Nomenclature

a_i = inertia terms
C = damping matrix
c_i = combined damping of link i and joint i
E[.] =expectation operator
f(t) = excitation vector
H(ω) = complex frequency response vector
$\mathbf{I}_i$ = Centroidal moment of inertia of link i
J = Jacobian matrix
K = stiffness matrix
k_i = combined stiffness of link i and joint i
L = lagrangian function
L_{ci} = distance of mass center of link i from joint i
L_i = length of link i
M = inertia matrix of robotic manipulator
m_i = mass of link i
n = number of links
q =dθ = joint elastic displacement vector
r(t) = normal coordinate vector
$\mathbf{R}_{qq^T}(0)$ = covariance matrix of joints displacement
$\mathbf{R}_{uu^T}(0)$ = covariance matrix of tip displacement
$\mathbf{S}_{qq^T}(\omega)$ =power spectral density matrix of joint displacement
$\mathbf{S}_0$ = intensity of white noise process
t = time variable
u = tip displacement vector
$\ddot{y}_0$ =vertically stochastic exited acceleeration of the base
θ = kinematic configuration of manipulatorreferences

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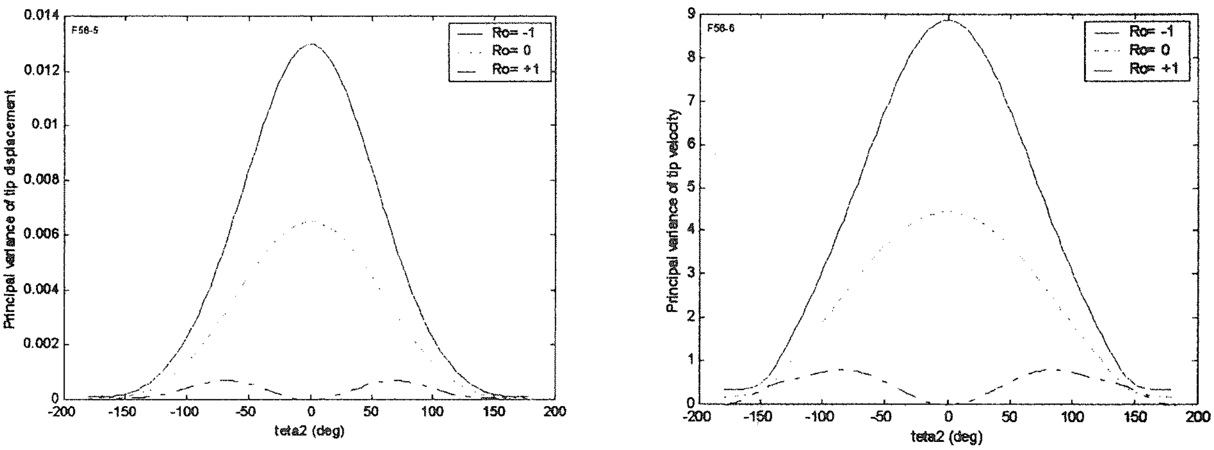

Fig. (4) Sensitivity of tip response to the base excitations parameters for $\alpha = 1$ and $\theta_1 = 45^\circ$ deg a) principal variance of tip displacement b) principal variance of velocity.

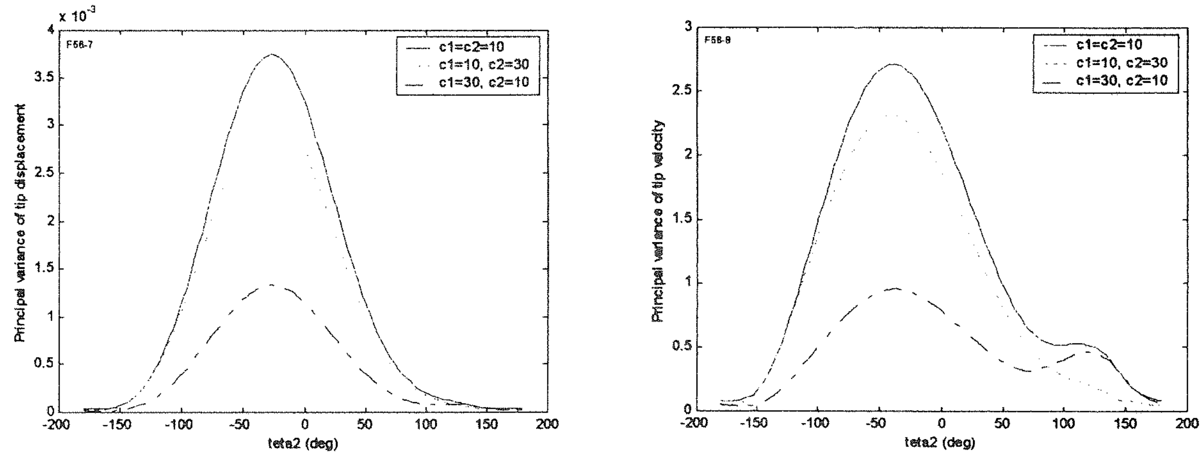

Fig. (5) Sensitivity of tip response to joints damping for $\rho = \alpha = 0$ and $\theta_1 = 45^\circ$ deg a) principal variance of tip displacement b) principal variance of velocity.

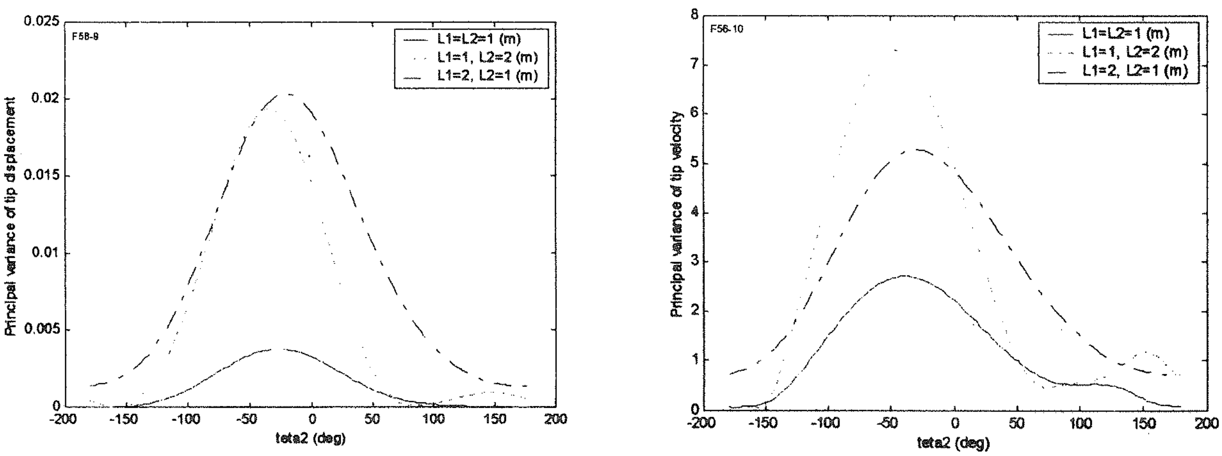

Fig. (6) Sensitivity of tip response to links length for $\rho = \alpha = 0$ and $\theta_1 = 45^\circ$ deg a) principal variance of tip displacement b) principal variance of velocity.

concluded that to minimize the stochastic vibration of the manipulator tip: the damping effects should be concentrated in the lower joints; the lower links should be smaller than the upper links and using from both horizontal and vertical base excitations with $\alpha=1$ and $\rho=+1$. It is noted that one of the base excitation can be produced virtually.

Table (1) Parameter values used for simulation [6].

Parameter	*Value*	*Unit*
m_1	2	Kg
m_2	2	Kg
L_1, L_2	1	m
L_{c1}, L_{c2}	0.5	m
I_1, I_2	1/6	Kg.m^2
k_1, k_2	4000	N/rad
c_1	10	N.s/rad
c_2	10	N.s/rad

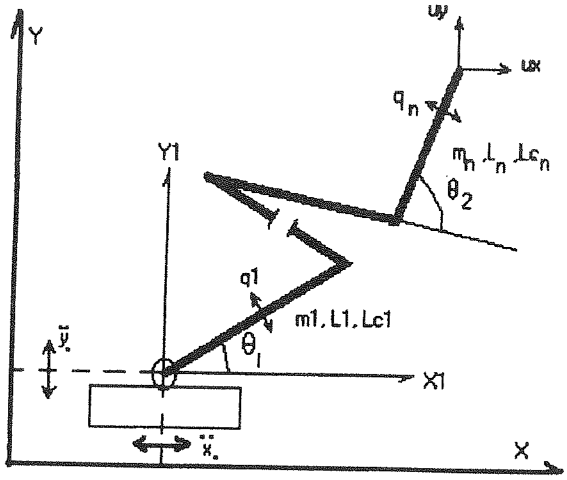

Figure (1) n-link planner robotic manipulator

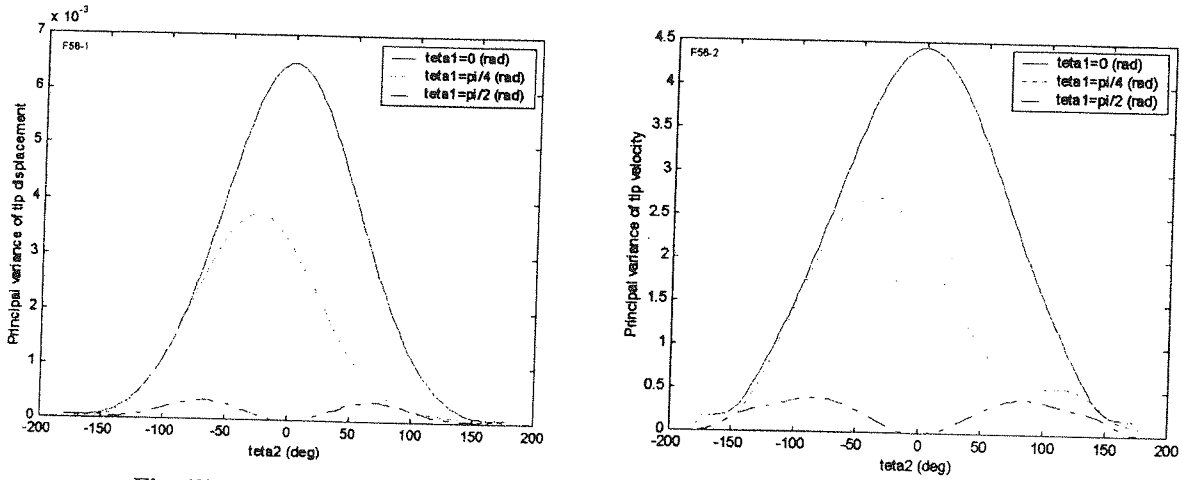

Fig. (2)- Sensitivity of tip response to manipulator configuration for $\rho = \alpha = 0$ a)
a) principal variance of tip displacement b) principal variance of tip velocity.

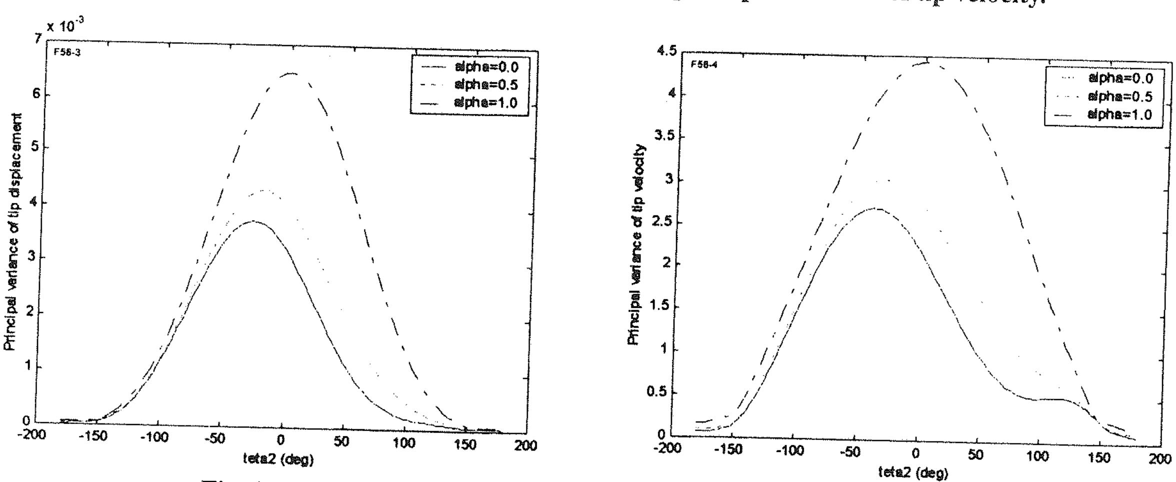

Fig. (3) Sensitivity of tip response to the base excitations parameters for $\rho = 0$ and $\theta_1 = 45°$ deg a) principal variance of tip velocity, b) principle variance sfvelacity.

5-Simulation Results and Discussions

Numerical simulations to compute the principal variance of the tip displacement and velocities to the both horizontal and vertical stochastic base excitations for the two-link planner robotic manipulator were performed. The results show sensitivity of the tip excitation to manipulator configuration, structural damping which concentrated at joints, base excitation parameters and manipulator links length. Table (1) gives the values of the parameters used for numerical simulation. Also, it is assumed that the stochastic vibration of the base acts simultaneous in horizontal and vertical direction with zero mean white noise process and unit intensity.

In the following discussion the term displacement refers to the major principal variance of the tip displacement, while the term velocity refers to the major principal variance of the tip velocity.

Figures 2 to 6 show results for the specific values of θ_1, and for $\pm$ 180 deg range of θ_2 and $S_0=1$. Figure 2 shows the response sensitivity of the tip displacement and velocity to manipulator configuration for $\theta_1 = 0, 45, 90$ deg and $\rho = \alpha = 0$. It is observed that the more perpendicular the manipulator structure to the excitation direction, the tip response is higher. Thus, the highest tip principal response occurs at $\theta_1 = \theta_2 = 0$ deg. At this configuration the links of the manipulator are perpendicular to the stochastic base excitation on vertical direction. The displacement and velocity at $\theta_1=90$ and for $\theta_2=0$ deg are zero.

Figures 3 and 4 show the sensitivities of the tip response to the excitation parameters in the horizontal and vertical directions for $\theta_1 = 45$ deg, $\rho = 0$, $\alpha = 0$ respectively. Figures 4 show that if the horizontal and vertical excitations act simultaneous with same excitation intensity; $\alpha = 1$ and $\rho=0$; the tip displacement and velocity response increase. Also, figure 4 shows the value of the tip displacement and velocity are double for $\alpha=1$, $\rho=-1$ and are zero for $\alpha=1$, $\rho=+1$. In the other word, we have maximum tip displacement when the ex($= S_0$, $S_{\ddot{x}_0\ddot{y}_0} = S_{\ddot{y}_0\ddot{x}_0} = -S_0$

$S_{\ddot{x}_0\ddot{x}_0} = S_{\ddot{y}_0\ddot{y}_0}$

and we have minimum tip displacement when $S_{\ddot{x}_0\ddot{y}_0} = S_{\ddot{y}_0\ddot{x}_0} = +S_0$.

the excitation parameters are $S_{\ddot{x}_0\ddot{x}_0} = S_{\ddot{y}_0\ddot{y}_0} = S_0$

The influence of the manipulator joints damping on tip displacement and velocity is shown in figure 5. The displacement and the velocity response can be reduced with the presence of damping. But if only the upper links (link 2 in Two-link study) are damped, the responses have low sensitivity to joints damping. On the other hand,if only the lower links (link in Two-link study) are damped the responses are sensitive to damping. The influence of the relative length of the lower link to the upper link on the tip displacement and velocity is shown in figure 6.

6-Conclusions

The stochastic vibration tip subjected to both horizontal and vertical randomly base excitation has been studied. A two-link articulated planner manipulator has been used to compute the sensitivities of principal variance of the tip motion with respect to manipulator configuration, damping, base excitation parameters and manipulator links lengths. The magnitude of the response has been found to vary significantly for different configurations and different parameters of system. Also, it was observed that the valuse of major variances could be reduced in joints damping and the base excitation parameters.

Similarly the covariance matrix of the manipulator tip velocity in the base frame is

$$\mathbf{R}_{\dot{u}\dot{u}^T}(0) = E\left[\dot{\mathbf{u}}\,\dot{\mathbf{u}}^T\right] = \mathbf{J} \cdot \mathbf{R}_{\dot{q}\dot{q}^T}(0) \cdot \mathbf{J}^T \quad (19)$$

A rotated frame $\mathbf{u}_*$ in which the covariance matrix of the tip motion is diagonal can be used to compute the covariance matrix of the tip response. This frame will be referred to as the principal variance of tip displacement [5].

To compute the principal variance matrix $\mathbf{R}_{u_* u_*^T}(0)$, singular value decomposition of the tip motion covariance matrix in a known frame, for example $\mathbf{R}_{uu^T}(0)$, is employed and this process leads to:

$$\mathbf{R}_{uu^T}(0) = \mathbf{U}_u \cdot \Delta_u \cdot \mathbf{U}_u \quad (20)$$

The $\mathbf{U}_u$ is an orthonormal matrix. The diagonal matrix Δ_u contains the eigenvalues of $\mathbf{R}_{uu^T}(0)$ and its elements represent the principal variance of the tip displacements $\mathbf{R}_{u_* u_*^T}(0)$, i.e.

$$\mathbf{R}_{u_* u_*^T}(0) = \Delta_u \quad (21)$$

Similar representation can be made for the tip velocity.

4-Case Study: Two-Link Planner Manipulator

Consider a two-link planner articulator manipulator to simultaneous horizontal and vertical stochastic excitation of the base. The Lagrangian L and Rayleigh dissipation function R for the system is

$$L = \frac{1}{2} a_1 \cdot \left(\dot\theta_1 + \dot q_1\right)^2 + \frac{1}{2} a_2 \cdot \left(\dot\theta_1 + \dot\theta_2 + \dot q_1 + \dot q_2\right)^2$$

$$+ a_3 \cdot \left(\dot\theta_1 + \dot q_1\right) \cdot \left(\dot\theta_1 + \dot\theta_2 + \dot q_1 + \dot q_2\right) . \mathrm{Cos}\left(\theta_2 + q_2\right)$$

$$+ a_4 \cdot \dot y_0 \cdot \left(\dot\theta_1 + \dot q_1\right) . \mathrm{Cos}\left(\theta_1 + q_1\right) - a_4 \cdot \ddot x_0 . \left(\dot\theta_1 + \dot q_1\right) . \mathrm{Sin}\left(\theta_1 + q_1\right)$$

$$+ a_5 \cdot \dot y_0 . \left(\dot\theta_1 + \dot\theta_2 + \dot q_1 + \dot q_2\right) . \mathrm{Cos}\left(\theta_1 + \theta_2 + q_1 + q_2\right)$$

$$- a_5 \cdot \dot x_0 . \left(\dot\theta_1 + \dot\theta_2 + \dot q_1 + \dot q_2\right) . \mathrm{Sin}\left(\theta_1 + \theta_2 + q_1 + q_2\right)$$

$$+ \frac{1}{2} a_0 \cdot \left(\dot x_0^2 + \dot y_0^2\right) - \frac{1}{2} k_1 q_1^2 - \frac{1}{2} k_2 q_2^2 \quad (22)$$

$$R = \frac{1}{2}\left(C_1 \dot q_1^2 + C_2 \dot q_2^2\right)$$

where

$$a_0 = m_1 + m_2,$$

$$a_1 = I_1 + m_1 . L_{c1}^2 + m_2 . L_1^2, \quad (23)$$

$$a_2 = I_2 + m_2 . L_{c2}^2$$

$$a_3 = m_2 . L_1 . L_{c2}, \quad a_4 = m_1 . L_{c1} + m_2 . L_1,$$

$$a_5 = m_2 . L_{c2}$$

The equatio of motion (1) can be obtained for small elastic displacement by using Lagrangian's method, where

$$M_{11} = a_1 + a_2 + 2 . a_3 . \mathrm{Cos}(\theta_2)$$
$$M_{12} = M_{21} = a_2 + a_3 \mathrm{Cos}(\theta_2), \quad M_{22} = a_2$$
$$C_{11} = c_1, \quad C_{22} = c_2, \quad C_{12} = C_{21} = 0$$
$$K_{11} = k_1, \quad K_{22} = k_2, \quad K_{12} = K_{21} = 0$$
$$F_{11} = a_4 . \mathrm{Sin}(\theta_1) + a_5 . \mathrm{Sin}(\theta_1 + \theta_2)$$
$$F_{12} = - a_4 . \mathrm{Cos}(\theta_1) - a_5 . \mathrm{Cos}(\theta_1 + \theta_2)$$
$$F_{21} = a_5 . \mathrm{Sin}(\theta_1 + \theta_2), \quad F_{22} = - a_5 . \mathrm{Cos}(\theta_1 + \theta_2) \quad (24)$$

Also the elements of the Jacobian matrix can be written as

$$J_{11} = - L_1 . \mathrm{Sin}(\theta_1) - L_2 . \mathrm{Sin}(\theta_1 + \theta_2),$$

$$J_{12} = - L_2 . \mathrm{Sin}(\theta_1 + \theta_2)$$

$$J_{21} = + L_1 . \mathrm{Cos}(\theta_1) + L_2 . \mathrm{Cos}(\theta_1 + \theta_2),$$

$$J_{22} = + L_2 . \mathrm{Cos}(\theta_1 + \theta_2) \quad (25)$$

using its mass normalized modal matrix **U**

$$\mathbf{q} = \mathbf{U} \cdot \mathbf{r} \qquad (3)$$

where $\mathbf{r}(t)$ is the vector of normal coordinate. Substituting the equation (3) into equation (1) and premultipling the result by the matrix U^T gives an uncoupled set of equations as [7]

$$\ddot{\mathbf{r}} + \beta . \dot{\mathbf{r}} + \lambda . \mathbf{r} = \gamma^{-1}\, \mathbf{u}^T \mathbf{f}(t) \qquad (4)$$

where β, λ and γ are diagonal matrix as

$$\mathbf{U}^T . \mathbf{M} . \mathbf{U} . \gamma,\ \mathbf{U}^T . \mathbf{K} . \mathbf{U} = \gamma.\lambda,\ \mathbf{U}^T . \mathbf{C} . \mathbf{U} = \gamma.\beta \qquad (5)$$

Therefore, the crosscorrelation matrix of joints displacement in matrix from are

$$\mathbf{R}_{qq^T}(\tau) = E\,[\mathbf{q}(t) . \mathbf{q}^T (t+\tau)] \qquad (9)$$

$$\mathbf{R}_{qq^T}(\tau) = \mathbf{U} . E\,[\mathbf{r}(t) . \mathbf{r}^T (t+\tau)] . \mathbf{U}^T$$

Also, the power spectral density of joints displacement q can be obtained as

$$S_{qq^T}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathbf{R}_{qq^T}(\tau) . e^{-i\omega\tau} . d\tau \qquad (10)$$

The power spectral density for a linear and stationary systems with multi-degree of freedom such as equation (4) are described by [8]

$$S_{qq^T}(\omega) = \mathbf{H}(\omega) . S_{ff^T}(\omega) . \mathbf{H}^T(\omega) \qquad (11)$$

where $\mathbf{H}(\omega)$; the harmonic transfer matrix can be written as

$$\mathbf{H}(\omega) = \mathbf{U} . \gamma^{-1} \left(\gamma - \omega^2\, \mathbf{I} + \mathbf{i} .\, \omega\, \beta\right)^{-1} . \mathbf{U}^T \qquad (12)$$

$\mathbf{I}$ = Identity matrix, $\mathbf{i} = \sqrt{-1}$

Also, the covariance matrix if the jonints displacement and the joints velocities can be written as

$$\mathbf{R}_{qq^T}(0) = E\left[\mathbf{q}.\mathbf{q}^T\right] = \int_{-\infty}^{+\infty} \mathbf{S}_{qq^T}(\omega).\, d\omega$$

$$\mathbf{R}_{\dot{q}\dot{q}^T}(0) = E\left[\dot{\mathbf{q}}.\dot{\mathbf{q}}^T\right] = \int_{-\infty}^{+\infty} \omega_0^2\, \mathbf{S}_{qq^T}(\omega).\, d\omega \qquad (13)$$

3. Tip Response Covariance

The covariance matrix of the tip response can be computed in the base frame; $X_1 = [X_1, Y_1]^T$. Since the joint motion vector **q** is assumed small (**q**=dθ) a Jacobian The displacement of the manipulator tip $u = [u_x, u_y]^T$ to the base frame $\mathbf{X}_1$ are related to the small joint displacement vector **q** as

$$\mathbf{u} = \mathbf{J} . \mathbf{q} \qquad (14)$$

The elements of the Jacobian matrix **J** can obtained as

$$\mathbf{J} = \frac{\partial \mathbf{R}_T(\theta)}{\partial \theta} \qquad (15)$$

where $\mathbf{R}_T(\theta)$ is the position of the manipulator tip as a function of joint angle vector θ.
The correlation matrix of the tip displacement in the base frame is defined as

$$\mathbf{R}_{uu^T}(\tau) = E\,[\mathbf{u}(t) . \mathbf{u}^T (t+\tau)] \qquad (16)$$

Substituting equation (14) into equation (16), we obtain

$$\mathbf{R}_{uu^T}(\tau) = \mathbf{J} . \mathbf{R}_{qq^T}(\tau) . \mathbf{J}^T \qquad (17)$$

Therefore, the covariance matrix of the tip displacement in the base frame can be defined as

$$\mathbf{R}_{uu^T}(0) = E\left[\mathbf{u}\mathbf{u}^T\right] = \mathbf{J} . \mathbf{R}_{qq^T}(0) . \mathbf{J}^T \qquad (18)$$

rigid manipulator were studied. Kujath and Akpan [5] studied dynamic responses of the planner manipulator tip with two links to a vertical random base excitation. They also studied tip stochastic vibration of a mobile robotic manipulator with same approach [6]. This paper discusses tip displacement dynamic of a planner manipulator excited simultaneously by a horizontal and vertical random motion of the base. It is assumed that structural flexibility of links is concentrated in the joints, and dry friction, gear backlash and actuator dynamics are neglected.

First, with using the lagrangian formulation we derive the dynamic equations of the robotic manipulator under both horizontal and vertical base excitations, and then the equations of motion are linearized and decoupled. The dynamic response variance of the links with respect to the base vibration are determined by employing power spectral density approach. Then, the covariance matrix of the tip displacement is computed in coordinate frame attached to the base by the Jacobian matrix. The principle variance of tip displacement is computed by covariance matrix of the tip displacement. Finally, simulation results for a 2R-planner robotic manipulator are presented. Then sensitivity of the principal variance of tip displacement and tip velocity to manipulator configuration, damping, excitation parameters and manipulator links length are investigated.

2-Dynamic Equations

Consider an n-link manipulator with revolute joints, Figure (1). It is assumed that dry friction, gear backlash and actuators dynamic are neglected, and the structural flexibility is constructed in the joints. Motion of manipulator can be described in the configuration vector $\theta=[\theta_1,\theta_2,\ldots,\theta_n]^T$ that representing the kinematics configuration of manipulator and $q=[q_1,q_2,\ldots,q_n]^T$ that representing the small elastic motion about a given kinematics configuration (both coordinates θ and **q** are measured at the joints). Therefore the total joint motion is given by θ+**q**. We study the small elastic motion **q** for a given θ when it is assumed that the structural and viscous damping of links concetrated at the joints and invariant to changes of θ.

By using Lagrangin's method, the dynamic equations of n-link manipulator to simultaneous horizontal and vertical random base excitations can be written in terms of the small elastic joint motion vector **q** as

$$\mathbf{M}\,(\theta).\,\ddot{\mathbf{q}} + \mathbf{C}.\,\dot{\mathbf{q}} + \mathbf{K}.\,\mathbf{q} = \mathbf{f}\,(t)$$

$$\mathbf{f}\,(t) = \mathbf{F}.\left[\ddot{x}_0\,,\,\ddot{y}_0\right]^T \qquad (1)$$

where **q** is the n × 1 vector of small elastic joint motion; **M** (θ) is the n × n Symmetric positive definite manipulator inertia matrix with respect to configuration vector θ; **C** is the n × n diagonal damping matrix; **K** is the n× n diagonal matrix representing the joint and structural flexbility concentrated in the joint; $\ddot{x}_0$ is the horizontal and $\ddot{y}_0$ is the vertical stoch

$$\mathbf{M}\,(\theta).\,\ddot{\mathbf{q}} + \mathbf{C}.\,\dot{\mathbf{q}} + \mathbf{K}.\,\mathbf{q} = \mathbf{f}\,(t)$$

$$\mathbf{f}\,(t) = \mathbf{F}.\left[\ddot{x}_0\,,\,\ddot{y}_0\right]^T$$

n × 2 vector

Consider a st

process for th

$$E[\ddot{x}_0] = 0,\quad S_{\ddot{x}_0\ddot{x}_0} = \alpha^2 . S_0\,,\ S_{\ddot{y}_0\ddot{y}_0} = S_0$$

$$E[\ddot{y}_0] = 0,\ S_{\ddot{x}_0\ddot{y}_0} = S_{\ddot{y}_0\ddot{x}_0} = \rho . S_0 \qquad (2)$$

where S_0 is intensity of white noise; α and ρ are constant parameters. Since **C** and **K** are diagonal matrices, equation (1) can decoupled

Tip Displacement Variance of Manipulator to Simultaneous Horizontal and Vertical Stochastic Base Excitations

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Abstract

The tip displacement variance of an articulated robotic manipulator to simultaneous horizontal and vertical stochastic base excitation is studied.The dynamic equations for an n-links manipulator subjected to both horizontal and vertical stochastic excitations are derived by Lagrangian method and decoupled for small displacement of joints. The dynamic response covariance of the manipulator links is computed in the coordinate frame attached to the base and then the principal variance of tip displacement is determined. Finally, simulation for a two-link planner robotic manipulator under base excitation is developed. Then sensitivity of the principal variance of tip displacement and tip velocity to manipulator configuration, damping, excitation parameters and manipulator links length are investigated.

Key Words

Base Vibration, Stochastic Vibration, Tip displacement variance and Robotic Manipulator.

1- Introduction

The study of dynamic response and maximum displacement of robotic manipulators tip under stochastic base excitatioin is an important research topic in the area of robotic engineering In practical applications, the study on motion of the manipulator tip is of primary importance because the tool and end effector is attached to this point. The results of investigation can be used in many applications such as space explorations, space macro-micro robotic manipulators, toxic waste clean up, mobile robotic manipulators and industrial manipulators which are installed on a randomly fluctuating base. Understanding the vibration responses of a robotic manipulator tip to stochastic excitations of the base and its sensitivity to the system parameters is also important for the robot structural design and control strategy development.

The excitation due to base vibration can be characterized as a deterministic or random motion. Streit et al. [1,2] used Floquet theory to study the dynamic response of manipulator excited by a deterministic base vibration. Dynamic behaviour of a single pendulum with randomly fluctuating base only, which is the simplest robotic model [3], and two degree-of-freedom manipulator [4] have been reported. In both cases, joint responses of