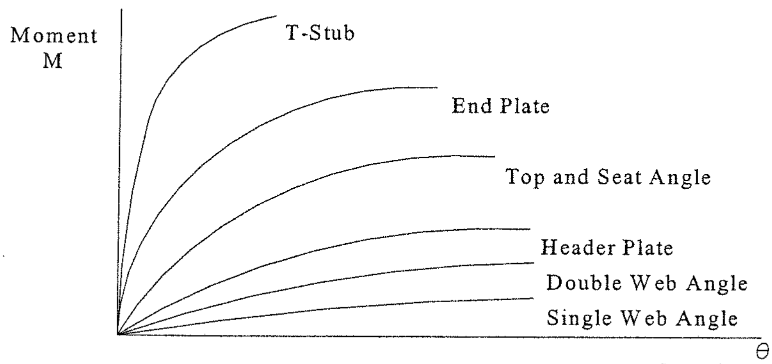


Figure (1) Moment-rotation curves of semirigid connections (Xu and Grierson 1993).

Among a great number of mathematical models for prediction of connection moment-rotation behavior, the power model presented by Kishi et al. (1987) is employed in this paper. This model has been widely utilized by researchers due to its convenience in calculation. The power model depends on the ultimate moment capacity, M_u, the initial connection stiffness, R_{ki}, and the shape factor, n, of each type of connections, as follow:

$$R_k = \frac{dM}{d\theta_r} = \frac{R_{ki}}{\left[1 + (R_{ki}\,\theta_r / M_u)^n\right]^{(n+1)/n}} \qquad (1)$$

in which M, θ_r and R_k are flexural moment, rotation and connection stiffness, respectively, at a certain time.

Structural model of beam-columns with semirigid connections

An elastic beam-column element connecting to two nonlinear flexural springs at its ends, springs AA' and BB', is assumed as shown in Fig. 2(a). The flexural moments, M_a and M_b, the shear forces, V_a and V_b, and the axial force, P, as well as the corresponding DOFs are depicted in Fig. 2.

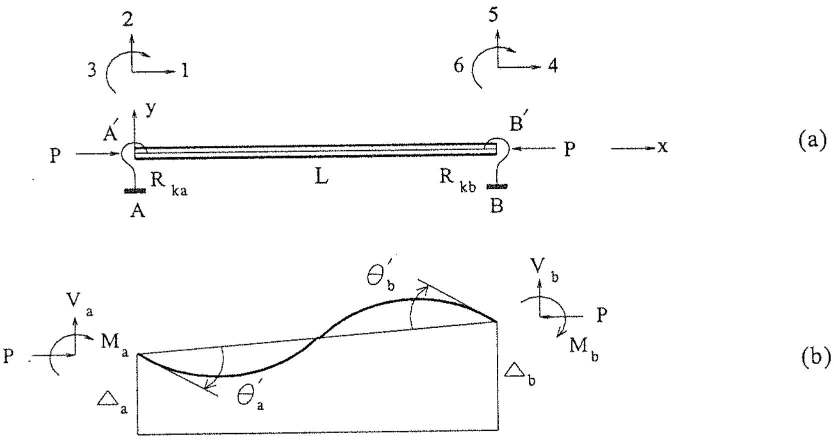


Figure (2) The elastic beam-column element, (a) the structural model of 2D prismatic element; (b) the end forces and DOFs.

Two dimensionless parameters, $k_a = R_{ka}/(EI/L)$ and $k_b = R_{kb}/(EI/L)$, representing springs stiffness, are defined; where E = modulus of elasticity, I = moment of inertia about the principal axis, L = element length, and R_{ka}, R_{kb} = flexural stiffness of springs AA' and BB', respectively.

Flexural stiffness matrix

The classical stability functions (Timoshenko and Gere 1961) are used to form flexural stiffness matrix of a prismatic beam-column element with rigid connections. Considering Fig. 2, these functions are as follows:

$$M_a = \frac{EI}{L}(r\theta'_a + s\theta'_b) \qquad (2a)$$

$$M_b = \frac{EI}{L}(s\theta'_a + r\theta'_b) \qquad (2b)$$

In which θ'_a, θ'_b are the reletive rotation between the beam-column and the end springs, AA' and

BB′, respectively (shown in Fig. 2(b)); and $u=\sqrt{|PL^2/EI|}$. In the case of compressive axial force ($P>0$):

$$r=\frac{u\sin u-u^2\cos u}{2-2\cos u-u\sin u} \quad ; \quad s=\frac{u^2-u\sin u}{2-2\cos u-u\sin u} \tag{3a}$$

and, for tensile axial force ($P<0$):

$$r=\frac{-u\sinh u+u^2\cosh u}{2-2\cosh u+u\sinh u} \quad ; \quad s=\frac{-u^2+u\sinh u}{2-2\cosh u+u\sinh u} \tag{3b}$$

Due to the end moments, M_a and M_b, rotational deformations are taken place in each spring with the value of $\theta_{ra}=M_a/R_{ka}$ and $\theta_{rb}=M_b/R_{kb}$. Since the flexural deformation induced in an element depends on its pure end rotations, the reletive rotations in Eq. (2), θ'_a and θ'_b, must be replaced by (θ_a-M_a/R_{ka}) and $(\theta_b=M_b/R_{kb})$, in which θ_a and θ_b are the overall rotations at each end; i.e.,

$$M_a=\frac{EI}{L}[r(\theta_a-\frac{M_a}{R_{ka}})+s(\theta_b-\frac{M_b}{R_{kb}})] \tag{4a}$$

$$M_b=\frac{EI}{L}[s(\theta_a-\frac{M_a}{R_{ka}})+r(\theta_b-\frac{M_b}{R_{kb}})] \tag{4b}$$

By solving Eq. (4) with respect to M_a and M_b, modified stability functions are resulted in:

$$M_a=\frac{EI}{L}(r_{ii}\theta_a+s_{ij}\theta_b) \tag{5a}$$

$$M_b=\frac{EI}{L}(s_{ij}\theta_a+r_{jj}\theta_b) \tag{5b}$$

where,

$$r_{ii}=\frac{1}{K}\left[r+(r^2-s^2)/k_b\right] \tag{6a}$$

$$r_{jj}=\frac{1}{K}\left[r+(r^2-s^2)/k_a\right] \tag{6b}$$

$$s_{ij}=\frac{s}{K} \tag{6c}$$

$$K=1+r(\frac{1}{k_a}+\frac{1}{k_b})+(r^2-s^2)\frac{1}{k_ak_b} \tag{6d}$$

By using Eq. (5), the 2D flexural stiffness matrix, [k], of the assumed beam-column is obtained as follow:

$$[k]=\begin{bmatrix} \frac{EI}{L^3}(r_{ii}+2s_{ij}+r_{jj}) & -\frac{EI}{L^2}(r_{ii}+s_{ij}) & -\frac{EI}{L^3}(r_{ii}+2s_{ij}+r_{jj}) & -\frac{EI}{L^2}(r_{jj}+s_{ij}) \\ & \frac{EI}{L}r_{ii} & \frac{EI}{L^2}(r_{ii}+s_{ij}) & \frac{EI}{L}s_{ij} \\ \text{SYM.} & & \frac{EI}{L^3}(r_{ii}+2s_{ij}+r_{jj}) & \frac{EI}{L^2}(r_{jj}+s_{ij}) \\ & & & \frac{EI}{L}r_{jj} \end{bmatrix} \tag{7}$$

Similar to the method described above, the 3D flexural stiffness matrix can be also derived.

Axial stiffness

In the absence of the flexural moment, the axial stiffness of the beam-column equals $k_{11}=k_{44}=-k_{14}=EA/L$. However, the axial stiffness is influenced by the flexural deformations about principal axes which produce additional axial deformations. Ekhande et al. (1989) has derived the modified axial stiffness, $s_1\,EA/L$, where

$$s_1=\frac{1}{1+\frac{EA}{4P^3L^2}H_z} \tag{8}$$

and H_z are given as follows:

-For $P>0$ (compression)

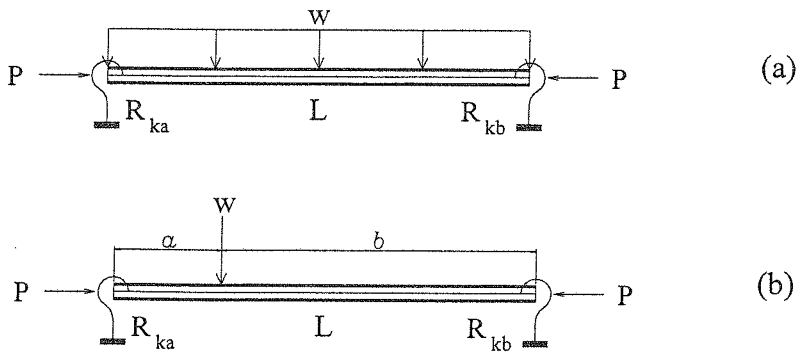
$$H_z=u(M_a^2+M_b^2)(\cot u+u\,\mathrm{cosec}^2 u)-2(M_a+M_b)^2 \\ +2uM_aM_b(1+u\cot u)\,\mathrm{cosec}\,u \tag{9a}$$

-For $P>0$ (tension)

$$H_z=u(M_a^2+M_b^2)(\coth u+u\,\mathrm{cosech}^2 u)-2(M_a+M_b)^2 \\ +2uM_aM_b(1+u\coth u)\,\mathrm{cosech}\,u \tag{9b}$$

External load vector

Two case of distributed and concentrated loads, as well as the effect of axial force on the fixed-end moment of a beam-column with rigid connection are studied by Beaufait et al. (1970). In the following, the corresponding results are pointed out, then the effect of semirigid connections is included in the formulations.



FIFigure (3) Beam-column element with semirigid connections; (a) under axial and uniform distributed loading; (b) under axial and concentrated loading.

The fixed-end moments of the beam-column with rigid connections (R_{ka} , $R_{kb} \rightarrow \infty$) due to uniformly distributed and concentrated loads are given in the following,

1-Uniformly distributed loading over the whole length of the element:

-For $P > 0$ (compression)

$$M'_{fa} = -wL^2[\frac{1}{2}(1-\cos u) + (\frac{\sin u}{u} - 1)(\frac{\sin u}{u} - \cos u) + (\frac{1-\cos u}{u})(\frac{1-\cos u}{u} - \sin u)] \Big/ (2 - 2\cos u - u\sin u) \qquad (10a)$$

$$M'_{fb} = -M'_{fa} \qquad (10b)$$

-For $P < 0$ (tension)

$$M'_{fa} = -wL^2[\frac{1}{2}(1-\cosh u) + (\frac{\sinh u}{u} - 1)(\frac{\sinh u}{u} - \cosh u) + (\frac{1-\cosh u}{u})(\frac{1-\cosh u}{u} + \sin u)] \Big/ (2 - 2\cosh u + u\sinh u) \qquad (10c)$$

$$M'_{fb} = -M'_{fa} \qquad (10d)$$

-and, for $P = 0$

$$M'_{fb} = -M'_{fa} = -wL^2/12 \qquad (10e)$$

2-Concentrated loading:

-For $P > 0$ (Compression)

$$M'_{fa} = -WL[a(1-\cos u) - (1-\cos au)(\frac{\sin u}{u} - \cos u) + (\frac{1-\cos u}{u} - \sin u)\sin au] \Big/ (2 - 2\cos u - u\sin u) \qquad (11a)$$

$$M'_{fb} = WL[b(1-\cos u)-(1-\cos bu)(\frac{\sin u}{u}-\cos u)+ (\frac{1-\cos u}{u}-\sin u)\sin bu] \Big/ (2-2\cos u - u\sin u) \quad (11b)$$

-For $P < 0$ (tension)

$$M'_{fa} = -WL[a(1-\cosh u)-(1-\cosh au)(\frac{\sinh u}{u}-\cosh u)+ (\frac{1-\cosh u}{u}+\sinh u)\sinh au] \Big/ (2-2\cos u + u\sin u) \quad (11c)$$

$$M'_{fa} = -WL[b(1-\cosh u)-(1-\cosh bu)(\frac{\sinh u}{u}-\cosh u)+ (\frac{1-\cosh u}{u}+\sinh u)\sinh bu] \Big/ (2-2\cos u + u\sin u) \quad (11d)$$

-and, for $P = 0$

$$M'_{fb} = -Wab^2L \; ; \quad M'_{fb} = Wa^2bL \quad (11e)$$

With the help of the fixed-end moments in the case of rigid connections, those related to the beam-column with semirigid connections can be readily derived.

The fixed-end moments M_{fa} and M_{fb} are those required to maintain the overall rotations at each end, θ_a and θ_b, equal to zero. Under a certain loading, the end springs rotate with the value of $\theta_{ra} = \theta_a - \theta'_a$ and $\theta_{rb} = \theta_b - \theta'_b$. Consequently, The fixed-end conditions are fulfilled when $\theta_{ra} = -\theta'_a$ and $\theta_{rb} = -\theta'_b$. Therefore, the fixed-end moments can be obtained by applying the principle of superposition, as follows:

$$M_{fa} = M'_{fa} - \frac{EI}{L}(r\theta_{ra} - s\theta_{rb}) \quad (12a)$$

$$M_{fb} = M'_{fb} - \frac{EI}{L}(s\theta_{ra} - r\theta_{rb}) \quad (12b)$$

By substituting $\theta_{ra} = M_{fa}/R_{ka}$ and $\theta_{rb} = M_{fb}/R_{kb}$ in Eq. (12) and solving resulted equations with respect to M_{fa} and M_{fb}, the fixed-end moments for the beam-column with semirigid connections can be obtained,

$$M_{fa} = \frac{1}{K}[(1 + r/k_b)M'_{fa} - (s/k_b)M'_{fb}] \quad (13a)$$

$$M_{fb} = \frac{1}{K}[(1 + r/k_a)M'_{fb} - (s/k_a)M'_{fa}] \quad (13b)$$

The end shear forces can be also obtained from the equilibrium conditions of the beam-column.

The local stiffness equations of each element are achieved by means of the stiffness matrix and the external loads vector determined earlier, as follow:

$$\{r\} = [k]\{u\} + \{r_f\} \tag{14}$$

where, $\{r\}$ = the end load vector, $\{u\}$ = the end deformations vector, and $\{r_f\}$ = the fixed-end vector, of each element. After transferring the local stiffness matrix and external load vector of each element to global coordinates, the stiffness equations of the whole structure are obtained by superimposing the global stiffness matrices and external load vectors of all the elements,

$$\{R\} = [S]\{U\} + \{R_f\} \tag{15}$$

where, $\{R\}$ = the structural load vector, $\{U\}$ = the structural deformation vector, and $\{R_f\}$ = the structural fixed-end load vector.

Procedure of numerical analysis

In order to analyse the framed structures with rigid connections by aforementioned method (by making use of Eq. 15), the loads as a whole can be applied instantaneously contrary to the methods based on finite element analysis, because the nonlinear issues including the effect of axial load or flexural moment on the stiffness matrices and the external load vectors are involved. For this respect, firstly, the framed structure is analysed linearly, then the corresponding results are taken into account to modify previous stiffness matrices and the external load vectors. The structural nonlinear behavior is obtained by iterative analyses and successive modification of current stiffness matrices and the external load vectors using the previous results, until attaining the required convergence. It should be noted that, in this case, it is not necessary to modify successively joints coordinates. By means of this method the structural nonlinear solution can be achieved in a relativly short time. However, in the presence of semirigid connections, because of the connections nonlinearity, The external loads must be divided into a number of small load increments, leading to a series of incremental structural stiffness as follow:

$$\{\Delta R\} = [S]\{\Delta U\} + \{\Delta R_f\} \tag{16}$$

where, $\{\Delta R\}$ = the incremental structural load vector, $\{\Delta U\}$ = the incremental structural deformation vector, and $\{\Delta R_f\}$ = the incremental structural fixed-end load vector. The analysis procedure can be summarized step by step as follows:

1-Dividing the external loads into a number of small load increments.

2-Calculation of the incremental structural load vector, $\{\Delta R\}$.

3-Calculation of the local stiffness matrix for each element and transferring to the global coordinates.

4-Assembling the resulted stiffness matrices in order to calculate the overall structural stiffness matrix, $\{S\}$.

5-Solving the incremental structural stiffness equation (Eq. 16) and calculation of $\{\Delta U\}$.

6-Calculation of fixed-end load vector for each element.

7-Calculation of the connection tangent stiffness by Eq. (1).

8-Modifying current stiffness matrices and the external load vectors using the recent results.

9-Modifying current connection tangent stiffness as shown in Fig. 4.
10-Repeating steps 1-9 until reaching the required convergence.
11-Repeating steps 1-10 for all the load increments.
12-Calculation of accumulated displacements and member forces.

By introducing the nonlinear effects in the stiffness matrices and the external load vectors, it is not needed to divide the external forces to very small load increments, resulting in reletively less iteratives than the method in which the nonlinear parameters are not involved. In addition, in this method, the joints coordinates are not required to modify successively.

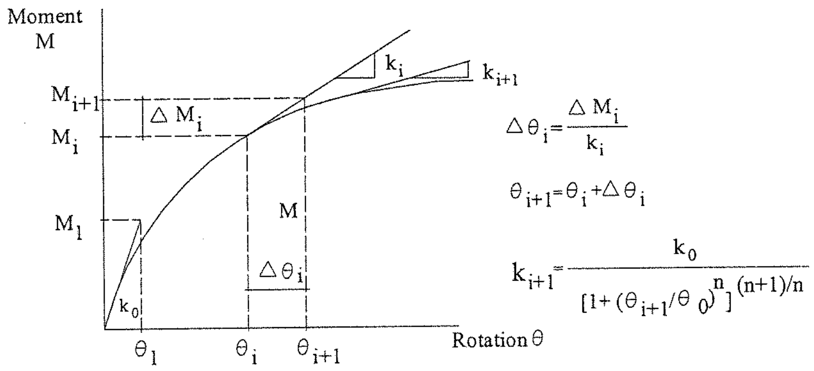


Figure (4) Connection tangent stiffness and its modification procedure.

Examples

In the following, three examples are included for three purposes: (1) verifying the validity of formulations, (2) comparing the results obtained from the proposed second order analysis with those resulted from AISC/LRFD method, and (3) investigating the effect of semirigid connections on frame behavior.

Example 1- One story-one bay frame with rigid connections

A one story-one bay frame with fixed supports and rigid connections is assumed as shown in Fig. 5. It is supposed that all the elements are of elastic material with modulus of elasticity of $E = 30{,}000$ ksi and with the following geometric properties: $A = 8\ \text{in}^2$ and $I = 250\ \text{in}^4$. the frame is considered to be continuously braced in the transverse direction.

The given frame was analysed by the presented method which contains the interaction between the axial and flexural forces on the corresponding stiffnesses. After three iterative structural solutions, an acceptable convergence was achieved. In Table 1, the results obtained from the proposed method are compared with linear solutions and the nonlinear solutions given by Beaufait et al. (1970).

The negligible differences between the results from the proposed method and those from the nonlinear analysis by Beaufait et al. (1970) are due to the effect of the flexural deformations on the axial stiffness which is ignored in the latter analysis. However, there are major discrepancies between the linear and two nonlinear results. The differences reach about 20% that indicate nonlinear sources can significantly affect the structural analysis.

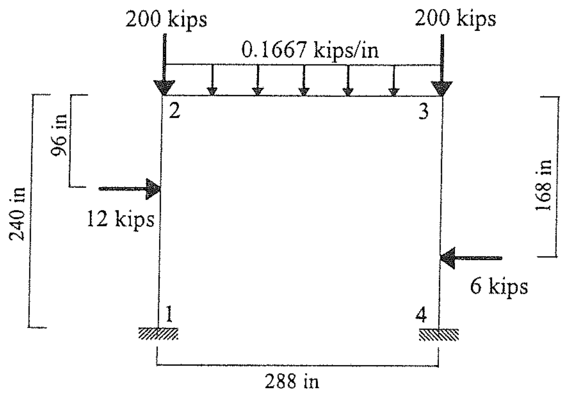


Figure (5) One story-one bay frame with rigid connections.

Table (1) Linear and nonlinear moments (in.-kips)-Example 1.

Joint No.	Linear analysis	Nonlinear analysis (Beaufait et al. 1970)	Nonlinear analysis (proposed method)
1	388.41	480.24	485.58
2	697.73	610.32	607.37
3	1075.26	1153.32	1150.30
4	530.05	653.40	647.76

Example 2- Second order analysis of two story-one bay frame and comparison with AISC/LRFD (1994)

A two story-one bay frame with the prorerties shown in Fig. 6 , is considered. The frame is subjected to the external loads which reach five times the design loads given in Fig. 6.

The maximum moment of each column was computed by the proposed method and reported in Table 2. The results presented by Goto and Chen (1987) and the amplified moments based on AISC/LRFD (1994) are given in Table 2 ,as well. In the approach employed by Goto and Chen (1987), nonlinear sources were estimated by a Taylor series expansion, and as a consequent, only first few terms were involved in the formulations. It should be noted that all the results are normalized by those corresponding to the linear analysis.

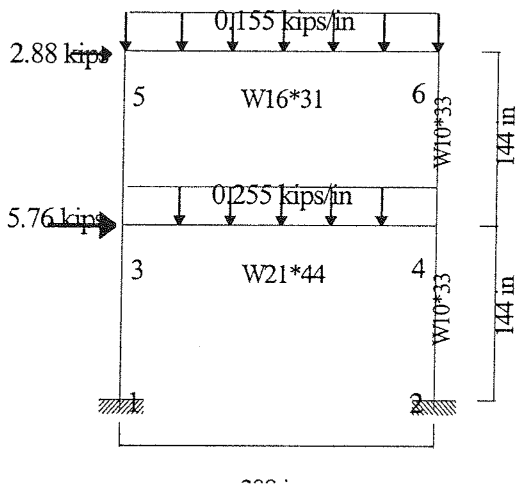


Figure (6) two story-one bay frame with rigid connections.

As observed in Table 2, the maximum moments obtained from the proposed method and those reported by Goto and Chen (1987) are approximately the same, whereas they differ rather significantly from the results based on AISC/LRFD (1994).

Table (2) Maximum normalized moment of columns - Example 2

Applied load/design load	Nonlinear analysis (Goto and Chen 1987)	Nonlinear analysis (proposed method)	Moment amplification (LRFD 1994)
(a) The first story (joint no. 4)			
1	1.006	1.005	1.010
2	1.012	1.012	1.021
3	1.019	1.018	1.032
4	1.027	1.026	1.044
5	1.036	1.034	1.057
(b) The second story (joint no. 6)			
1	1.005	1.004	1.002
2	1.009	1.008	1.005
3	1.014	1.012	1.008
4	1.019	1.019	1.010
5	1.025	1.021	1.013

The noteworthy drawback with the AISC/LRFD (1994) method is that the amplified moments based on this method does not always provide enough safety factor, comparing with nonlinear solutions which are supposed to be more closer to reality. As an example, in the first story, the amplified moments obtained by AISC/LRFD (1994) were overestimated as compared with the nonlinear solutions. However, in the second story, underestimate solutions are resulted in.

Example 3- Second order analysis of two story-one bay frame with semirigid connections

In order to investigate the effect of semirigid connections on the nonlinear behavior of framed structures, a two story-one bay frame with semirigid connections, used by King and Chen (1993), is employed as shown in Fig. 7(a). The double bolted web angles whose moment-rotation relationship is plotted in Fig. 7(b), is used in this example. This type of flexible semirigid connection was modeled by Eq. (1) with the relating parameters given in Fig. 7(b).

Two linear analysis were carried out by replacing the semirigid connections with the simple and rigid connections. The framed structure with semirigid connections was also analysed nonlinearly by the proposed method. The corresponding results from the three types of analyses as well as the nonlinear solutions by King and Chen (1993) are compared in Table 3.

Table (3) Maximum moments of each element (in.-kips)-Example 3.

Elem.	Linear solution (Rigid connections)	Linear solution (simple connections)	Nonlinear analysis (King and Chen 1993)	Nonlinear analysis (proposed method)
1-3	88	433	176	173
2-4	324	431	278	267
3-5	283	108	188	177
4-6	400	108	343	326
3-4	673	1037	728	720
5-6	400	726	461	472

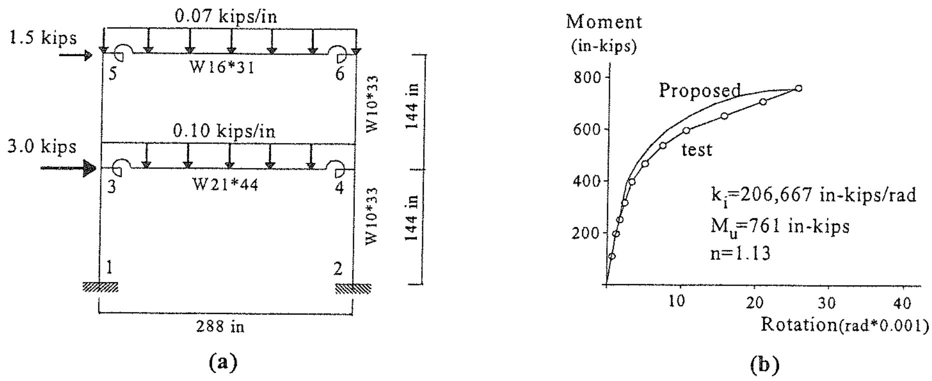


Figure (7) (a) two story-one bay frame with semirigid connections; (b) connection moment-rotation relationship

The discrepancies between the date from the proposed method and that from the nonlinear analysis by King and Chen (1993) arise from the differences in the semirigid connection modeling and the differences in the nonlinear parameters included in each method. But, by comparing the nonlinear solutions with the linear analyses with the assumption of simple and rigid connections, it is recognized that the semirigid connections influence noteably the structural behavior. It seems the conventional analyses in which the flexible semirigid connections are replaced with the simple connections, may be invalid.

Conclusions

Second order analysis of a prismatic steel beam-column of double symmetrical cross section with semirigid end connections is derived. For this regard, the stability functions were utilized to form the stiffness matrix in which the effect of axial force on the flexural stiffness and the effect of flexural deformations on axial stiffness are involved. The semirigid connections is included in the stiffness matrix by a power model. Through the examples described in this study, the following remarkable concerns are pointed out:

1-The result obtained from the proposed method are in an acceptable agreement with those by other researchers.

2-There are sometimes major discrepancies between linear and nonlinear solutions that indicate nonlinear sources can significantly affect the structural behavior and they should be taken into account..

3-The moments obtained from the nonlinear analyses differ rather significantly from the results based on AISC/LRFD (1994). The AISC/LRFD (1994) method does not always provide enough safety factor.

4-The semirigid connections influence noteably the structural behavior. It seems the conventional analyses in which the flexible semirigid connections are replaced with the simple connections, may be invalid.

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