Table (1) Direction cosines and weight coefficients of integration points.

Direction cosines of integration points			Weights W_i
l_i	m_i	n_i	
$\sqrt{1/3}$	$\sqrt{1/3}$	$\sqrt{1/3}$	27/840
$\sqrt{1/3}$	$-\sqrt{1/3}$	$\sqrt{1/3}$	27/840
$-\sqrt{1/3}$	$\sqrt{1/3}$	$\sqrt{1/3}$	27/840
$-\sqrt{1/3}$	$-\sqrt{1/3}$	$\sqrt{1/3}$	27/840
$\sqrt{1/2}$	$\sqrt{1/2}$	0.0	32/840
$-\sqrt{1/2}$	$\sqrt{1/2}$	0.0	32/840
$\sqrt{1/2}$	0.0	$\sqrt{1/2}$	32/840
$-\sqrt{1/2}$	0.0	$\sqrt{1/2}$	32/840
0.0	$-\sqrt{1/2}$	$\sqrt{1/2}$	32/840
0.0	$+\sqrt{1/2}$	$\sqrt{1/2}$	32/840
1.0	0.0	0.0	40/840
0.0	1.0	0.0	40/840
0.0	0.0	1.0	40/840

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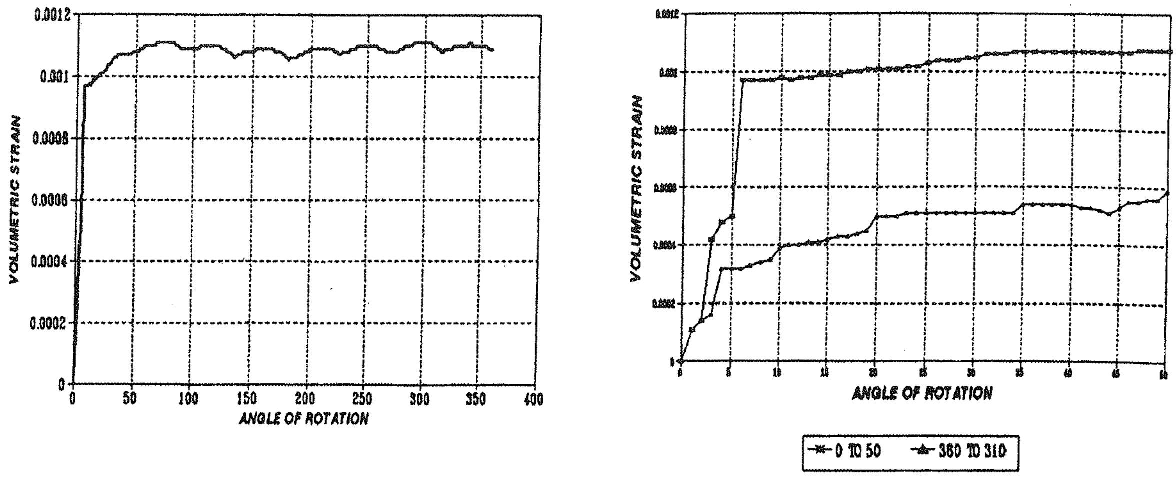

Figure (4) a) εv versus zero to 50° and 360 to 310° of β b) εv versus zero to 360° to 310° of β

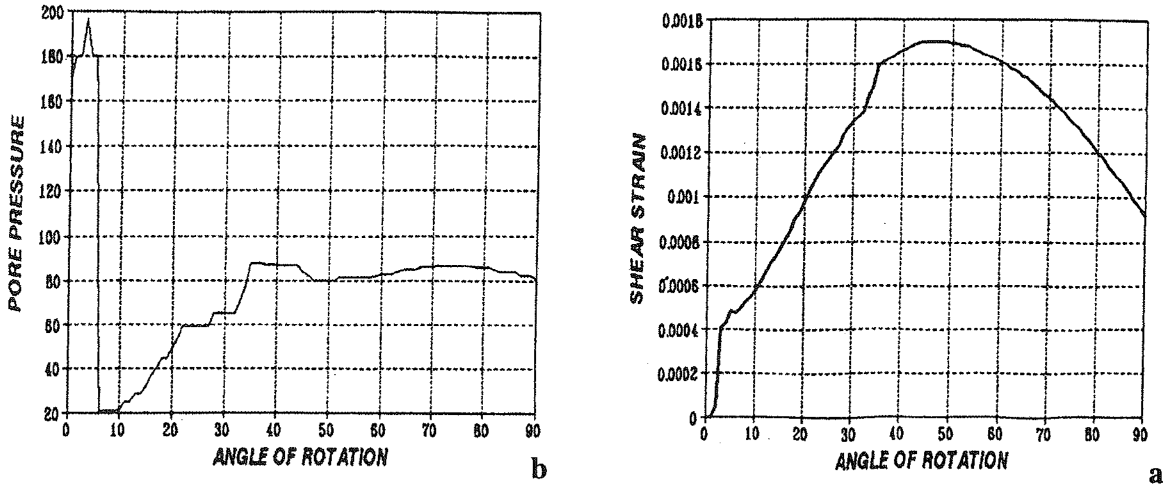

Figure (5) a) shearing strain, γoct, versus zero to 90° of β b) pore water pressure versus β

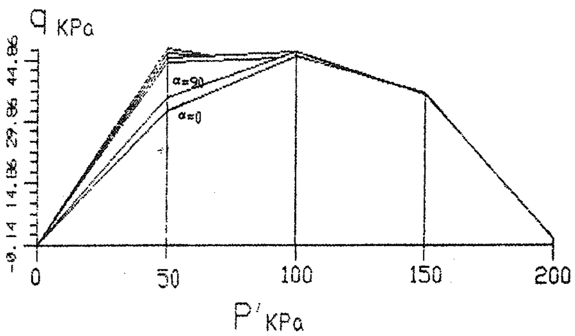

Figure (7) projection of SBS on q versus p'

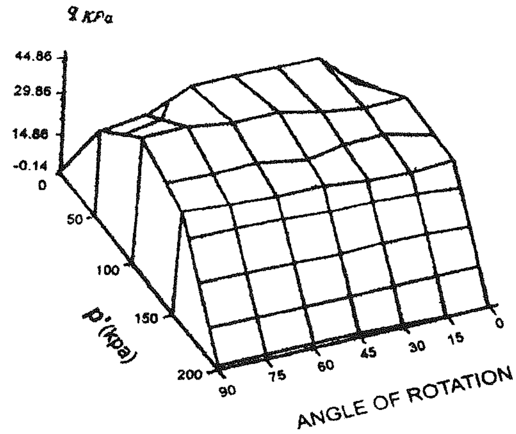

Figure (6) q (stress deviator), p' (mean effective stress) versus β

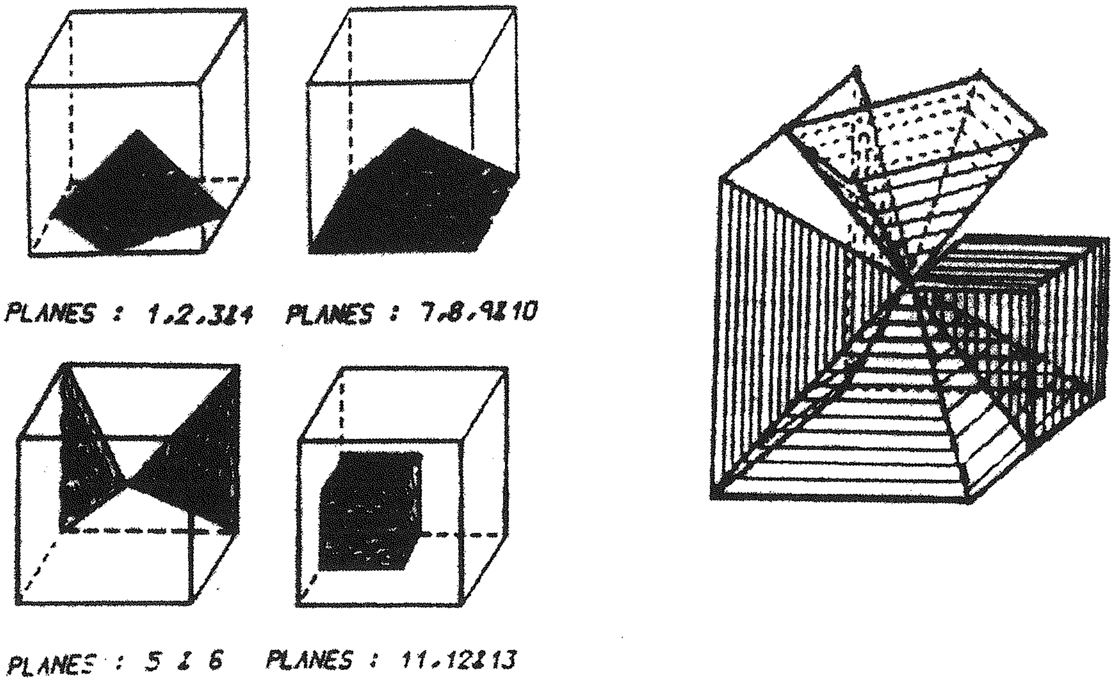

Figure (2) Demonstration of the orientation of 13 planes.

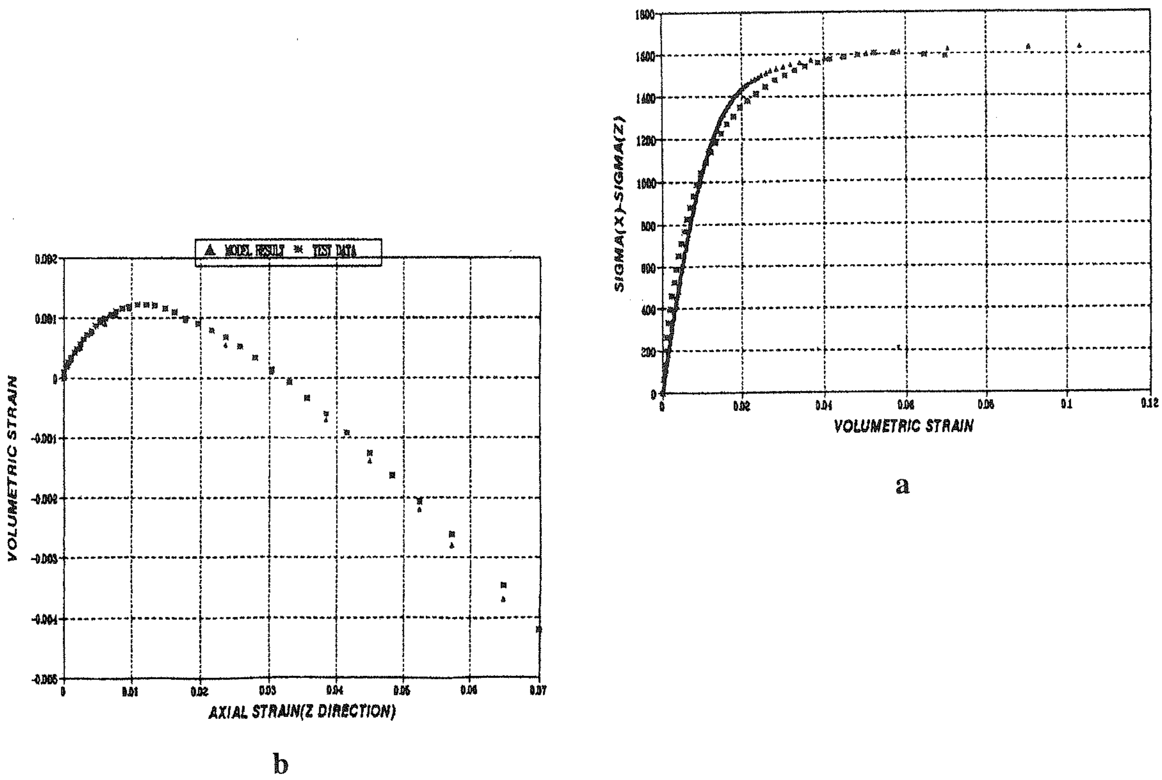

Figure (3) a) q - ε_v, Model Calibration b) εv - e_1, Model calibration ter pressure may even make liquefaction of sand.

Presentation of (SBS) Surface

To obtain SBS surface a combination of β and b changes can be defined and the obtained results can be presented in a three dimensional curve as shown in Figure 6. This Figure is a variation of q (stress deviator) versus p' (mean effective stress) versus β (angle of rotation of principal stress axes). Generally, b can obtained as follows:

$$b = (\sigma z - \sigma 3) / (\sigma 1 - \sigma 3) \qquad (21)$$

Accordingly, β, b, σ_z are kept constant, while σ_x and σ_z are increased. Therefore, τ_{xy} is obtained from equation 18. However, the value of b and σ_z are found as follows:

$$b = \frac{\sigma 2 - (\sigma x - \sigma z) / 2 + 0.5\,(\sigma x - \sigma z)\sqrt{1 + \tan^2(2\beta)}}{(\sigma x - \sigma z)\sqrt{1 + \tan^2(2\beta)}} \qquad (22)$$

$$\sigma z = \frac{\sigma 2 - \sigma z\,(0.5 + (b - 0.5)/\cos(2\beta))}{(0.5 - (b - 0.5)/\cos(2\beta))} \qquad (23)$$

After running the program several times and keeping b = 0, the provided SBS is shown in Figure 6. For more clarification, The projection of this three dimensional curve on q versus p' is shown in Figure 7.

Conclusion

From this study the capability of model to predict the effect of rotation of principal stress axes through plastic behavior of granular material upon undrained condition has been examined. Furthermore, the concept of behavior of granular material on the basis of sliding mechanisms and elastic behavior of particles has worked out.

The rotation of principal axes are included in such a way that there is a clear proof for it, instead of having only some hypotheses without enough reasons. Also, it has been demonstrated clearly that how much effect may be provided due to rotation of stress and strain axes. It has been shown that upon undrained condition, the volume change of soil skeleton and compressibility of water together can build up some pore water pressure. This increment of pore water pressure may be led to liquifaction in loose sands.

The undrained state boundary surface SBS has been numerically defined using q, p', and β. This SBS, which is the portrayal of the initial anisotropy of the material behavior, means that the volume change can be a state quantity for granular materials.

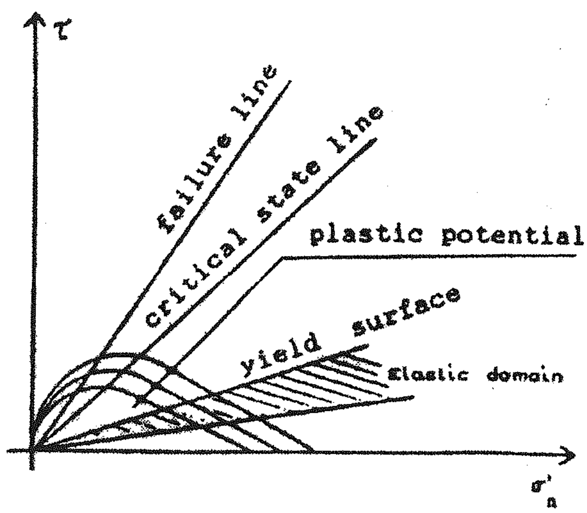

Figure (1) Yield, plastic potential, elastic zone, critical state line, and failure line in t-σ_n space.

$$D^{ep} = [C^{ep}]^{-1} \tag{16}$$

$$C^{ep} = C^{e} + C^{p} \tag{17}$$

where C^e, C^p, and C^{ep} are compliance elasticity , plasticity, and elasto-plasticity matrices.

According to incremental algorithm, C^{ep} computed in previous step can be used for current step, therefore , the solution will not remain indeterminate.

Accumulation of Pore Pressure

The excessive pore water pressure is obtained through assuming water as elastic medium without shear resistance. Under cyclic loading, hysteresis loop of dissipated energy takes place. Consequently, while plastic strain takes place, some volumetric strain shall be obtained in soil as well as water. The difference of these two strain values multiplied by bulk modulus of soil and water mixture can present pore water pressure increment.

Identification of Parameters

In a general case, for the most anisotropic, non-homogeneous material, 13 sets of material parameters corresponding to plastic sliding of each sampling planes are required. However, any knowledge about the similarity of the sliding behavior of different sampling planes reduces the number of required parameters.

The number of parameters required to be used in proposed model to obtain the behavior of an isotropic homogenous sand is five. Two of these parameters correspond to elastic behavior of soil skeleton and the rest to plastic flow on each sampling plane. These paramters are listed as follows:

1) Elastic modulus, E, 2) Poisson ratio, ν, 3) Slope of critical state line, η_c, 4) Constant value in hardening function, Ao, 5) Peak angle of internal friction, ϕ_f.

E and υ are found in the usual way as for any other model. The other three parameters correspond to the plastic behavior of one plane. In this research, these three parameters have been assumed to be the same for all 13 defined planes because of initial isotropic conditions.

Rotation of Principal Stress and Strain Axes

In order to demonstrate this capability, some numerical tests were planned in which the values of principal stresses are constant.

The only change which takes place in these numerical tests is the direction of principal stress axes. Figure 3-a , and 3-b show the obtained model result and experiment in calibrating the model. Figure 4 shows the variation of volumetric strain versus zero to 50° and 360 to 310° of β in undrained test. Clearly, the variation of ε_v upon change of same value of β, although the trends are different, are not same. Therefore, the trend for change of β or on the other words, the stress path can affect on the volume change of soil. Figure 5-a shows the variation of shearing strain invariant, γ_{oct}, versus zero to 90° of β in undrained test. Figure 5-b shows the variation of pore water pressure during stated rotation path.

Consequently, it has been shown from the numerical results that the strains due to rotation of principal stress direction can be significantly large and the built up pore waceptual numerical integration of multilaminate framework presents the following summation for computing C^p.

$$C^p = 4\prod . \sum_{i=1}^{n} W_i . L^T C_i^p L \qquad (8)$$

Where Wi are weight coefficients and C^p is the global plastic compliance matrix corresponds to a single point in the medium and L is transformation matrix for corresponding plane.

Definition of Planes in Three Dimensional Media

The choice of 13 independent planes for the solution of any three dimensional problem is a fair number. The orientation of the sampling planes as given by their direction cosines and the weight coefficients for numerical integration rule are given in table 1.

One of the important features of multilaminate framework is that it enables identification of the active planes as a matter of routine. The application of any stress path is accompanied with the activities of some of the 13 defined planes in three dimensional media. The value of plastic strain on all the active planes are not necessarily the same. These priorities and certain active planes can change due to any change of direction of stress path, a number of active planes may stop activity and some inactive ones become active and some planes may take over others with respect to the value of plastic shear strain. Thus the framework is able to predict the mechanism of failure.

Figure 2 shows the orientation of all 13 planes in similar cubes. In order to clarify their positions, they have been presented in four cubes.

The Model Response Under Undrained Conditions

The principle of effective stress for a saturated soil element in incremental form is stated as follows:

$$d\sigma = d\sigma + m . dU \qquad (9)$$

where d used for representing small increments and m = (1, 1, 1, 0, 0, 0).

It can also be assumed that in a fully undrained case, the skeleton volume change is precisely equal to change in the volume of pore water. Therefore, dU is calculated as:

$$dU = K_f . \{m\}^T . d\varepsilon \qquad (10)$$

where K_f is obtained [9] as follows:

$$\frac{1}{K_f} = \frac{1}{K_w} + \frac{1-\upsilon}{K_s} \qquad (11)$$

where K_w is bulk modulus of water and υ is initial porosity.

$$dU = K_f . \{m\}^T . d\varepsilon . \{m\} \qquad (12)$$

Retaining the elastolastic consititutive law, it is presented as follows:

$$d\sigma = D^{ep} . d\varepsilon \qquad (13)$$

$$d\sigma = D^{ept} . d\varepsilon \qquad (14)$$

where D^{ep} and D^{ept} are effective and total stress-strain matrices. Substituting equations 26, 27, and 28 in 23 simply the result is written as:

$$D^{ept} = D^{ep} + K_f . \{m\}^T . \{m\} \qquad (15)$$

where,

control such as yielding should be checked at each of the planes and those of the planes which are sliding will contribute to plastic deformation. Therefore, the granular material mass has an infinite number of yield functions usually one for each of the planes in the physical space.

The Constitutive Equations of Multilaminate Model

The classical decomposition of strain increments under the concept of elasto-plasticity in elastic and plastic parts are schematically written as follows:

$$d\varepsilon = d\varepsilon^{e} + d\varepsilon^{p} \tag{1}$$

The increment of elastic strain ($d\varepsilon^e$) is related to the increments of effective stress ($d\sigma$) by:

$$d\varepsilon^{e} = C^{e} . d\sigma \tag{2}$$

where , C^e is elastic compliance matrix, usually assumed as linear.

For the soil mass, the overall stress-strain increments relation, to obtain plastic strain increments ($d\varepsilon^p$), is expressed as:

$$d\varepsilon^{p} = C^{p} . d\sigma \tag{3}$$

where, C^p is plastic compliance matrix.

Clearly, it is expected that all the effects of plastic behavior be included in C^p. To find out C^p, the constitutive equations for a typical slip plane must be considered in calculations. Consequently, the appropriate summation of all provided compliance matrices corresponding to considered slip planes yields overall C^p.

The equation of yield function is formulated as follows:

$$Fi\ (\tau i\ , \sigma_{ni}\ , \eta i) = \tau i \pm \eta i\ \sigma_{ni} \tag{4}$$

where, τi and σni are stress components on ith plane and $\eta i=\tan(\alpha i)$ is a hardening parameter and assumed as a hyperbolic function of plastic shear strain on the ith plane. αi is the slope of yield line.

Feda [8], derived the plastic potential function which is used in this research. This function is stated in terms of τi and σi for the τ versus σ_n space as follows:

$$\psi\ (\tau i\ , \sigma_{ni}) = \tau i \pm \eta_{c} . \sigma_{ni}\ , \ln\ (\sigma_{ni} / \sigma_{nio}) \tag{5}$$

where, η_c is the slope of critical state line and σ_{ni} is the initial value of effective normal stress on ith plane. Typical presentations of this function are shown in Figure 1.

In theory of plastic flow, consistency condition is a necessary condition which requires that a yield criterion be satisfied as far as the material is in a plastic state. Upon the use of general flow rule, consistency condition and a simple shear hardening, plastic compliance matrix of ith plane is obtained as follows:

$$C^{p}_{i} = \{1/Hp_{i}\} . \{\partial\psi_{i} / \partial\sigma_{i}\} \{\partial F_{i}/\partial\sigma_{i}\}^{T} \tag{6}$$

where Hpi is defined as hardening modulus of ith plane and is obtained as follows:

$$Hpi = - \{\partial F_{i} / \partial \varepsilon_{i}^{pt}\}.\{\partial\psi_{i}/\partial\tau_{i}\} \tag{7}$$

where εi^{pt} is plastic shear strain on ith plane.

C^p_i as a whole , represent the plastic resistance corresponds to ith plane and must be summed up as the contribution of this plane with the others. Accordingly, the concal axis. The medium loose Ham river sand exhibited partial liquefaction due purely to the rotation of principal stress directions. The dominance of the initial anisotropy during principal stress rotations has been incorporated in the concept of the state boundary surface (SBS).

For a granular mass such as sand that supports the overall applied loads through contact friction, the overall mechanical response ideally may be described on the basis of micro-mechanical behavior of grains interconnections. Naturally, this requires the description of overall stress, characterization of fabric, representation of kinematics, development of local rate constitutive relations and evaluation of the overall differential constitutive relations in terms of the local quantities.

In recent years, another class of models called' multilaminate model' was developed by Zienkiewics and Pande (1977) [3] for jointed rock masses and by Pande and Sharma [4] for clays. Bazant and Oh [5] have developed a similar model for fracture analysis of concrete under the name microplane model'.

An elastoplastic model named reflecting surface model developed by Pande and Pietruszczak [6] and used to predict cyclic loading behavior of normally consolidated and lightly over-consolidated clays. Sadr-*nejad* [7] also developed a model for the prediction of elastic-plastic behavior and liquefaction of sand.

This paper presents a multilaminate model capable of predecting the behavior of granular material under monotonic, cyclic loading and other respectively complex stress paths.

The concept of proposed model is natural, physically meaningful and extremely simple. According to this formulation which is based on a simple numerical integration, an appropriate connection between averaged micro and macro-mechanical behavior of material has been presented. The inclusion of the rotation of principal stress and strain axes, induced anisotropy and the possibility of supervising and even controlling any variation through the medium are the significant of the model.

Basic Assumptions and Discussions

Multılaminate framework by defining the small continuum structural units as an assemblage of particles and voids which fill infinite spaces between the sampling planes, has appropriately justified the contribution of interconnection forces in overall macro-mechanics. Plastic deformations are assumed to occur due to sliding, separation/closing of the boundaries and elastic deformations are the overall responses of structural unit bodies. Therefore, the overall deformation of any small part of the medium is composed of total elastic response and an appopriate summation of sliding, Seperation/closing phenomenon under the current effective normal and shear strsses on sampling planes.

According to these assumptions overall sliding, separation/closing of intergranular points of grains included in one structural unit are summed up and contribured as the reslult of sliding, separation/closing surrounding boundary planes. This simply implies yielding, failure or even ill-conditioning and bifurcation response to be possible over any of the randomly oriented sampling planes. Consequently, plasticity

Numerical Evaluation of State Boundary Surface (SBS) Through Rotation of Principal Stress Axes in Sand

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Abstract

In applying shear stress to saturated soil with arbitrary stress paths, the prediction of the exact value of strains is difficult because of mainly its stress path dependent nature.

Rotation of the principal stress axes during shearing of the soil is a feature of stress paths associated with many field loading situations. A proper understanding of the effects of principal stress rotation on soil behavior can be provided if the anisotropy existing prior to stress rotation and induced anisotropy due to plastic flow in soil are clearly understood and modeled.

A multilaminate based model for soil is developed and used to compute and present the influence of rotation of principal stress axes on the plastic behavior of soil. This is fulfilled by distributing the effects of boundary condition changes into several predefined sampling orientations at one point and summing the micro-results up as the macro-result. The validity of the presented model examined by comparing numerical and test results showing the mentioned aspect. In this paper, the state boundary surface (SBS) is numerically obtained by a multilaminate based model capable of predicting the behavior of sand under the influences of rotation of the direction of principal stress axes and induced anisotropy. The predicted numerical results are tally in agreement with experiments.

Introduction

The volumetric strain in drained tests as a state quantity in strains which should posses an essence of stress path independency by definition is a unique quantity determined when the origin and the destination of stress paths are given. The concept of the SBS state boundary surface was first introduced by Roscoe and Poorooshasb [1]. This surface in (ω, p', q) (note, ω is moisture content of soil) space is that surface confining a space between itself and the origin, within which a point can represent a state of an element of the soil but outside which a point cannot represent such a state. A unique surface named SBS [2] interrelates the action of volumetric strain in drained shear and generated pore water pressure in undrained shear as a boundary surface in q, p', e space. The letters q, p', e stand for stress deviator, mean effective stress and void ratio repectively. Thereafter, Symes [3] experimentally demonstrated state boundary surface for isotropically consolidated sand under undrained condition in q, p', β space. β stands for the angle of rotation of principal stress axes respect to verti-