-As shown by experimental and analytical results, temprature variations are limited during charging process.

#### References

- [1] M. Gonzales, M. A. Perez, A. Diaz, F.J. Ferrero, Ni-Cd and Ni-MH battery optimized fast-charge method for portable telecommunication applications, 522-529, IEEE Proceeding of INTELEC (1996).
- [2] K. E. Yeager, Electric vehicles and solar power: enhancing the advantages of electricity, IEEE Power Engineering Review, Vol. 12, No. 10 (1992).
- [3] T. Robbins, Fuse model for over-current protection simulation of DC distribution systems, 336-340, Proceedings of the INTELEC (1978).
- [4] T. Robbins, J. Hawkins, Battery model for over-current protection simulation of DC distribution systems, 307-314, 16<sup>th</sup> International Telecom. Energy Conference (1994).
- [5] J. A. Gow, C. D. Manning, Photovoltaic converter system suitable for use in small scale stand-alone or grid connected applications, IEE Proc. Elec. Power Appli., No. 6, pp. 535-543 (2000).
- [6] E. Van Dijk, J. B. Klaassens, H. J. N. Spruijt, D. M. O'Sullivan, Battery charge design for the columbus MTFF power system, IEEE Trans. on Aerospace and Electronic Systems, Vol. 33, pp. 29-37 (1997).
- [7] J. R. Delaney, J. R. Lippert, W. L. Herald, Air force armament laboratory battery power supply operation and maintenance, IEEE Trans. on Magnetic, Vol. 27, No. 1 (1991).
- [8] Y. H. Kim, H. D. Ha, Design of interface circuit with electrical battery models, IEEE Trans. on Industrial Electronics, Vol. 44, No. 1, pp. 81-86 (1997).
- [9] J. P. Kajs, R. C. Zowarka, Bus high current battery model, IEEE Trans. on Magnetics, Vol. 29, No. 1, pp. 1003-1008 (1993).
- [10] R. Nailen, Battery protection-where do we stand?, IEEE Trans. on Industry Applications, Vol. 27, No. 4, pp. 658-667 (1991).
- [11] D. Fan, R. E. White, A mathematical model of a sealed Nickel-Cadmium battery, Journal of Electrochemical Society, Vol. 138, No. 1 (1991).
- [12] D. Fan, R. E. White, Mathematical modeling of a Nickel-Cadmium battery: effects of intercalation and oxygen reaction, Journal of Electrochemical Society, Vol. 138, No. 10 (1991).
- [13] P. D. Vidts, R. E. White, Mathematical modeling of a Nickel-Cadmium cell: Proton diffusion in the nickel electrode, Journal of Electrochemical Society, Vol. 142, pp. 1509-1519 (1995).
- [14] N. Kato, K. Yamamoto, Estimation of the capacity of Nickel-Cadmium batteries by measuring impedance using electrolyte-deficient battery characteristics, pp. 772-777, IEEE Proceedings of INTELEC (1995).
- [15] A. F. Burke, A method for the analysis of high power battery designs, 2252-2257, Proceedings of the 32<sup>nd</sup> Intersociety Energy Conversion Engineering Conference (1997).
- [16] B. Y. Liaw, Numerical simulation of coupled electrochemical and transport processes in battery systems, Vol. 2, 899-903, Proceedings of the 32<sup>nd</sup> Intersociety Energy Conversion Engineering Conference (1997).
- [17] D. Linden, Handbook of batteries, McGraw-Hill, New York, Chapter 28 (1994).
- [18] M. A. S. Masoum, M. Ghassemi, Electrical and thermal analysis of an environmentally safe microprocessor-based Ni-Cd fast battery charger, Accepted for Publication in International Journal of Environmentally Conscious Design and Manufacturing.
- [19] C. K. Krishnaprakas, Efficient solution of spacecraft thermal models using preconditioned conjugate gradient methods, Journal of Spacecraft and Rockets, Vol. 35, No. 6 (1998).
- [20] A. Bejan, Heat transfer, John Wiley & Sons Inc. (1993).

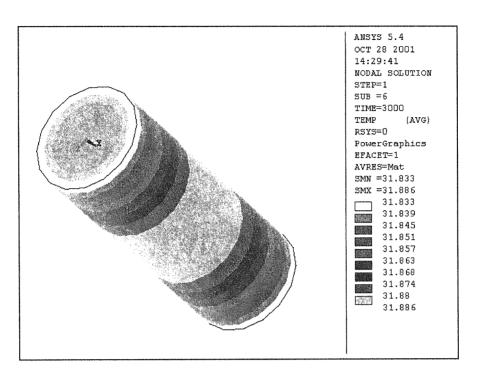


Figure (9) Thermal contour generated by ANSYS.

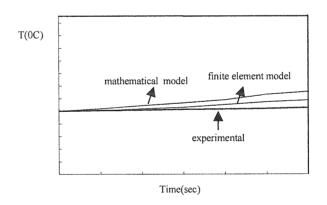


Figure (10) Comparison of simulated (Fig.5 and Eqs.4-6) and measured (Figure7) battery temprature for a charge rate of  $I_c = 3.5A$ .

#### 6- Conclusion

Electrical and thermal behaviours of Ni-Cd batteries are investigated. Based on the computed and measured results, the following conclusion are made:

- -Due to electro-chemical characteristics of Ni-Cd batteries, parameters of electrical model are linear and nonlinear functions of charge current.
- -Linear and nonlinear dependencies of electrical parameters are simulated using measured characteristics at two and three different charge rates, respectively.
- -Comparison of computed and measured results shows good agreements and demonstrates the high accuracy of the proposed electrical and thermal models for a 7Ah Ni-Cd battery.

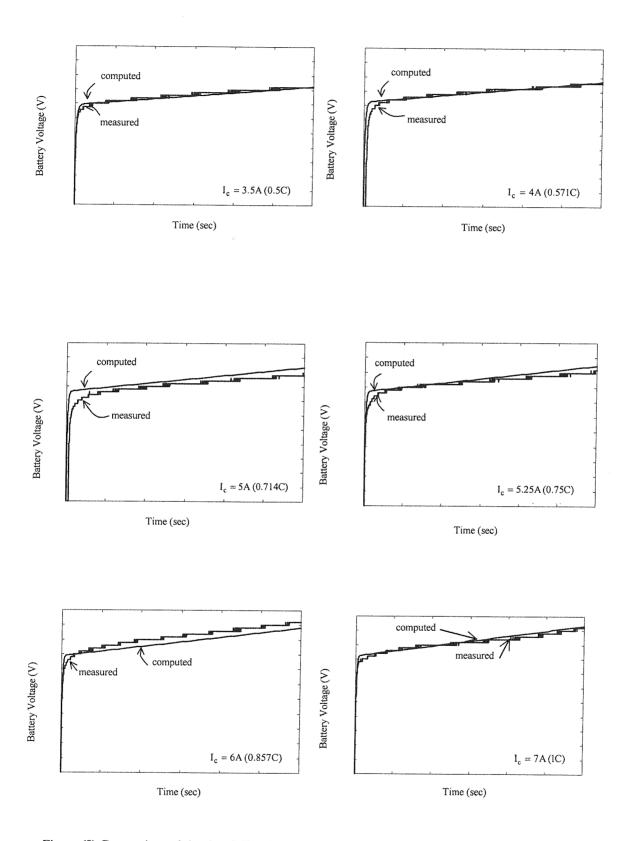


Figure (8) Comparison of simulated (Fig.4) and measured (Fig.7) battery voltages at different charge rates.

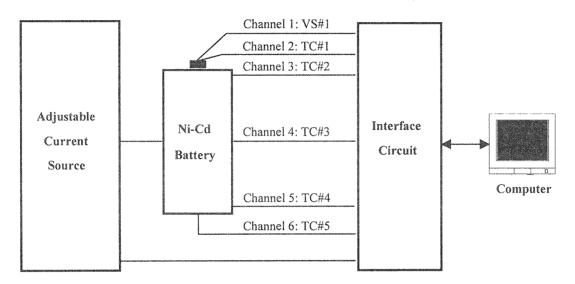
$$\overset{\bullet}{Q} = V I (1 - \eta) \tag{6}$$

where  $\eta$  is battery efficiency.

A finite difference code is developed and used for the solution of instantaneous sets of equations (see Fig.6).

# 4- Measuring Electrical and Thermal Characteristics of Ni-Cd Battery

In order to validate proposed electrical and thermal models and to investigate their accuracy, an experimental setup consisting of an adjustable current source, a personal computer, an interface board, voltage and temprature sensors and a 7AH Ni-Cd battery is used as shown in Figure 7.



TC: Thermocouple, VS: Voltage Sensor

Figure (7) The experimental setup used to measure battery electrical and thermal characteristics.

# 5- Comparison of Simulated and Measured Results

Comparison of computed and measured results are presented for two main reasons:

-to validate the proposed electrical model at charge currents used for computing battery parameters (e.g., 3.5A and 7A),

-to investigate the accuracy of electrical and thermal models at other charge currents.

As demonstrated by figure 8, computed and measured battery voltages show very good agreements for six values of charge currents. Similar results were noticed at other charge currents. A maximum error of 5.5% is noticed at the knee of battery voltage for  $I_C = 5A$ .

Thermal behaviour of battery is depicted by Figs.9-10. Figure 9 shows the thermal contour generated by the finite element code, ANSYS. As shown, the temperature stays at a constant value (31  $\pm$  1 °c). Results of lumped parameter model are depicted by Figure 10 which also indicates a constant temperature around 31  $\pm$  1 °c. Comparison of results obtained from lumped parameter method, ANASYS, and experimental analysis are compared in Figure 10. As shown by these figures, all results are in good agreements.

node is required [18]. The energy equation for each node is given by:

$$m C_p \frac{dT_W}{dt} = hA(T_{\infty} - T_W) + Q$$
 (4)

where

m = mass of battery, Kg

 $C_P$  = heat capacity, KJ/Kg K

Q = power generation, W

 $A = battery area, m^2$ 

 $h = convective coefficient, W/m^2K$ 

 $T_{\infty}$  = environment temperature, K

t = time, sec

 $T_W$  = battery surface temperature, K

To solve energy equation, we need to compute values for convective coefficient and power generation [19-20]. The convective coefficient is calculated by:

$$h = \frac{k_f}{D} \left\{ 0.6 + 0.387 \left[ \frac{Gr \, Pr}{1 + (0.559 / Pr)^{\frac{9}{16}}} \right]^{\frac{1}{6}} \right\}^2$$
 (5)

where

$$Gr = Grashof number = \frac{g\beta(T_W - T_{\infty})l^3}{v^2}$$

 $T_f\!=\!$  final temperature =  $(T_W+T_\infty)/2$  , K

D = battery diameter, m

 $K_f = air thermal conductivity, W/mK$ 

$$\beta = 1/T_f$$
,  $K^{-1}$   
 $\nu = (0.1014 T_f - 14.73)(10)^{-6}$ ,  $m/s^2$   
 $l = battery height, m$ 

 $Pr = -0.00022T_f + 0.774$ 

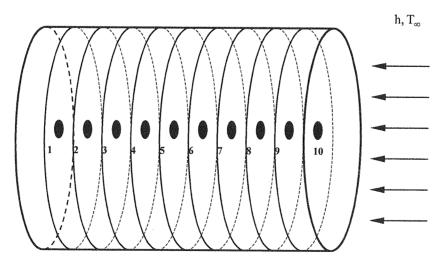


Figure (6) Nodal schematic of the Ni-Cd battery used for thermal and electrical analysis.

Measured voltage (V) and current (I) amplitudes calculate power generation as follows:

# 3- Thermal Analysis of Ni-Cd Battery

As known, a battery operates in a narrow allowable temperature range. If the temperature of Ni-Cd battery increases beyond its limit, it may cause temporary or permanent damages to the battery. Thus, thermal consideration becomes an essential issue in battery applications. Important design parameters for battery are; thermo-physical and optical properties, interface conductance, internal heat dissipation, external heat flux and etc. A Ni-Cd battery is made up of a Ni-Cd Elctrolyte type electrode, which is covered by a layer of nickel. The dimension and thermo-physical property of the 7Ah Ni-Cd battery under consideration is given in Table 3.

Table (3) Dimension and thermo-physical property of the Ni-Cd battery used for the analysis [17].

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Parameter	Value
L [mm]	33
D [mm]	88
M [gr.]	250
K <sub>Ni</sub> [W/mK]	58
K <sub>Cd</sub> [W/mK]	94
$\rho_{\rm e}\left[\Omega.m\right]$	2.1x10 <sup>-8</sup>
$\rho_{Ni}[kg/m^3]$	8900
$\rho_{Cd}$ [kg/m <sup>3</sup> ]	8600
Cp <sub>Ni</sub> [kj/kgK]	448
Cp <sub>Ni</sub> [kj/kgK]	232

#### 3-1- Finite Element Model

To analyze the battery, 2750 hexagonal type elements with uniform mesh distribution are used (Fig.5). These elements are part of the physical structure and analytical model. The exterior surface temperature is at 30° and Eq.4 is utilized for heat transfer coefficient (h). The rate of charge is C/2 and battery efficiency is assumed 90 percent. Then, the boundary condition, thermo-physical properties and heat generation are incorporated into the model and a finite element based code, ANSYS, is utilized to analyze the battery.

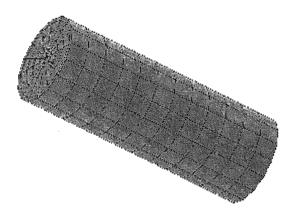


Figure (5) Finite element schematic of the Ni-Cd battery used for electrical and thermal analysis.

# 3-2- Mathematical Lumped Parameter Model

This method is utilized because the Biot number is low (e.g., less than 0.01). In this method, the battery is divided into 10 nodes as shown by Fig.6. Earlier studies show that no more than 10

Table (1) Computed model parameters (Fig.2) for the 7Ah Ni-Cd battery, used for analysis.

I <sub>C</sub> [A]	3.5	5.25	7
$R_s[\Omega]$	0.046	-	0.0314
$R_p[\Omega]$	0.0529	0.0347	0.0273
$C_{s}[F]$	159091	-	307.7
C <sub>p</sub> [F]	285.7	-	307.7

Table (2) Computed parameters for a 7AH Ni-Cd battery model (Eq.2) at  $\, {\rm I}_{\circ} = 3.5 {\rm A} \, .$ 

Parameter	Value	
$K_{R_s}[\Omega/A]$	-0.0042	
$K_{C_s}[\Omega/A]$	1088.3	
$K_{C_p}[F/A]$	6.2857	
$R_{S_o}[\Omega]$	0.046	
C <sub>s<sub>o</sub></sub> [F]	159091	
C <sub>Po</sub> [F]	285.7	
$K_{R_{pl}}[\Omega/A^2]$	0.0018	
$K_{R_{\mathfrak{p}^2}}[\Omega/A]$	-0.0258	
$K_{R_{p3}}[\Omega]$	<sub>p3</sub> [Ω] 0.1217	
V <sub>o</sub> [V]	1	

#### 2-2- Electrical Simulation

The proposed model of Fig.2 is simulated using SIMULINK facility of MATLAB software package. In order to consider the linear and nonlinear dependencies of battery voltage with charge current, measured model parameters (Tables 1 and 2) are used in simulations.

Figure 4 shows model diagram in SIMULINK. The desired charge current is generated by a controllable current source. Since parameters of battery model change with charge current, a MATLAB subroutine generating  $R_p$ ,  $C_p$ ,  $R_s$  and  $C_s$  is linked with SIMULINK diagram. In order to simulate a battery characteristics at different charge currents, the value of  $I_c$  is changed. Computed waveforms are shown in Figure 8.

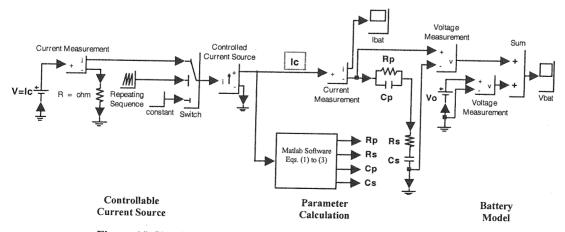


Figure (4) Simulation of Ni-Cd battery (Fig.2 and Eqs.1 to 3) by SIMULINK.

where  $I_c$  is charge current,  $I_s$  is the biasing current level for computing parameters and  $R_{so}$ ,  $C_{so}$  and  $C_{po}$  are the values of  $R_s$ ,  $C_s$  and  $C_p$  at  $I_o$ , respectively. Constants of  $K_{R_s}$ ,  $K_{c_s}$ ,  $K_{c_p}$  are obtained from measurements at two charge rates (e.g., C/2 and C) while  $K_{R_{pl}}$ ,  $K_{R_{p2}}$  and  $K_{R_{p3}}$  are computed from measured characteristics at three different charge rates (e.g., C/2, 3/4C, C). In order to compute parameters of Eq.2, the following steps are taken:

Step 1: Compute  $\Delta V_{R_s}$  and  $\Delta V_{C_s}$  from measured characteristics (Fig.1) at charge rates of C/2 and C.

Step 2: Compute  $\Delta V_{R_p}$  from measured characteristics (Fig.1) at three different charge rates (e.g., C/2, 3/4C and C). Figure 3 demonstrates how these computations are performed for a biasing current  $I_o = 3.5A$ .

Step 3: Compute parameters of Eq.2 at different charge currents using:

$$R_{s} = \Delta V_{R_{s}} / I_{c}$$

$$C_{s} = I_{c} \Delta t / \Delta V_{C_{s}}$$

$$R_{p} = (\Delta V_{R_{p}}) \Big|_{max} / I_{c}$$

$$\Delta V_{R_{p}} (t = \tau_{p}) = (1 - exp(-1)) \Delta V_{R_{p}} \Big|_{(max)}$$

$$C_{p} = \tau_{p} / R_{p}$$
(3)

These parameters are listed in Table 1 for a 7Ah Ni-Cd battery manufactured by SANYO.

Step 4: Calculate battery parameters of Fig.2 by solving the set of linear and nonlinear equations of Eq.2 and Table 1.

For the 7Ah Ni-Cd battery under consideration, parameters of Fig.2 are computed using Equations.1-3 and are shown in Table 2.

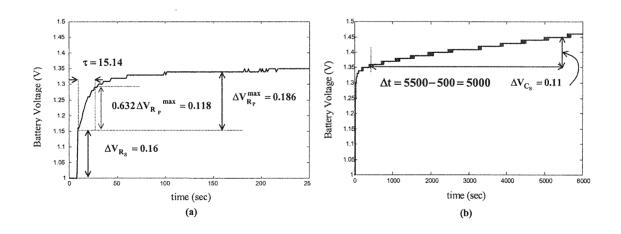


Figure (3) Extraction of Ni-Cd battery parameters from measured voltage waveform (Eq.3) at a charge current of 3.5A; (a) starting characteristic, (b) total characteristic.

battery voltage can be divided to four terms. A voltage step  $\Delta V_{R_s}$  that can be modeled with a resistor in series with the step current, an exponential voltage  $\Delta V_{R_p}$  that can be modeled as a RC parallel network, a linear voltage drop  $\Delta V_{C_s}$  and a constant voltage  $V_{\circ}$ , as shown in Figure 1. Therefore, battery voltage can be computed as:

$$V_{bat}(t) = V_{o} + \Delta V_{R_{s}}(t) + \Delta V_{R_{p}}(t) + \Delta V_{C_{s}}(t)$$
(1)

Based on Eq.1, a nonlinear battery model is proposed as shown in Fig.2. The nonlinear nature of the model is due to the dependency of its parameters  $(R_p, C_p, R_s \text{ and } C_s)$  on charge current  $(I_c)$ .

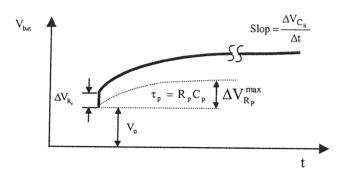


Figure (1) Typical step response of a Ni-Cd battery.

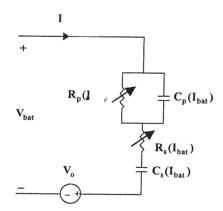


Figure (2) The proposed nonlinear model for Ni-Cd batteries with variable parameters (Eq.2).

## 2-1- Computing Model Parameters

Parameters of Fig.2 are extracted from measured characteristics at different charge rates. Measurements show that  $R_s$ ,  $C_s$  and  $C_p$  are linear functions of charge current, while  $R_p$  changes nonlinearly with the rate of charge. Therefore,

$$R_{s}(I_{c}) = R_{so} + K_{R_{s}}(I_{c} - I_{o})$$

$$C_{s}(I_{c}) = C_{so} + K_{C_{s}}(I_{c} - I_{o})$$

$$C_{p}(I_{c}) = C_{po} + K_{C_{p}}(I_{c} - I_{o})$$

$$R_{p}(I_{c}) = K_{R_{p_{1}}}I_{c}^{2} + K_{R_{p_{2}}}I_{c} + K_{R_{p_{3}}}$$
(2)

# Electrical and Thermal Analysis of Ni-Cd Batteries

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### **Abstract**

A new nonlinear model for electrical analysis of Ni-Cd batteries is proposed. Model parameters are considered to be linear and nonlinear functions of charge current. To generate these parameters, battery characteristics of different charge currents are measured and used. Linear dependency of parameters are generated using two charge rates while nonlinear variations are modeled based on three charge currents. Thermal characteristics are computed with results obtained by a finite element based code, ANSYS, and a generated finite difference (lumped parameter) model. For experimental analysis, a labratory setup capable of measuring electrical and thermal behaviour of Ni-Cd batteries at different charge rates is used. The Computed and measured results show good agreements for a 7Ah, size F, Ni-Cd battery, manufactured by SANYO.

## Keywords

Ni-Cd, Nonlinear Model, Electrical, Thermal.

#### Introduction

The ultra-fast charging capability, fine performance and high capacity of Nickel Cadmium batteries along with their limited weight and size are very attractive and have distinct properties for many light and compact equipment. They have found many applications including cordless and portable devices, emergency and standby power, telecommunication equipment, photovoltaic systems, electric vehicle, satellites and spacecraft and power plant supporting equipment [1-7].

In order to use Ni-Cd batteries more efficiently, their response to various operating conditions must be understood. There are urgent needs to optimize battery performance and to further understand their characteristics and limitations in the processes of scaling up and manufacturing. There is also a renewed interest in optimizing charging algorithms for rapid and safe charging techniques to enhance battery performance and life time. Numerical battery modeling is a very cost-efficient approach to address these requirements. Numerous papers have been published on battery models [8-16]. However, most of them ignore the nonlinear electrochemical characteristics and thermal properties of the battery.

The subject of this paper is to introduce nonlinear electrical and thermal models for Ni-Cd batteries and to investigate their accuracy through different measurements.

## 1- The Proposed Electrical Model for Ni-Cd Batteries

In this section, a new nonlinear electrical model for Ni-Cd batteries is presented. A typical Ni-Ni-Cd battery response to a step current is shown in Figure 1. Based on this characteristic,