[7] Deuermeyer, B., and Schwarz, L. B., 1981, A model for the analysis of system service level in warehouse/retailer distribution systems: The identical retailer case, Schwarz . L. B, ed. Multi-level production inentory control systems, TIMS studies in Management Science,Vol.16 North-HollandAmesterdam, The Netherland. 163-193 .

[8] Graves, S. C., 1985, A multi-echelon inventory model for a repairable item with one- for-one replenishment , Management Science 31, 1247- 1256 .

[9] Hopp, W, Spearman,M., and Zheng, R.Q., 1997, Easily implementable inventory control policies Operations Research 45, 327-340.

[10] Lee, H. L, 1987, A muti-echelon inventory model for repairable items with emergency lateral transshipments,Management Science 33, 1306-1316.

[11] Little, J. D.C, 1961, A proof of the queueing formula: L= .W., Operations Research 9

[12] Moinzadeh, K,and Lee. H. L., 1986, Batch size and stocking levels in multi-echelon repairable systems , Management Science 20, 472-481.

[13] Palm, C., 1938, Analysis of the Erlang traffic formulae for busy signal arrangements, Ericsson Technique 4, 39-58.

[14] Scarf, H. E ., 1958, Stationary operating characteristics of an inventory model with time lag, Arrow, K. J et al,studies in the mathematical theory of Inventory and Production, Stanford University Press, Stanford, CA.

[15] Shanker, K., 1981, Exact analysis of a two-echelon inventory system for recoverable itemunder batch inspection poicy , Naval Research Logistics Quarterly 28, 579-601.

[16] Sherbrooke, C. C., 1968, METRIC: A multi-echelon technique for recoverable item control, Operations Research 16, 122-141.

[17] Simon, R. M., 1971, Stationary properties of a two-echelon inventory model for low demand items, Operations Research 19, 761-777.

[18] Svoronos, A. ,and Zipkin, P., 1988, Estimating the performance of multi-echelon inventory systems, Operations Research 36, 57-72 .

[19] Svoronos, A., and Zipkin, P., 1991, Evaluation of the one-for-one replenishment policies for multi-echelon inventory systems, Management Science 37, 68-83.

[20] Wang,Y., Cohen, M. A, and Zheng,Y. S., 2000, A two-echelon repairable inventory system with stocking-center-dependent depot replenishment leadtimes Management Science 46,

[21] Zipkin, P., 1999, Foundation of inventory management, Mc Grow Hill, NewYork.

5-Conclusion Remarks

We have analyzed a two-echelon system for repairables with a central repair depot and multiple inventory stocking centers. Our model extends the literature by allowing depot replenishment lead times to be stoching center dependent and to have two kinds of defectives.

Our analysis of the model led to an exact evaluation of system performances. We have shown that existing models have errors in evaluation of performance measures errors are more meaningful when the depot stocking level is low.

6-Appendix A List of Notation

N: number of stocking centers in the system.
n: index of a stocking center, n = 1, 2, .., N.
O: index of the central depot.
λ_n : demand arrival rate at center n.
λ_o : The replenishment order arrival rate at the depot.
r_n :The per cent of type B defectives arrival at centern.
S_n: base-stock level at center n.
S_o: base-stock hevel at the depot.
O_o: depot outstanding orders in steady state.
O_n: Out standing orders of stocking center n in steady state.
I_o: Inventory level at the depot.
I_n: Inventory level at center n.
L_n: depot replenishment lead time corresponding to orders from Stocking center n.
$G_n(.)$: probability distribution function of L_n.
L'_n : stocking center "n" replenishment lead time for type B defective.
G'_n (.): Probability distribution function of L'_n.
$\hat{L}_n$: trarsportation time from the depot to center n.
$\hat{G}_n(.)$: probability distribution function of $\hat{L}_n$.
W_n: random delay at the depot of replenishment orders from stocking center "n".
$F_n(.)$: Probability function of W_n.
$\hat{W}_n$: waiting time of customers at stocking center n.
Y_n: $W_n + \hat{L}n$,
$H_n(.)$: probability distribution function of Yn.
β_o : fill rate at the depot of replenishment orders from stocking centers.
β_n= Customer fill rate at stocking center n.
B_o: backorders at the depot of replenishment orders from stocking centers.
B_n: Customer back orders at stocking center n.

References

[1] Axsater, S, 1990, Simple solution procedures for a class of two-echelon inventory problems Operations Research 38, 64-69.

[2] Axsater, S., 1990, Modeling emergency lateral transshipments in inventory systems, Management Science 36, 1338.

[3] Axsater, S., 1991, Exact and approximate evaluation of batch-ordering policies for two-level inventory systems, Operations Research 39, 777-785.

[4] Chen, F., and. Zheng,Y. S., 1997, One warehouse multi-retailer systems with centeralized information , Operations Research 45, 275-287.

[5] Cohen, M. A., Kamesam, P., Kleindorfer, P., Lee,H.and Tekerian, A., 1990, OPTIMIZER: Amulti-echelon inventory system for service logistics management, Interfaces 20, 65-82.

[6] Dada, M ., 1992, A two-echelon inventory system with priority shipments, Management Science 38, 1140-

The fill rate service is equal to the probability of instock at a center, that is:

$$\beta_n = \Pr\{I_n \geq 1\} = \sum_{k=o}^{S_n - 1} \Pr\{O_n = k\} \tag{24}$$

4-Numerical Analysis

We use some numerical comparisxons to illustrate how significantly the difference in DRLTs impacts the performance of the system. For simplicity, we consider a network with only two stocking centers, which are identical in all respects except for their corresponding DRLTs. Specifically the DRLTs from stocking center 1 are assumed to be 50 days (Constant) and those from stocking center 2 to be 10 days. The demand rates at two centers are assumed to be the same, i.e., $\lambda_1 = \lambda_2 = 5$ units per year. We also assume that $r_1 = r_2 = 0$.

For the system given above, we compare our exact model with two approximation approaches we derived based on two existing models. Under both approaches, the stocking center-dependent DRLTs are approximated as i.i.d. random variables across all of the stocking centers That is, a DRLT, independent of from which local center a depot replenishment is triggered, takes eieher 50 days of 10 days with equal probability. The difference between the two approximate approaches is in how they treat stocking-center replenishment lead times. The first approach uses sequentlal stocking-center lead times as proposed in svoronos and zipkin (1991) [19], and thus will be labled as the SZ approaeh; and the second uses i.i.d approximation for the stocking-center lead times as suggested in the METRIC model, and will be labled as the METRIC approach..

Our model shows that the service levels at center 1 are worse than those at center 2; center 1 has a higher expected customer back order level and a lower customer fill rate. Assuming that the trans protation time from depot to both stocking centers takes 2 days, Table 1 lists the expected back orders and fill rates at the two stocking centers evaluated by the three methods. Note that while $E[B_1]$ and $E[B_2]$ also (β_1 and β_2) are different under our model, they are identical under each of the two approximation models. Therefore, the table lists only $E[B_1]$ and β_1 for the latter two models. One can see from the table that the SZ approach consistently overestimates the service levels for center 1 and understimates those for center 2. Compared with the SZ approach, the METRIC approach generates, in general, overly optimistic estimates for the service levels.

Table (1) Performance Metrics and comparison.

			Our model				Sz approach		METRIC APPROACH	
S_1	S_1	S_2	$E[B_1]$	$E[B_2]$	β_1	β_2	$E[B_1]$	β_1	$E[B_1]$	β_1
0	0	0	0.612	0.279	0.000	0.000		0.445	0.438	0.000
0	1	1	0.162	0.045	0.550	0.766		.000	0.083	0.645
0	2	2	0.032	0.006	0.871	0.962	0.104	0.659	0.011	0.928
0	3	3	0.005	0.001	0.973	0.995	0.020	0.916	0.001	0.99
0	4	4	0.001	0.000	0.995	0.999	0.003	0.983	0.000	0.999
1	0	0	0.207	0.118	0.000	0.000	0.000	0.997	0.158	0.000
1	1	1	0.038	0.012	0.831	0.894	0.162	0.000	0.012	0.854
1	2	2	0.006	0.001	0.968	0.989	0.025	0.863	0.001	0.989
1	3	3	0.001	0.000	0.995	0.999	0.004	0.979	0.000	0.999
2	0	0	0.068	0.052	0.000	0.000	0.000	0.997	0.059	0. 000
2	1	1	0.007	0.003	0.939	0.951	0.06	0.000	0.002	0.943
2	2	2	0.001	0.000	0.994	0.997	0.005	0.945	0.000	0.998
							0.001	0.996		

We have 4 Performance measures, we can use AHP to achieve the best allocation.

replenishment lead time Yn consists of a random delay at the depot, I .e.,Wn and a transportation time $\hat{L}_n$. The probability distribution of $\hat{L}_n$, denoted by $\hat{G}_n(.)$, is known, and that of W_n denoted by $F_n(.)$, has been obtained from our analysis of the depot problem. Thus, the distribution of Yn, denoted by $H_n(.)$, can be obtained simply by a convolution, That is,

$$H_n = F_n * \hat{G}_n \tag{16}$$

Where * denotes the convolution operator.

Svoronos and Zipkin (1991) [19] show that the steady-state outstonding order caused by type A defectives, denoted by O_{nl}, has the same distribution as the replenishment-lead-time demand, that is, the number of customer demands (type A defectives) during a random time period with distribution $H_n(.)$. As a consequence, the distribution of O_{nl} can be derived as follows: conditioning on Y_n=y, O_{nl} is a poisson random variable with mean value of $(1-r_n)\lambda_n y$, that is,

$$\Pr\{O_{nl} = k \mid Y_n = y\} = \frac{((1-r_n)\lambda_n y)^k}{k!}\exp(-(1-r_n)\lambda_n y), k = 0,1,.. \tag{17}$$

Simply by unconditioning, we have:

$$\Pr\{O_{nl} = k\} = \int_0^\infty \frac{((1-r_n)\lambda_n y)^k}{k!}\exp(-(1-r_n)\lambda_n y)dH_n(y), k = 0,1,... \tag{18}$$

The steady-state outstanding order caused by type B defectives denoted by O_{n2}, is a poisson random variable with mean value of μ_n.

$$\mu_n = rn \times \lambda_n \times E[L'_n]$$

$$P_r\{O_{n2} = k\} = \frac{e^{-\mu_n} \times (\mu_n)^k}{k} \tag{19}$$

Now, we can obtain the distribution of O_n, That is, the number of Outstanding orders of stocking center n in steady state.

$$\Pr\{O_n=k\} = P_r\{O_{nl}= k\} \times P_r\{O_{n2}=0\}+\ldots+ P_r\{O_{nl}=0\} \times P_r\{O_{n2}=k\}$$

$$= \sum_{x=0}^{k} P_r\{O_{nl} = x\} \times P_r\{O_{n2} = k - x\} \tag{20}$$

Having specified the steady – state outstanding orders, the inventory level distribution can be computed through:

$$I_n = S_n - O_n \tag{21}$$

For any given stocking level S_n; and we can further compute the expected on-hand inventory and expected backorders by:

$$E[I_n^+] = \sum_{J=O}^{S_n} (S_n - J)p_r\{O_n = j\} \tag{22}$$

$$E[B_n] = \sum_{j=s_n}^{\infty} (S_n - j)Pr\{O_n = j\} \tag{23}$$

characteristics of the system can be obtained by letting $t_n \to \infty$. Let $A_n(\tau)$ denote the resulting limit of $A_n(t_n, t_n + \tau)$. From (7), we then have,

$$A_n(\tau\tau = S_o - \sum_{i=1}^{N} Q_i(\tau\tau + \sum_{i=1}^{N} R_i(\tau\tau + \delta_n(\tau\tau \qquad (10)$$

where, $Q_i(\tau)$ and $R_i(\tau)$ for i,i=i, 2, .., N, are the limits of $Q_{i,t_n}(\tau)$ and $R_{i,t_n}(\tau)$, respectively. $Q_i(\tau)$ and $R_i(\tau)$ are mutually independent poisson random variables with mean values of $(1-r_i)\lambda_i q_i(\tau)$ and $(1-r_i)\lambda_i r_i(\tau)$, respectively, where,

$$q_i(\tau\tau = \int_{\tau}^{\infty}[1 - G_i(s)]ds \qquad (11)$$

and

$$r_i(\tau\tau = \int_{0}^{\tau} G_i(s)ds \qquad (12)$$

Furthermore, because the sum (convolution) of independent poisson random variables is again poisson, we can write (10) as

$$A_n(\tau\tau = S_o - Q(\tau(+ R(\tau(+ \delta_n(\tau\tau \qquad (13)$$

where, $Q(\tau)$, $R(\tau)$, and $\delta_n(\tau)$ are mutually independent; $Q(\tau)$ and $R(\tau)$ are poisson random variable with mean values of $\sum_{i=1}^{N}(1-r_i)\lambda_i q_i(\tau)$ and $\sum_{i=1}^{N}(1-r_i)\lambda_i r_i(\tau)$, respectively.

The steody-state version of (6) is given by:

$$\{W_n \le \tau\} \Leftrightarrow \{A_n(\tau\tau \ge 1\} \qquad (14)$$

It is easy to see from (13) and (14) how the random delay experienced by a stocking – center replenishment order is affected by the distribution of its own correspohding DRLT. Because both Q and R are common to all n (n=1, 2, …, N), W_n, for any given n, is a function of n only through $G_n(\tau)$ in (13). Specifically, if the depot replenishment lead time from stocking center n is (stochastically) shorter, the probability that the depot will get an additionel unit of inventory τ time units after an order arrival from stocking center n (i. e., $\delta_n(\tau)=1$) is higher and, as a consequence, the probability that a replenishment order from stocking center n is to be delayed move than τ is smaller. This reinforces out intuition as discussed in the introduction section.

Formally, the distribution of the delay enperienced by orders placed by centern can be computed as:

$$\begin{aligned} F_n(\tau\tau &= \Pr\{W_n \le \tau\} = \Pr\{A_n(\tau\tau \ge 1\}; \\ &= \Pr\{S_o - Q(\tau(+ R(\tau(+ \delta_n(\tau\tau \ge 1\}; \\ &= \Pr\{S_o - Q(\tau(+ R(\tau(\ge 1\} + G_n(\tau\tau)\Pr\{_o - Q(\tau(+ R(\tau(= 0\}. \end{aligned} \qquad (15)$$

3-Performance Measureds-The Stocking Center Problem

3-1- The steady-state Distribution of outstanding orders

Consider any stocking center n, n=1, 2, …, N, for type A defectives. In steady-state, its

2-2-Probability Distribution of the Random Delay at the Depot

In this section, we derive probability distribution of the random delay experienced by replenishment orders from any given stocking center n. To that end, we will derive first the transient properties of the system and then the steady-state results.

For transient analysis, we assume that the depot starts with so good parts on hand at time 0. Focus is made on a typical order from center n arriving at the depot at time t_n, (i. e., t_n is a demand epoch of center n).

Let $W_n(t_n)$ denote the random delay experienced by this order. For $\tau > 0$, we wish to obstain pr $\{wn\ (t_n) \leq \tau\}$. Let $A_n\ (t_n,t_n+\tau)$, be the depot inventory level at time $t_n+\tau$ plus the demands arrived in $[t_n,t_n+\tau]$, which can be interpreted as the invntory available at time $t_n+\tau$ at the depot for This specific replenishment order. Thus, we have (Wang et al 2000) [10]

$$\{W_n(t_n) \leq \tau\} \leftrightarrow \{A_n(t_n, t_n+\tau) \geq 1\}. \tag{6}$$

$A_n\ (t_n,t_n+\tau)$ can also be considered as the depot inventory level at time $t_n+\tau$, assuming that the depot stops satisfying all the center orders which arrive onof after time t_n. To obtain the distribution of $A_n(t_n,t_n+\tau)$, we first note that each center order that arrived before time t_n reduces $A_n(t_n,t_n+\tau)$ by one unit. Upon their arrival, whereas each depot replenishment order filled by time $t_n+\tau$ increases $A_n\ (t_n,t_n+\tau)$ by one unit. For each center order, there is a corresponding depot replenishment order. Thus, we define

$Q_{i,t_n}\ (\tau)$ = the number of orders that arrived during (o,t_n) from stocking center i and whose corresponding depot orders have not been filled by $t_n+\tau$;

$R_{i,t_n}\ (\tau)$ = the number of orders that arrived during $(t_n,t_n+\tau)$from stocking center i and whose corresponding depot orders have been filled by $t_n+\tau$.

$\sigma_n(\tau)$= 1, if the depot order corresponding to the specific order at time t_n is filled by $t_n+\tau$;

$\sigma_n(\tau)$= o,otherwise

Then, according to (Wang et al 2000) [20] we have the following inventory balance equation:

$$A_n(t_n, t_n+\tau) = S_o - \sum_{i=1}^{N} Q_{i,tn}(\tau\tau + \sum_{i=1}^{N} R_{i,tn}(\tau\tau + \delta_n(\tau\tau \tag{7}$$

where

1) $Q_{i,tn}(\tau), R_{i,tn}(\tau), i = 1,2,..,N, \text{and} \delta_n(\tau)$ are all mutually independent.

2) $Q_{i,tn}(\tau), \text{and} R_{i,tn}(\tau), i = 1,...,N,$ are poisson random variables with mean values of $(1\text{-}r_i)\ \lambda_i q_{i,t_n}(\tau)(\text{and}(1-r_i)\lambda_i r_{i,t_n}(\tau)$, respectively, where

$$q_{i,t_n}(\tau\tau = \int_0^{t_n}[1-G_i(t_n+\tau-s)]ds = \int_{\tau}^{t_n+\tau}[1-G_i(s)]ds \tag{8}$$

$$r_i, t_n(\tau\tau = \int_0^{\tau} G_i(\tau-S)ds = \int G_i(s)ds \tag{9}$$

3) $\delta_n(\tau) = \begin{cases} 1, \text{ with probability of } G_n(\tau), \\ 0 \text{ with probability of } 1\text{-}G_n(\tau). \end{cases}$

Because Equation (7) holds for every demand arvival epoch t_n of center n, the steady-state

2-Performance Measures-Th Depot Problem

2-1- The steady-state Depot Inventory Level Distribution

We start by analyzing the steady-state behaviour of the depot inventory system whose outstanding replenishment orders correspond physically to the defective items going through their defective lead times and their repair processes. Let Oo be the number of the depot's outstanding orders in steady state. Consider the depot orders triggered by the demands from stocking center n. Because the replenishment lead times are assumed to be independent and indentically distributed, the outstanding orders form a queue in an M/G/∞ system (scarf 1958) [14]. The number of outstanding orders corresponds to the queue length, i. e., the number of busy servers. Applying palm's theorm (Palm 1938) [13], the steady-state number of depot outstanding orders from stocking center n is simply a poisson random number with a mean value of $(1-r_n) \times \lambda_n . \times E[L_n]$. Note that although in aggregation the repair facility serves N streams of orders with arrival rate $\lambda_n(n=1, 2, .., N)$, the order streams may be seen as N independent M/G/∞ queues. Therefore, the total number of depot outstanding orders is the independent sum/convolution of those corresponding to all the stocking centers, which is again a poisson random variable with the mean value of $\mu_o = \sum_{n=1}^{N}(1-r_n).\lambda_n.E[L_n]$. Given the distribution of the outsanding orders, the steady-state distribution of the inventory level can be found using the relationship of:

$$I_o = S_o - O_o \tag{1}$$

Therefore, the expected on-hand inventory at the depot is:

$$E[I_o^+] = E[(S_o - O_o)^+] = \sum_{j=o}^{S_o-1}(S_o - J)\frac{\mu_o^j}{J!}\exp(-\mu_o) \tag{2}$$

and the expected backorders are:

$$E[B_o] = E[(S_o - O_0)^-] = \sum_{J=S_o+1}^{\infty}(J - S_o)\frac{\mu_o^j}{j!}\exp(-\mu_o) \tag{3}$$

The average delay at the depot of the replenishment orders from all stocking centers can be calculated, applying Little's Law, as:

$$E[W_o] = \frac{E[B_o]}{\lambda_o} \tag{4}$$

But, as we mentioned in the introduction, the replenishment orders from different stocking centers actually experience different delays at the replenishment ordrs from different stocking centers actually experience different delays at the depot, which will be derived in the next subsetion.

It is important to calculate the probability of an order from any stocking center being satisfied without delay, it is called fill rate:

$$\beta_o = pr\{I_o \geq 1\} = \sum_{J=o}^{S_o-1}\frac{\mu_o^j}{j!}\exp(-\mu_o) \tag{5}$$

Note that although replenishment orders from difeeerent stocking centers will experience different random delays at the depot, they actually have the same fill rate.

Because the stocking centers replenishment orders are triggered by independent poisson demand processes, they merge into a poisson demand process at the depot with the arrivel rate of $\lambda_o = \sum_{n=1}^{N}(1-r_n)\lambda_n$'. Whenerer a type A defects occurs, it is sent back to the depot, whenever the depot receives an order from a stocking center, a corresponding depot replenishment order is automatically placed with the repair facility and this replenishment order will be delivered to the depot good-part inventory system when the corresponding defects part is received and repaired. A depot replenishment leadtime (DRLT) includes the repair time and the defective lead time (as defined in the Introduction). We assume that the DRLTS are "stocking-centerwise" i.i.d.random variables. Let L_n denotes the DRLT corresponding to the orders triggered by the type A demands at stocking center n , and $G_n(.)$ be the probability distribution function of L_n.

A stocking center's replenishment lead time for type A defects, consists of a random delay at the depot and a transportation time from the depot to the center. While the latter can be random, it is assumed to be sequential, i. e., orders shipped to the same stocking center donot cross in transportation. We denote the transportation time to center n and its probobility distribution function by $\hat{L}_n$ and $\hat{G}_n$. respectively.

A stocking center's replenishment lead time for type B defectives, consists of a random delay at the same stocking center, and it is shown by L'_n and its probability distribution function is shown by G'_n (.).

Our main goal is to characterize the system performance for any given stocking policy {Sn: n= 0,1, ..., N}. We will follow the usual approaeh of decomposing the two-eehelon system into a depot subproblem and a stocking center problem. The connection between the two subsystems is the random delay at the depot experienced by replenishment orders (from type A) from the stocking centers. The distribution of this delay will be obtained through the analysis of the depot problem. For reader's convenience, all notation defined in this paper is listed in Appendix A.

Figure 1 illustrates the topology and material flows of a service part network obout which the following assumptions are made.

Assumption 1. The service network has a two-echelon structure with a central depot at the top echelon and multiple inventory stocking centers at the bottom echelon.

Assumption 2. Each stocked item is considered in isolation.

Assumption 3. Demands for good units occur at the stocking centers, and are generated by independent poisson processes which capture random failures of parts in the installed base of machines supported by the stocking centers.

Assumption 4. Corresponding to each demand for a good part, Stocking centers receive a defects one from the customer. Type A defectives are passed on the central depot for repair and inventory replenishment. Type B defectives are repaired on the stocking centers.

Assumption 5. Customer demands are backlogged when a stocking center is out of stock.

Assumption 6. For type A defectives, The stocking centers replenish good (repaired) parts from the depot using a one-for-one replenishment policy.

Assumption 7. The depot fills stocking centers' replenishment orders on a first-come-first-serve basis, and a replenishment order is backlogged when the depot runs out of good ports.

Assumption 8. The DRLTs corresponding to a given stocking center are independent and identically distributed; DRLTs corresponding to different stocking centers are independent but may be distributed differently.

Assumption 9. Depot repair capacity and stocking centers repair capacity are unlimited.

Assumption 10. Lateral transshipments between stocking centers are not allowed.

alternative evaluation approach that is exact, albeit under the assumption of constant lead times. Svoronos and Zipkin (1991) [19] study a model with stochostic lead times that are generated exogenously, but preserve the order sequence (referred to as sequential lead times), they derive procedures to evaluate exact system performance. Very recently, wang et al. (2000) [20] study a two-echelon system with stocking-center dependent depot replenishment lead times for one type of defectives. Other extensions of the METRIC model frame work include Deuermeyer and schwarz (1981) [18], Axsater (1991) [3], and chen and zheng (1997) [4] all of whom cohsider batch-ordering inventory policies. Lee (1987) [10], Axsater (1990) [2], and Dada (1992) [6] who study lateral transshipments among stocking centers. This list is not complete. Interested readers are referred to the comprehensive reviews of zipnin (1999) [21].

The purpose of this paper is to generalize stocking center dependent DRLTs for two kinds of defects. The paper is organized as follows: the model and notation are defined in section 2.In section 3 we derive the performance measures for the depot problem, and in section 4 we derive the performance measures of stoching center problem, section 5 discusses our numerical and comparison.

1-The system, Assumptions, and Notations

Consider a two-echelon system for a single repairable part that consists of a single central repair depot and N inventory stocking centers. Customers arrive at center n (n=1, 2, …, N) according to a poisson process with rate λn demands across different stocking centers are independent. There are two kinds of defects, type A defects mustbe repaired at the central depot, and type B defects must be repaired at stocking centers. r_n percent of defectives that arrive at stocking center n, have type B defects and $(1-r_n)$ per cent of these defects have type A defects and must be repaired at the central depot. The system operates according to the so-called one-for-one replenishment policy, i. e., the continuous time base-stock policy. Under this policy, the inventory position, which is the on-hand inventory plus outstanding orders minus backorders, is kept at a constant base-stock level by replenishing one unit immediately upon receiving a unit demond. The base-stock level at stocking center n is s_n. Arrival center in a demand is satisfied with a good part if available, backlogged otherwise. Upon seeing the demand, the stocking center immediately places a replenishment order with the depot, which is satisfied immediately with a good part if available, backlogged otherwise. The depot satisfies orders on the FCFS basis. Also stocking centers satisfy orders on the FCFS basis for each type of defects separately. While demanding a good part, each customer returns a defective one to the stocking center with a possible delay. Type A defects parts are shipped back to the depot facility for repair. After being repired., these items replenish the depot good part inventory. Figure l depicts the material flows in the service parts net work.

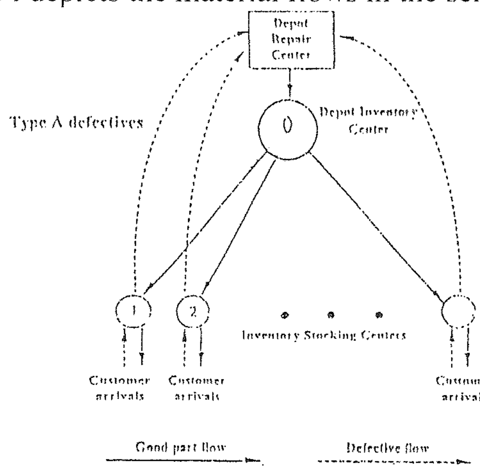

Figure (1) Material flows in the service network.

A Two-Echelon Repairable Inventory System with Two Kinds of Defectives, one kind Repaired at the Centeral Depot and the Other one Repaired at Stocking Centers

S. M. T. Fatemi Ghomi
Professor

F. Jolai
Assistant Professor

M. Mohammadi
M.Sc Degree

Department of Industrial Engineering,
Amirkabir University of Technology

Abstract

Consider a two-echelon repairable inventory system consisting of a centeral depot and multiple stocking centers. There are two kinds of defective, type A defects must be repaired at centeral depot, and type B defects at stocking centers. The centers provide parts replacement service to customers and the depot fills center replenishment orders on a FCFS basis.

Type A defects that are received at the centers are passed to the depot for repair and depot inventory replenishment. For this system, existing models (for exemple, METRIC model) usually assume that the depot replenishment lead times (DRLTs) (for type A defects) are i.i.d., which however, doesh't fit well into the service part logistics system, that motivated this research. Because the DRLTs consist of the sum of repair times, defective return times and transportation times, they are different across stocking centers which are located globally. We study the impact of such center-dependent DRLTs on system performance. We show that for such systems using the i.i.d. DRLT assumption introduces errors in estimating system performance.

Key words

Inventory; Multiechelon system; One for one policy; Repairables; service-parts.

Introduction

Multi-echelon service parts inventory management is one of the most successful areas of operations research application. Mathematical models have been implemented for many large-scale systems and have had a significant impact on performance, pioneered by the METRIC model (sherbrooke1968) [16] for military applications, the most recent work has addressed commercial service parts logistics systems (e. g., cohen et al. 1990 [5], Hopp et al. 1997 [9]).

The METRIC model uses results from queueing theory, (i. e., palm's Theorem (palm 1938) [13] and little's law (Little 1961) [11] to evaluate the system performance metrics of average inventory level and expected customer backorders at each inventory location. The key idea of the METRIC model approach is to approximate the distribution of a stocking center's outstanding orders, in steady state, by a poisson random variable whose mean is the product of the average replenishmet leadtime and the average demand rate. This approximation turns out to be robust.

Due to its implementation success, The METRIC model has received considerable academic attention. Simon (1971) [17] and shanker (1981) [15] provide an exact analysis of the model under the constant leadtime assumption. Graves (1985) [8] develops a two-moment approximation for stocking centers' outstanding orders. Axsater (1990) [1] develops an