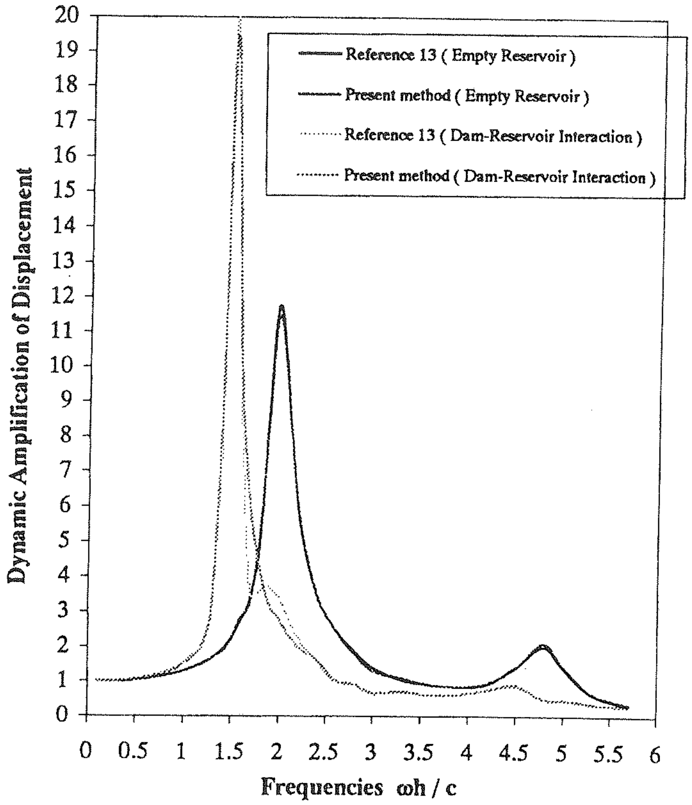

Figure (6) Steady state response for the flexible dam crest displacements in the frequency domain.

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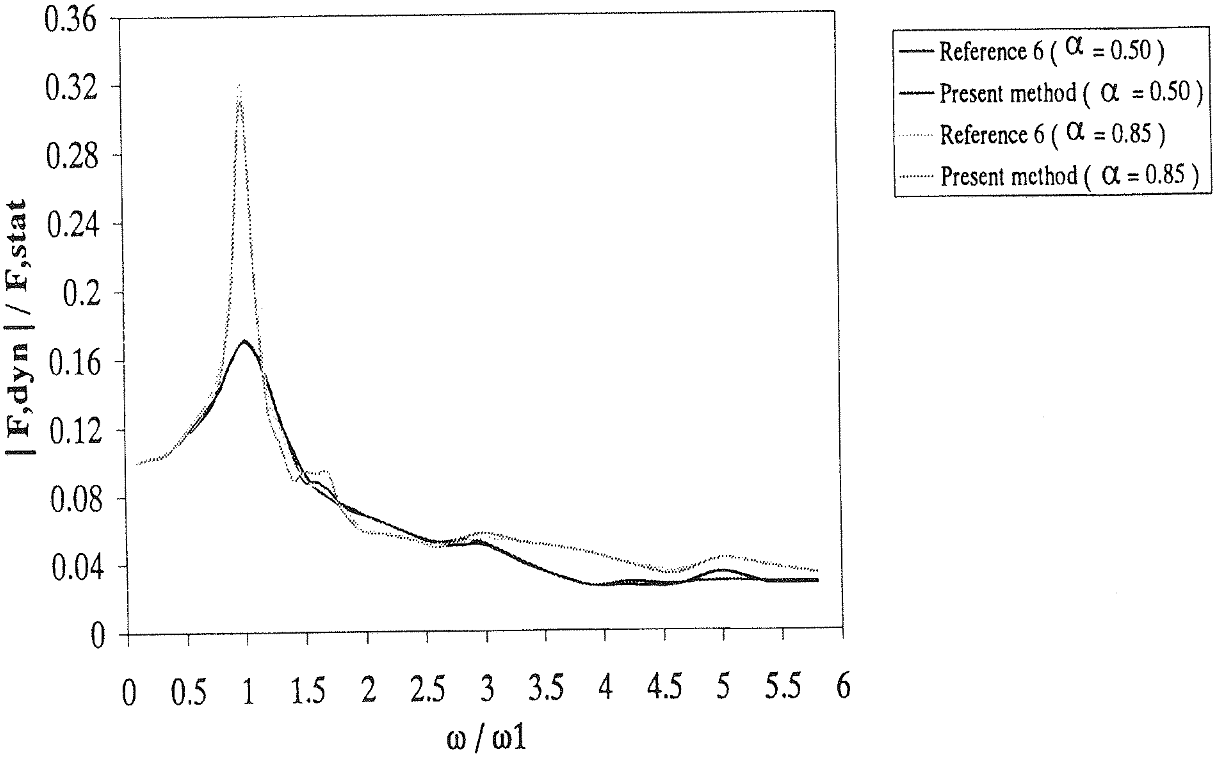

Figure (4) Hydrodynamic force (absolute value) on the upstream face of the rigid dam due to the harmonic excitation of dam support

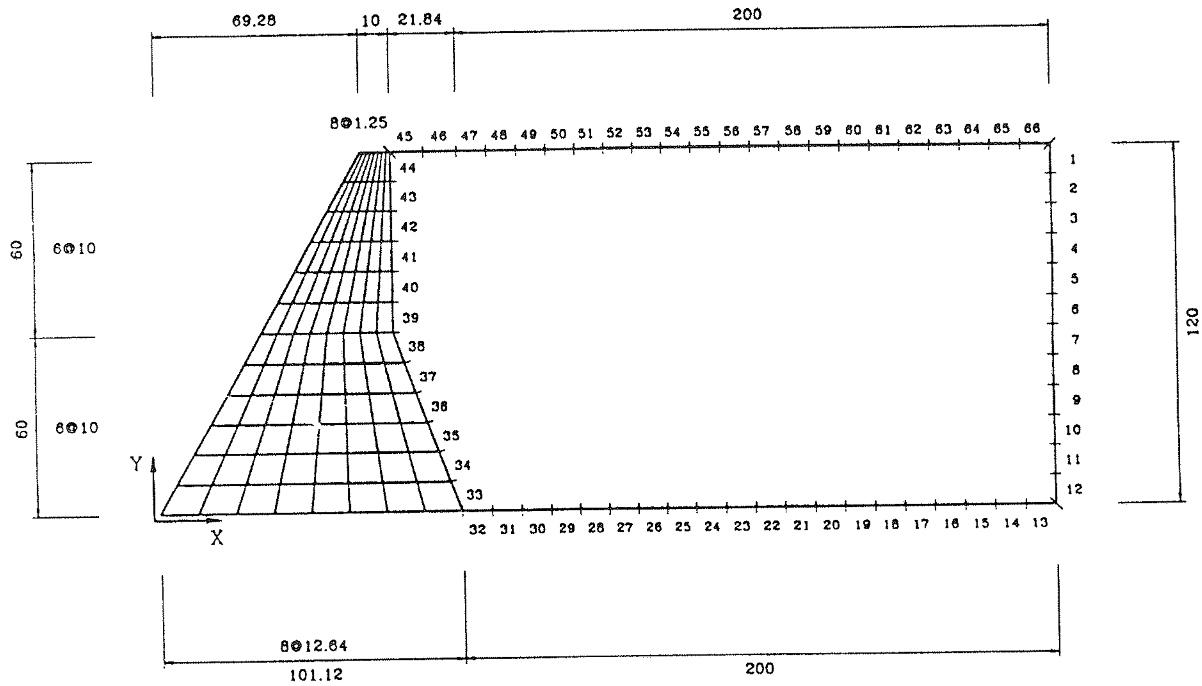

Figure (5) The BEM - FEM model for example 2 (response of a flexible dam - reservoir system).

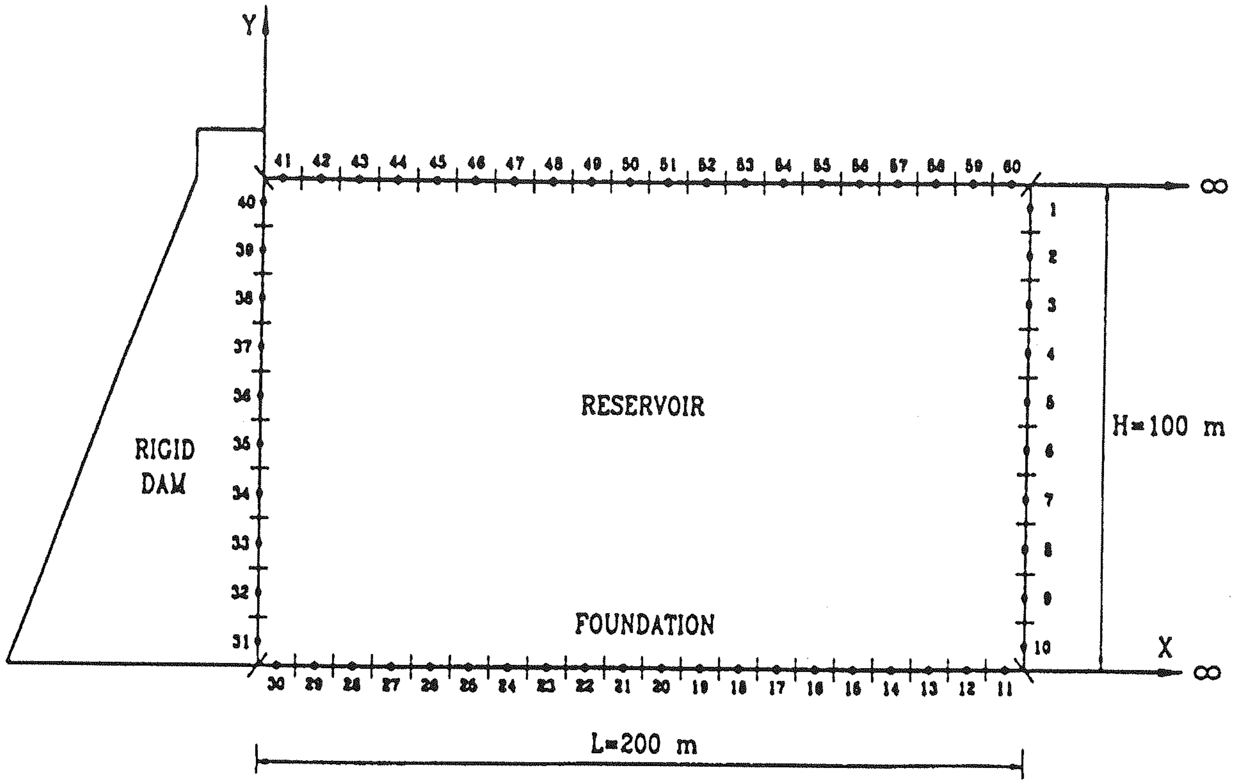

Figure (2) Reservoir boundary element model (60 constant elements) and a rigid dam.

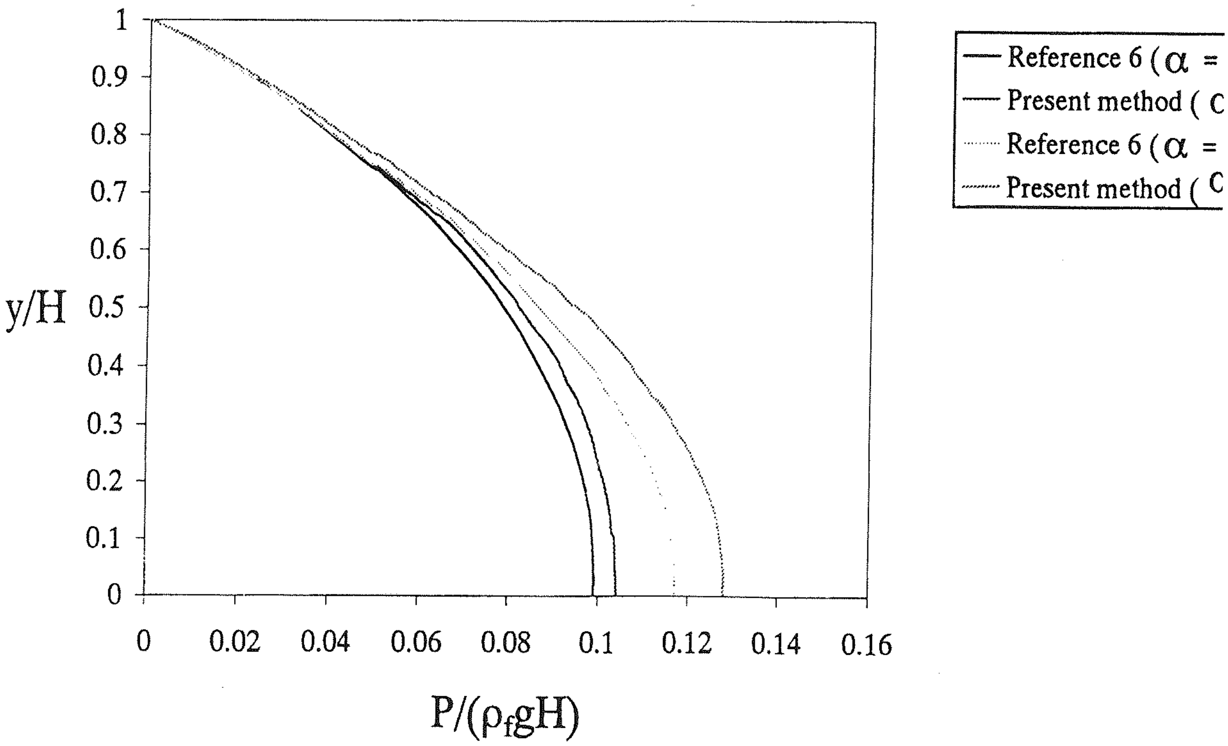

Figure (3) Hydrodynamic pressure distribution (absolute value) on the upstream face of the rigid dam (height=H) due to the unit harmonic exitation of dam sulpport (ω/ω_1=1.2)

reservoir cases and full reservoir cases. Static displacement of the dam is obtained first. Dynamic crest displacement relative to static value of displacement is then added to it.

Discussions

As Figures 3, 4 and 6 show, the results obtained from the present method have good agreement with the results of other researchers. But in frequency range between the first and second natural frequencies of reservoir, some errors occur in the results (not greater than 10~25%) because of approximate Sommerfield radiation boundary condition [12]. So it is well supposed that by using a more exact boundary condition, we can obtain completely proper results from the present method.

A special case

As for the results of this research (not shown here), in a 2D circular reservoir with a rigid structure, the system matrices will be necessarily symmetric. So in this particular case, we can solve the system of equations easily by simple methods and a limited memory space.

Conclusions

This paper presents an efficient method for 2D dynamic interaction analysis of gravity dam-reservoir systems, using Euler-Lagrangian approach, based on BEM-FEM formulation, in the frequency domain. The results obtained from the present research are as follows:

1- It is evident that the present derived formulation for the dam-reservoir dynamic interaction analysis, is a proper and successful formulation in Euler-Lagrangian approach. However unlike the FEM-FEM formulation for the velocity potential field in the reservoir, symmetry does not in general exist for the fluid - structure system. Only in a special case of circular reservoir, if the surrounding structure is rigid, symmetry is obtained in the fluid domain.

2- The weighing function in the BEM is the fundamental solution, whereas the weighing function in the FEM is the shape function of the trial function. So, the important result of this investigation is that considering basic difference between weighing functions used in BEM and FEM, in general, the governing equations set are nonsymmetrical equations and for obtaining symmetric equations, comprehensive research and techniques are required.

3- Considering the smaller input data and problem dimension and also for increasing the speed and exactness of analysis in BEM, the propsed method is much more versatile than FEM in the frequency domain.

4- To extend the work to time-domain analysis, standard Fourier synthesis, could be used in a straight forward manner.

5- Extension of the work using Hankel fundamental solutions would enhance the accuracy of the radiation boundaries.

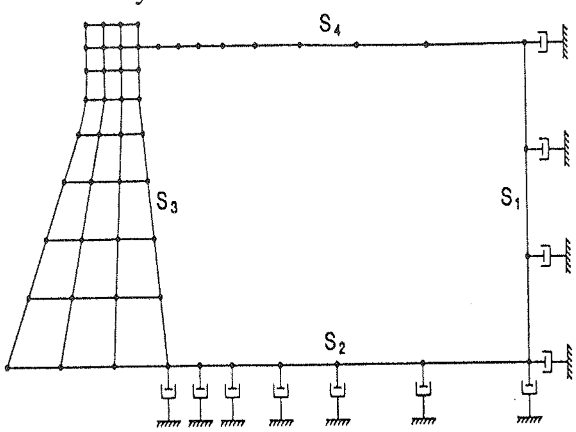

Figure (1) The general model of the dam-reservoir system.

G_1,G_2 and G_4 are submatrices of G, related to S_1, S_2 and S_4 boundary conditions, respectively, D is a matrix for dam effects on the reservoir [1].

Equations (6) are the governing equations set for the dam structure in the time domain. In the frequncy domain, the displacements and forces are as follows:

$$\bar{a} = a\, e^{i\omega t} \tag{14a}$$

$$\bar{F} = F\, e^{i\omega t} \tag{14b}$$

So, substituting equations (14) in equations (6), the following equations set is obtained as the governing equations set for dam structure in the frequency domain:

$$A_s\, a + \rho_f\, i\, \omega\, Q\, \varphi = F_{eq.} \tag{15}$$

in which,

$$A_s = -M_s\, \omega^2 + C_s\, i\, \omega + K_s \tag{15a}$$

$$F_{eq.} = -M_s\, r \tag{15b}$$

Combining equations (13) and (15), we obtain the governing equations set of the coupled dam-reservoir system based on BEM-FEM in the frequency domain:

$$\begin{bmatrix} A_s & \rho_f i\omega Q \\ \rho_f i\omega D & H_0 \end{bmatrix} \begin{bmatrix} a \\ \varphi \end{bmatrix} = \begin{bmatrix} F_{eq.} \\ O \end{bmatrix} \tag{16}$$

By solving the above equations set, the dam-reservoir interaction can be analyzed.

Numerical Results

Two test examples are examined to verify the presented formulation.

Example 1. Hydrodynamic Pressure on a rigid dam

Hydrodynamic pressure in the vertical upstream face of a rigid dam has been computed. The geometry of boundary element model of the reservoir is shown in Figure 2. However the boundary conditions and other assumptions for the fluid domain remain the same as expressed in the past sections. The acoustic velocity c and the water mass density ,ρ_f are 1440 m/s and 1000 kg/m^3, respectively. Figure 3 shows the hydrodynamic pressure distribution on the upstream face of rigid dam for two different values of α (see Equation 2a). In this Figure, ω/ω_1=1.2, in which ω_1 is the fundamental frequency of reservoir. This value being between the first and the second natural frequencies of the reservoir, is usually more demanding for accuracy [12]. As seen in Figure 3, the accuracy is quite good. Figure 4 shows the hydrodynamic forces on the upstream face of rigid dam for a range of frequencies ω/ω_1=0~6. In this figure, two different values of α have been considered.

Example 2. Response of a flexible dam-resevoir system

For evaluating abilities of the presented method for the dam-reservoir interaction analysis, a deformable dam analyzed by Tsai et al. [13] has been selected. Figure 5 shows BEM-FEM model of this dam (66 constant boundary elements and 96 four-node isoparametric finite elements). In this analysis, steady state response for dam crest displacements in the frequency domain has been shown in Figure 6.

For the concrete deformable dam material, it is assumed that, the elasticity modulus E=3.5×10^{10} N/m^2, Poisson ratio ν=0.2, concrete mass density ρ_c=2450kg/m^3 and damping coefficient ξ=5% of critical damping. For evaluating the resevoir effects on dynamic response of the dam, analysis is performed for two cases, namely the empty the Elliptic boundary value problems (such as static and steady-state problems). Nevertheless, for Hyperbolic boundary value problems (such as transient problems), the solution in each time step is dependent to the all previous values of the solution in the past time-history. So, the inherent reduction of space dimension is neutralized by an extra time dimension. In other words, it seems that BEM, when applied to modeling of infinite and semi-infinite domains, has no preference ability for dam-reservoir systems analysis in the time-domain due to complexities of the convolution integral and singularity of the kernels [2 and 4].

However considering the above diffculties first, BEM in the frequency-domain is used as a numerical method for the reservoir field and then it could be employed for the time - domain solutions through the Fourier synthesis.

The governing equation of the reservoir field is equation (7). In the frequency domain, velocity potential and displacement vary by time as follows:

$$\bar{\varphi} = \varphi e^{i\omega t} \tag{10a}$$

$$\bar{a} = a e^{i\omega t} \tag{10b}$$

where i is the imaginary number $\sqrt{-1}$ and ω is the loading frequency. Substituting equation (10) in equation (7), we have:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \varphi + k^2 \varphi = 0 \tag{11}$$

where $k(=\omega/c)$ is the wave number.

In BEM similar to FEM for numerical solving of equation (11), the weighted residual method is used. Then by using the weighted residual method, Green Lemma and using a special weighing function, the fundamental solution, an important boundary integral equation is obtained. It is approved that Bessel functions (as the fundamental solution) can be used for wave propagation problems in finite domains. For infinite domains, the Bessel functions have not a good behavior because of reflected wave terms from infinity and thus Hankel functions must be used as the fundamental solution. However, since in this research, the reservoir far field has been modeled by a proper boundary condition (Sommerfeld boundary condition), the Bessel functions have been adopted as the fundamental solutions [6]. On the other hand, the fundamental solutions in 3D problems have logarithmic forms whose calculation by numerical integration is much simpler than that of 2D problems where the Bessel/Hankel functions govern. Therefore, extension of the problem to 3D is very easy. By discretizing the reservoir boundaries by boundary elements, the following basic equations set is derived:

$$H \varphi = G q \tag{12}$$

H and G are coefficient matrices; φ is the vector of velocity potential nodal values; q the nodal vector of velocity potential normal derivatives.

After inserting the boundary conditions (equations 1 to 4) to equation (12), the following discrete set of equations is derived as the governing equations set for the reservoir field in the frequency domain:

$$H_0 \varphi + \rho_f i \omega D a = 0 \tag{13}$$

in which,

$$H_0 = -\rho_f \left(H + \frac{i\omega}{c} G_1 + \frac{i\omega}{\beta c} G_2 + \frac{\omega^2}{g} G_4\right) \tag{13a}$$

and the hydrodynamic force vector due to fluid - structure interaction is

$$F_I = -\rho_f \left(\int_{S_3} N_s^T\, n\, N_f\, ds \right) \dot{\bar{\varphi}} = -\rho_f\, Q\, \dot{\bar{\varphi}} \quad (6b)$$

M_s, C_s and K_s are the mass, damping and stiffness matrices of the dam respectively; $\bar{a}$, $\dot{\bar{a}}$ and $\ddot{\bar{a}}$ are nodal displacement, velocity and acceleration of the dam, respectively; N_s and N_f are the structure and fluid shape function matrices, respectively; ρ_f is the fluid mass density; r is the rigid body displacement vector of the structure; $\ddot{\bar{u}}_g(t)$ is the ground acceleration; n is the unit vector normal to the dam upstream face; $\bar{\varphi}$ is the nodal velocity potential of the fluid domain and Q is the fluid- structure interaction matrix.

The governing equation of the resrevoir field is the well-known Helmholtz equation as follows:

$$\nabla^2 \bar{\varphi} - \frac{1}{c^2} \ddot{\bar{\varphi}} = 0 \quad (7)$$

Similar to the structure field, using FEM based on the weighted residual method, the following discrete set of equations is derived as the governing equation set for the reservoir domain:

$$M_f \ddot{\bar{\varphi}} + C_f \dot{\bar{\varphi}} + K_f \bar{\varphi} = F'_I \quad (8)$$

in which,

$$M_f = \frac{1}{c^2} \int_{V_f} N_f^T \rho_f N_f\, dV + \frac{1}{g} \int_{S_4} N_f^T \rho_f N_f\, ds$$

(The mass matrix of reservoir)(8a)

$$C_f = \frac{1}{c^2} \int_{S_1} N_f^T \rho_f N_f\, ds + \frac{1}{\beta c} \int_{S_2} N_f^T \rho_f N_f\, ds$$

(The damping matrix of resrvoir)(8b)

$$K_f = \int_{V_f} \nabla^T N_f \rho_f \nabla N_f\, dV$$

(The stiffness matrix of reservoir)(8c)

$$F'_I = -\rho_f \left(\int_{S3} N_f^T n^T N_s\, ds \right) \ddot{\bar{a}} = -\rho_f Q^T \ddot{\bar{a}}$$

(The fluid-structure interaction effects)(8d)

Combining equations (6) and (8), we obtain the governing equations set of coupled dam-reservoir systems as follows:

$$\begin{bmatrix} M_s & 0 \\ 0 & M_f \end{bmatrix} \begin{bmatrix} \ddot{\bar{a}} \\ \ddot{\bar{\varphi}} \end{bmatrix} + \begin{bmatrix} C_s & \rho_f & Q \\ \rho_f & Q^T & C_f \end{bmatrix} \begin{bmatrix} \dot{\bar{a}} \\ \dot{\bar{\varphi}} \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_f \end{bmatrix} \begin{bmatrix} \bar{a} \\ \bar{\varphi} \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix} \quad (9)$$

The above equations system is a symmetric equations set.

BEM-FEM Formulation

Recent progresses in the BEM show that due to the low amount of effort needed for discretization and solution, this method is an efficient method for analysis. In dam-reservoir interaction analysis, hydrodynamic pressure distribution on the upstream face of dam is important for the dam designer though its distribution in other parts of the reservoir has no direct significance. As BEM is related only to the boundaries of the reservoir, this method reduces both problem dimensions and input data, and therefore could increase the speed and precision of the solution.

BEM is a perfectly successful method for

Basic Assumptions

1- The problem is 2D.
2- Only horizontal earthquake is exerted to the dam base.
3- Reservoir water is compressible, nonviscous (invicid) and irrotational.
4- The dam material is isotropic and linear-elastic.
5- Free surface linear waves are considered.
6- Dam-foundation interaction is neglected.
7- Dam-reservoir interaction is considered.
8- Mass density of water is constant.
9- Amplitude of displacements is small.
10- For radiation (transmitting) boundary, the Sommerfeld boundary condition is adopted.
11- Refraction boundary condition is considered for partial energy absorption of reservior bottom.
12- The veloctiy potential is considered as the reservoir variable.

Boundary Conditions

According to the above assumptions (also see Figure1), the boundary conditions for the reservoir field are as follows:

a) Radiation (truncation/transmitting/transparent) boundary condition (S_1 boundary):

$$\frac{\partial \bar{\varphi}}{\partial n} = -\frac{1}{c}\dot{\bar{\varphi}} \tag{1}$$

b) Refraction boundary condition for reservoir bottom (S_2 boundary):

$$\frac{\partial \bar{\varphi}}{\partial n} = -\frac{1}{\beta c}\dot{\bar{\varphi}} \tag{2}$$

$$\beta = \frac{1+\alpha}{1-\alpha} \tag{2a}$$

c) Interaction boundary condition (S_3 boundary):

$$\frac{\partial \bar{\varphi}}{\partial n} = -\dot{\bar{u}}_{sn} \tag{3}$$

d) Free surface boundary condition (S_4 boundary):

$$\frac{\partial \bar{\varphi}}{\partial n} = -\frac{1}{g}\ddot{\bar{\varphi}} \tag{4}$$

where $\bar{\varphi}$ is the velocity potential, c is the water pressure waves velocity, β is the relative acoustic impedance of reservoir bottom, n is the unit vector normal to the reservoir boundaries, $\dot{\bar{u}}_{sn}$ is the normal velocity of dam upstream face, g is the gravity acceleration and α the reservoir bottom reflection coefficient.

FEM-FEM Formulation

The governing equations set for the structure is the well-known equilibrium equations as follows:

$$\sigma_{ij,j} + b_i = 0 \tag{5}$$

where σ_{ij} are the components of the stress tensor and b_i are the body forces.

In FEM, for solving a differential equation with known boundary conditions, an approximate method (such as the weighted residual method)is used. Based on this approximate method and by using the Green Lemma, the following discrete set of equations is derived as the governing equations set for the structure:

$$M_s \ddot{\bar{a}} + C_s \dot{\bar{a}} + K_s \bar{a} = F_0 + F_I \tag{6}$$

in which the inertia force vector due to earthquake is

$$F_0 = -M_s\, r\, \ddot{\bar{u}}_g\ (t) \tag{6a}$$

An Efficient Method for Dam-Reservoir Seismic Interaction Analysis

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Abstract

An efficient method for two-dimensional dynamic interaction analysis of gravity dam-reservoir systems is presented. The Euler-Lagrangian approach, based on BEM-FEM formulation in the frequency domain, is adopted. Displacement-based finite elements for the structure and velocity potential-based boundary elements for the fluid domain are developed. Compressibility of the fluid is considered. In this formulation, symmetry of the equations system could only be achieved once the fluid domain is circular and interaction absent. The results show that this method can be offered as an appropriate method for dam-reservoir interaction analysis and indeed extendable to time-domain analysis.

Keywords

Boundary Element Method, Finite Element Method, and Fluid-Structure Interaction.

Introduction

Research on linear dynamic response of dam-reservoir systems, has made progress toward a completeness stage. The analytical methods are applicable only to simple geometry and boundary conditions, and in practical cases with irregular or complicated conditions, it is necessary to apply numerical methods such as finite element methods (FEM) and boundary element methods (BEM). For the fluid, two different approaches have been used by different researchers [2]; i.e., the displacement formulation (Lagrangian approach) and the velocity potential/pressure formulation (Eulerian approach). Due to its less degrees of freedom, the Eulerian approach has been employed for the fluid domain. For modeling of the structure domain, the Lagrangian approach has been adopted. In the Eulerian FE formulation of the fluid, pressure field would lead to nonsymmetrical interaction system [10]. L. G. Olson and K. J. Bathe [7] presented a method for obtaining a symmetric set of equations based on FEM for both fluid and structure domains (FEM-FEM). They used the velocity potential and hydrostatic pressure as the fluid variables and displacement as the structure variable. In this research, the above idea has been elaborated for obtaining a symmetric set of equations based on FEM for both reservoir water and dam structure (FEM-FEM) using velocity potential for the fluid and displacement for the dam. Furthermore the same approach is adopted for the FEM-BEM formulation to explore the possibility and conditions of achieving symmetry[9].