Table (2) Models used for comparing results with those of Xue et al. [8]

Model No.	R (cm)	h (cm)	b (cm)	a (cm)	b/a	β (cm)
SH 2510	100	2.5	10	10.02	- 1	0.57
SH 2520	100	2.5	20	20.14	0.99	1.15
SH 2530	100	2.5	30	30.47	0.98	1.74
SH 2540	100	2.5	40	41.15	0.97	2.35
SH 2550	100	2.5	50	52.36	0.95	2.99
SH 2560	100	2.5	60	64.35	0.93	3.68
SH 25 70	100	2.5	70	77.54	0.90	4.43

References

[1] Lekkerkerker, J. G., On the Stress Distribution in Cylindrical Shells Weakned by a Circular Hole, Uitgeverig Waltman, Delft, 1965.

[2] Van Dyke, P., Stress About a Circular Hole in Cylindrical Shell, A. I. A. A. J., 1965, 3, 1733.

[3] Tsai, C. J., and Sanders. J. L., Jr., Elliptical Cut-Outs in Cylindrical Shells, 1975, A.S.M.E. paper N75 APM 10.

[4] Guz, A. N., Stress Concentration Around Oirfices in Thin Shells (Survey), Soviet Applied Mechanics, 1969, 3, 217.

[5] Mizoguchi, K., Tanigawa, Y. and Yamamoto, F., Deformation and Strength of a Cylindrical Shell with Cut-Outs, Bulletin of JSME, 1972, 15, 413.

[6] Steele, C.R. and Steele, M. L., Stress Analysis of Nozzles in Cylindrical Shells with External Load, Trans. ASME, J. of Pressure Vessel Tech., 1983, 105, 191.

[7] Steele, C.R., Steele, M.L., and Kathian, A.J., An Efficient Computational Approach for a Large Opening in a Cylindrical Vessel, Trans. ASME, J. of Pressure Vessel Tech., 1986, 108, 436.

[8] Xue, M.D., Deng, Y., and Hwang, K.C., Some Results on Analytical Solution of a Cylindrical Shells with Large Openings, Trans. ASME, J. of Pressure Vessel Tech., 1991, 113, 297.

[9] Xue, M.D., Chen, W. and Hwang, K. C., Stresses at the Intersection of Two Cylindrical Shells, Nuc. Eng. Des., 1995, 154, 231.

[10] Mahdi, T. A. and Goltabar, A. R., On the Implementation of Finite Element for Cylindrical Shells with Holes, Proc. Int. Cong. Comp. Meth. Engg., Shiraz, 1993, 175.

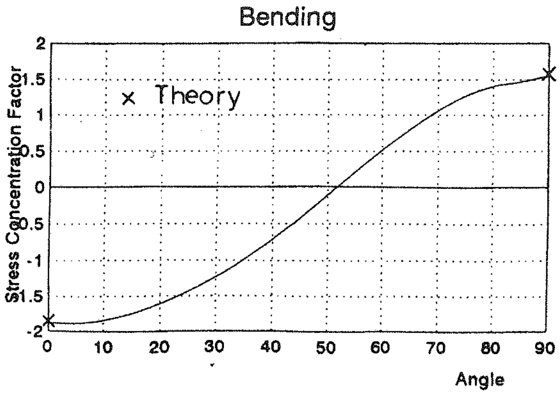

Figure (12) Tangential bending stresses around the hole for model SH 2540.

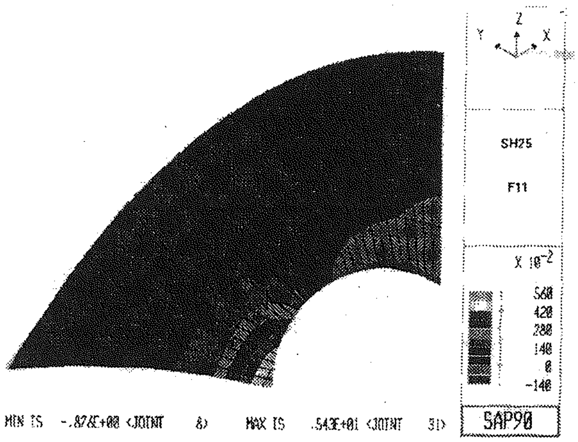

Figure (13) Tangential mebrane stresses near the edge of the hole for model SH 2540.

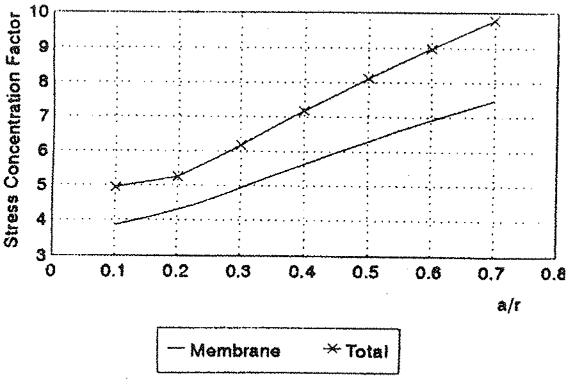

Figure (10) Menmbrane stress for the shells given in table (2).

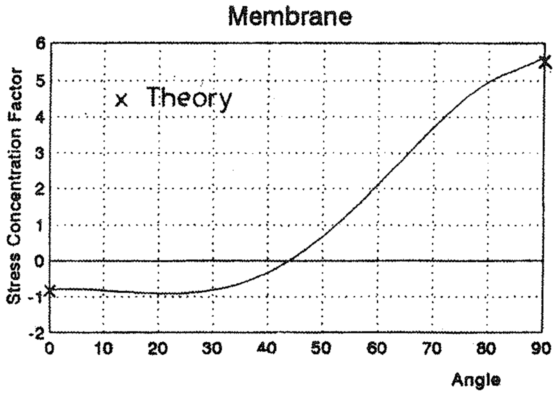

Figure (11) Tangential membrane stresses around the hole for model SH 2540.

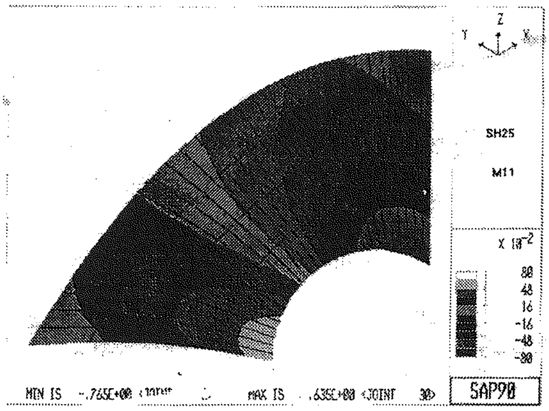

Figure (14) Tangential bending stresses near the edge of the hole for model SH 2540.

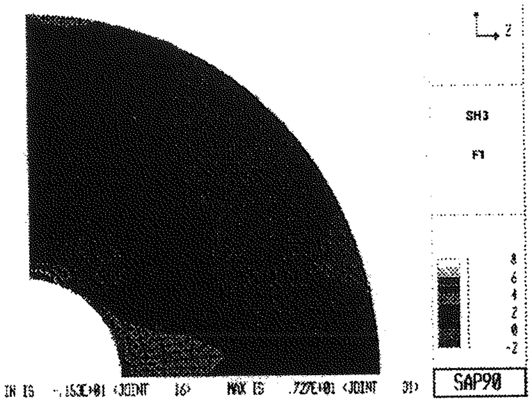

Figure (7) Tangential memberane stresses developed near the edge of the hole for a shell having $\beta = 6$ and $a/R = 0.1$.

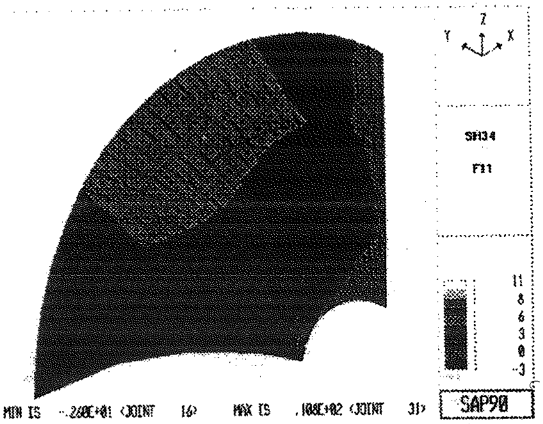

Figure (8) Tangential membrance stresses developed near the edge of the hole for a shell having $\beta = 6$ and $a/R = 0.4$.

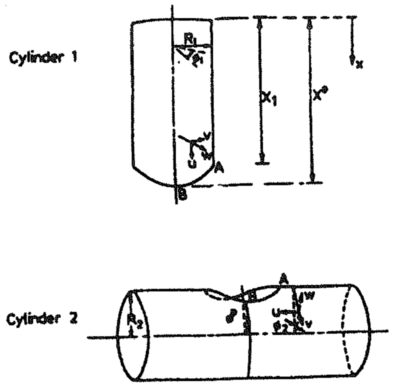

Figure (9) A Hole developed in cylinder 2 as a result of cylinder-to-cylinder intersections

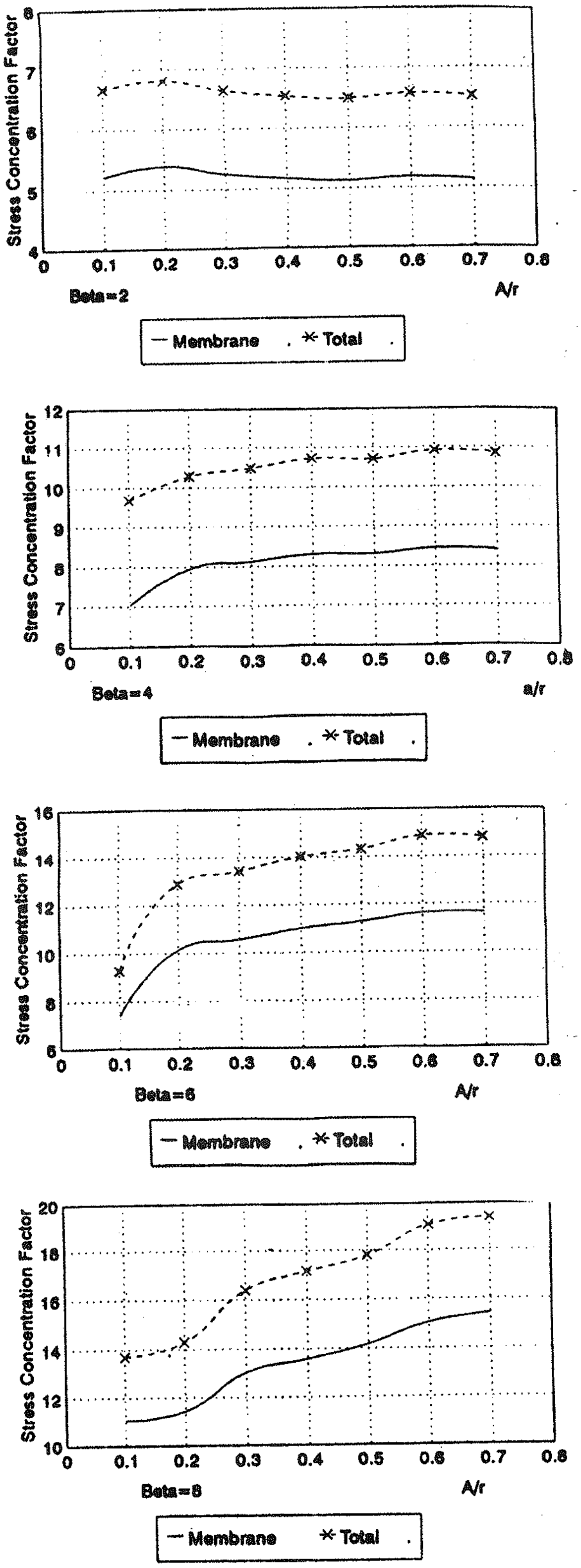

Figure (6) Stress concentration factors at $\theta = 90$ for shell with different (a/R) ratios.

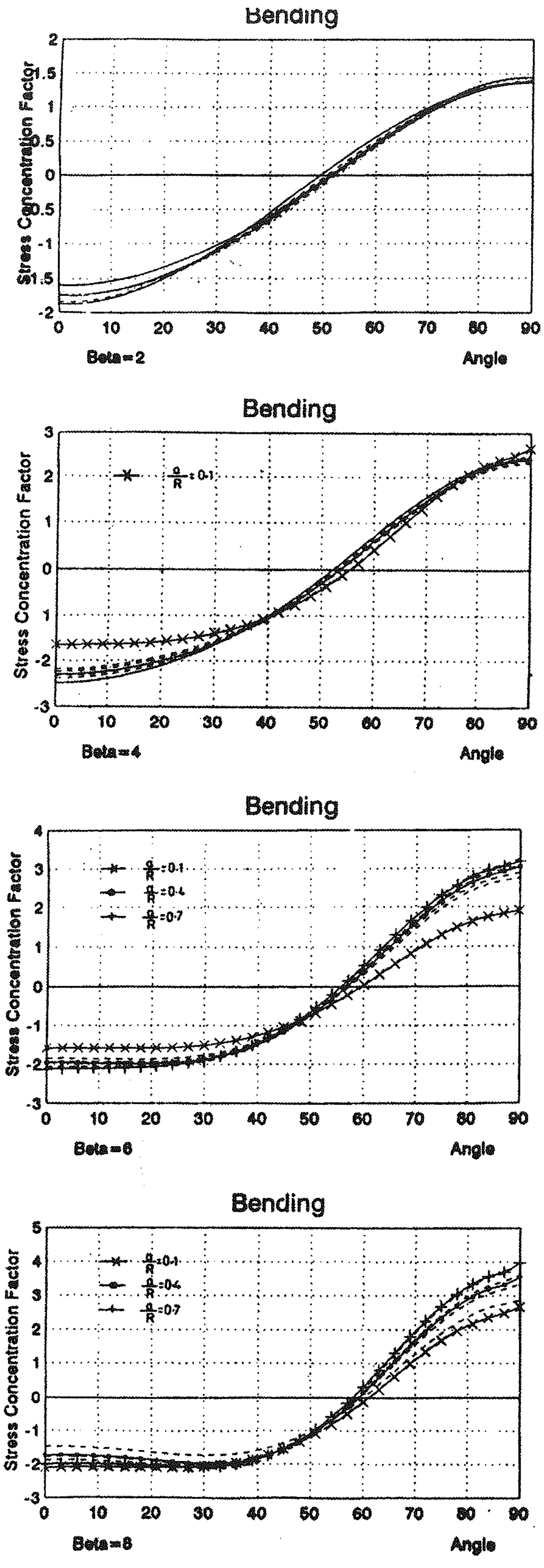

Figure (5) Tangential bending stresses around the hole for shell with different (a/R) Ratios.

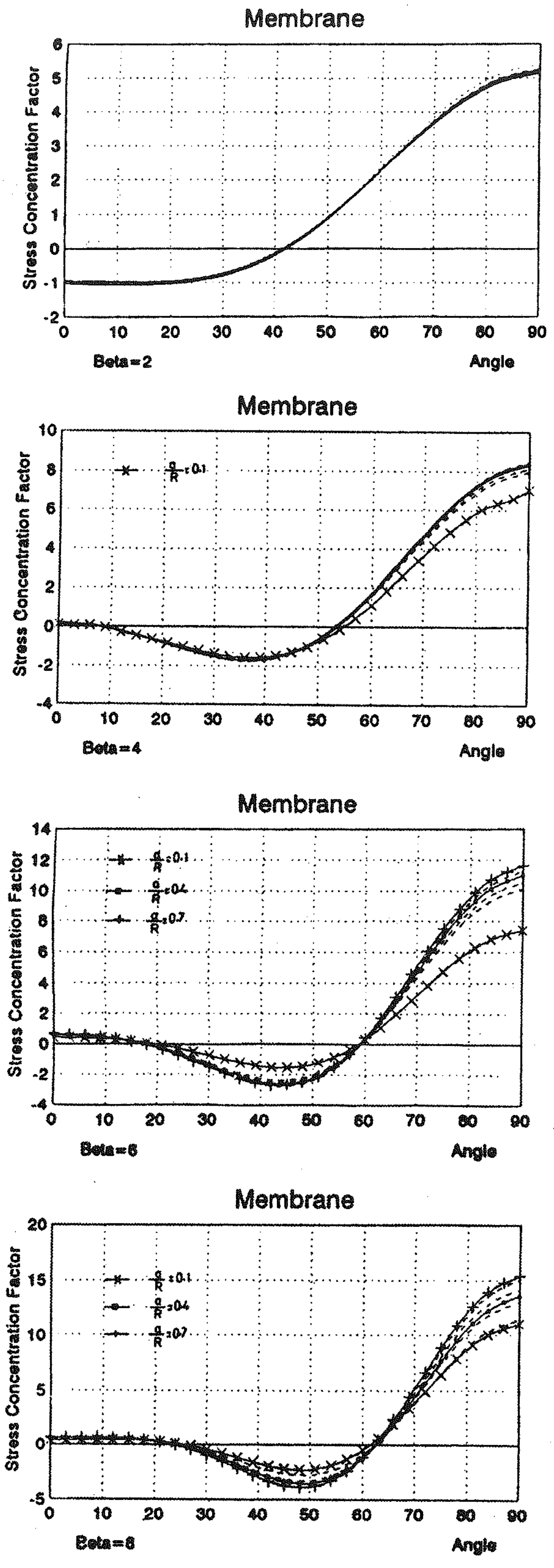

Figure (4) Tangential membrane stresses around the hole for shell with different (a/R) ratios.

has been tested. Table 2 gives the geometrical descriptions of these models. For shells subjected to uniform longitudinal tensions, the tangential stress factors for different (a/R) ratios are presented in figure 10.

For more detailed analysis, model SH2540 has been investigated. The results for tangential membrane stress and bending stress concentration factors along the edge of the hole, are shown in figures 11 and 12 respectively. In these two figures, it has been shown that the finite element method produced results that are quite comparable with the analytical one. The stresses produced near the hole for the tangential membrane and tangential bending cases are shown in figures 13 and 14, respectively.

Conclusion

In solving for cylindrical shells with large holes, it has been found that any increase in (a/R) ratio has produced a correspondent increase in the S.C.F. This is particularly true for higher values of β. In all the cases shown, the memberne stress factor is more important than the bending one. Clearly, the high stresses occurred as a result of the existence of the hole in the shell are limited to small areas, and such a phenomena has only a local nature. However, it has been noticed that any increase in (a/R) ratio has a direct effect on the increase of the area of the local zone compared to the radius of the hole (a).

Acknowledgement

The research reported in this paper was sponsored by University of Yazd whose assistance is gratefully acknowledged.

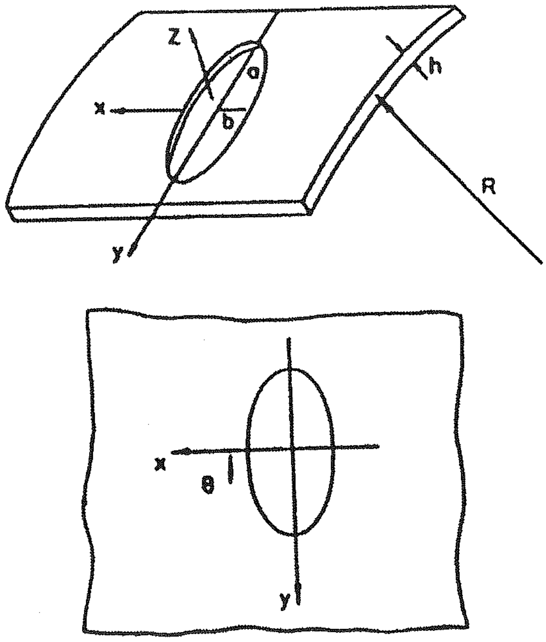

Figure (1) Typical shell with a curvlinear hole.

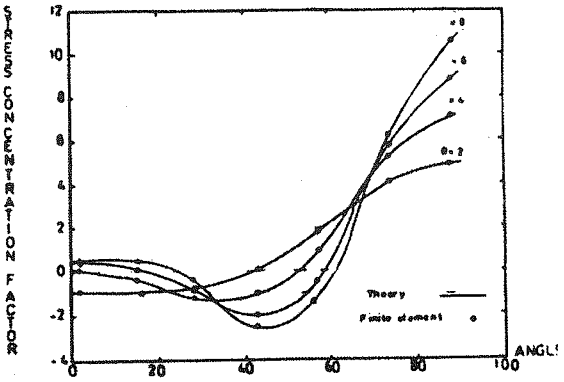

Figure (2) Tangential membrane stresses around the hole with a/R = 0.1.

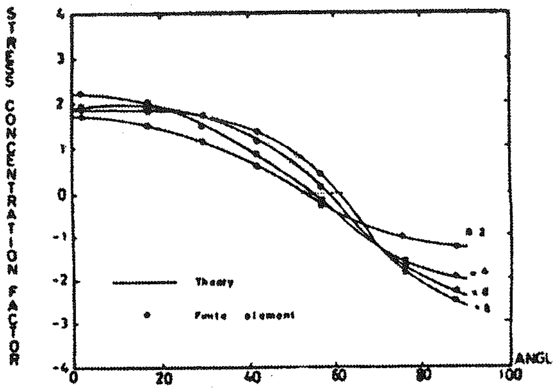

Figure (3) Tangential bending stresses Around the Hole with a/R = 0.1.

Table (1) Models used in the parametric study

Model	β	R (cm)	a = b (cm)	h (cm)	a/R	h / R
SHELL 12	2	100	20	0.8165	0.2	0.0082
SHELL 13	2	100	30	1.8371	0.3	0.0184
SHELL 14	2	100	40	3.266	0.4	0.0327
SHELL 15	2	100	50	5.1031	0.5	0.0510
SHELL 16	2	100	60	7.3485	0.6	0.0735
SHELL 17	2	100	70	10.0021	0.7	0.1000
SHELL 22	4	100	20	0.2041	0.2	0.0020
SHELL 23	4	100	30	0.4593	0.3	0.0046
SHELL 24	4	100	40	0.8165	0.4	0.0082
SHELL 25	4	100	50	1.2758	0.5	0.0128
SHELL 26	4	100	60	1.8371	0.6	0.0184
SHELL 27	4	100	70	2.5005	0.7	0.0250
SHELL 32	6	100	20	0.0907	0.2	0.0009
SHELL 33	6	100	30	0.2041	0.3	0.0020
SHELL 34	6	100	40	0.3629	0.4	0.0036
SHELL 35	6	100	50	0.5670	0.5	0.0057
SHELL 36	6	100	60	0.8165	0.6	0.0082
SHELL 37	6	100	70	1.1113	0.7	0.0111
SHELL 42	8	100	20	0.0510	0.2	0.0005
SHELL 43	8	100	30	0.1148	0.3	0.0011
SHELL 44	8	100	40	0.2041	0.4	0.0020
SHELL 45	8	100	50	0.3189	0.5	0.0032
SHELL 46	8	100	60	0.4593	0.6	0.0046
SHELL 47	8	100	70	0.6251	0.7	0.0063

Stress concentration Factors for Large Holes: Verification of The Results

As has been mentioned earlier, Xue et al. [8] has suggested an analytical solution for cylindrical shells with large holes. The geometrical shape of the hole has been made as a result of intersecting two cylindrical shells. Accordingly its projection in x - y plane is a circular one as shown in figure 9. To investigate this problem, seven models those papers are only limited to certain loads and geometrical configurations. To overcome these difficulties , the present paper has used a finite element method with a "mesh generation" procedure . The general finite element program "SAP90" has been used to solve different shell problems with different loading conditions. The present effort is aimed to prove that the current suggested procedure can retain both computational efficiency and minimum user time.

Finite Element Idealisation

Past experience [10] has shown that using large number of elements and fine meshes near the hole is necessary to get the true values of stresses near the hole, especially for high values of β. In all the meshes used, ratio of element width to element length has been chosen to be approximately a unity. Furthermore, the shape of element has been chosen to approach a square one. In this procedure , sharp angles have been generally avoided. This rule has been strictly appled to these elements more closely to the hole. However , in regions away from the hole, where the stresses are nearly uniform, these rules are relaxed.

Stress Concentration Factors for Small Holes

Numerical calculations have been carried out covering the ranges of values of β between 2 and 8. In all calculations, the ratio of radius of the hole, a , to the radius of the cylinder, R, has been assumed to be 0.1. The thickness of the shell, h, has been calculated from equation (1) and found to be sufficiently small compared to R. The geometrical properties of such group of shells are within the range of applicability of the shallow shell theory [1, 2]. Accordingly, the perturbed stresses are expected to die within a shallow region of the shell. The stress concentration factors (S. C. F) for shells with different values of β are presented in figures 2 and 3. It can be shown from these figures that excellent agreement between the finite element and the shallow shell theory has been obtained.

Stress Concentration Factors for Large Holes: Parametric Study

To understand the behaviour of cylindrical shells with large holes, the influence of different geometrical parameters on maximum stresses need to be studied. Important indpendent non-dimensional geometrical parameters include the curvature parameter β which is directly connected to the S.C.F., as shown in the previous sections, and the a/R ratio. The shape of the hole is assumed to be a circular one in the developed surface of the shell. The models used in this parametric study are given in table 1.

Numerical calculations, for shells subjected to uniform longitudinal tensions and for different (a/R) ratios, are presented in figures 4 and 5. The stress concentration factors at $\theta = 90$ for different (a/R) ratios for $\beta = 2, 4, 6, 8$ are shown in figure 6. Comparing the results for large (a/R) ratios with those obtained from the shallow shell theory, the differences are found within the order of $(\beta a^2/R^2)$ expected by Lekkerkerker [1]. In another word, such differences are within the error produced by the shallow shell theory when applied to non - shallow shell prolems. Furthermore, it has been noticed that the perturbed stresses are not vanishing within a small region as shown in figure 7 and 8.

Finite Element Analysis of Cylindrical Shells With Large Holes

T. Mahdi
Assistant Professor
Civil Engineering Department,
University of Yazd

Abstract

The presence of a hole in a cylindrical shell causes a significant increase in the magnitude of stresses. By utilizing a parametric study, the paper investigates the effect of the size of the hole, on the stresses in cylindrical shells. It also compares some of its results with analytical solutions that have been published recently.

Introduction

For the last four decades, the finite element method has played a leading role in the analysis and design of structures. In dealing with shell problems, many successful shell elements have been developed. However, serious difficulties have been encountered when stress concentration problems are considered. The presence of holes in a cylindrical shell is one of such problems.

Many analytical methods [1-3] have been suggeted to tackle these difficulties. However, most of these methods were found limited to small holes and only suitable for shallow shells. In such methods, the changes of stresses around the holes were only related to a single parameter, β, which is called the curvature parameter and is given as follows:

$$\beta = \frac{\sqrt[4]{12\,(1-\upsilon^2}}{2} * \frac{\alpha}{\sqrt{Rh}} \qquad (1)$$

where a, R and h are shown in figure 1.

For shells with large holes, some analytical methods have been reported in Reference [4]. However, such methods were performed under various restrictions. One of the best methods suggested in this respect is that attributed to Mizoguchi et.al. [5] . An eight-order system describes the shell response, has been solved using Fourier series. More recently, Steele et.al. have suggested another analytical method that has been applied mainly to large openings in cylindrical shells [6, 7]. In this method; asymptotic solutions based on Donnell's shallow shell equation and "cut" solutions based on sander's shell equation, have been used. Some doubts have been cast on the validity of this method [7, 8], when applied to shells with openings having (a/R) ratios approach unity. On the other hand, Xue et. al. [8, 9] have developed a new analytical procedure based on Morley's equation that makes a considerable improvement on steel's results. However, the results given in