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List of Symbols

1- Mean radius of sphere	R
2- Thickness of sphere	T
3- Mean radius of cylinder	r
4- Thickness of cylinder	t
5- Angle of obliquity	α
6- Dimensionless parameter	ρ
7- Internal pressure	P
8- Maximum Principal stress	σ_{max}
9- Maximum shear stress	τ_{max}
10- Stress concentration factor	SCF, K, K_1, K_2
11- Factors for simple rule	k, A, B

Nevertheless, if the isolated points, which may appear due to approximation or possible mistakes in calculating the initial data are discarded, then the data points will locate within a narrow band with a small slope. In this case the variation of k with r/R, t/T, and $\sqrt{(R/T)}$ is so that an average magnitude can be considered for k. Accepting an average for k is useful only to derive a rule-of-thumb relation which is a simple tool to estimate the stresses at early stages of design. It is not adviseable to replace a detailed analysis of important structures by simple rules.

The scattered values of k may be correlated in a better way by the help of statistical and numerical methods but the approximate results will not change substantially. The existance of approximations is inherent in simple rules and can not be avoided. The simplicity and speed in using these rules explain their application. For instance, according to investigation reported in [10], the approximation of results obtained from relation (1) which is recommended by ASME Code to be used for oblique connection analysis, ranges from - 8% (unconservative) to +27% (conservative) compared to experimental results.

Summary and Conclusion

To estimate theoretical stress concentration factors for an oblique nozzle K_2, in terms of the corresponding quantity for radial connection K_1, a simple relation can be derived. The relation which is applicable for angles of obliquity less than 45°, and may be considered as extension of a similar relation in ASME Code, is written as:

$$K_2=K_1(1+k.\ \sin^2\alpha) \qquad (2)$$

Where:

$$k = 3.3 \text{ for vessel} \qquad (5a)$$

$$k = 4.5 \text{ for nozzle} \qquad (5b)$$

To obtain more precise values for k the following linear relations must be used:

$$k = 0.08\sqrt{(R/T)} + 2.2 \text{ for vessel} \qquad (6a)$$

$$k = 0.17\sqrt{(R/T)} + 2.2 \text{ for nozzle} \qquad (6b)$$

Whenever the requirements set by Code are not fulfilled to use relation (1), the simple rule(2) along with suitable values for k from (5a) and (5b) or from (6a) and (6b) helps the designer to estimate the theoretical stress concentration factors for oblique nozzle in terms of the same quantity for radial connection within the prescribed ranges of geometrical ratios. For critical structures the results of this rule must be used cautiously. In such situations it is advised to use separate values of k in Table 5 or to use relation (4) along with separate valuse for A and B listed in Table 6.

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If the isolated points on the diagrams are neglected, the data points are scattered in such a way that can be considered within a narrow band with a small slope. For each diagram a mean value for k can be assigned and hence Tables 1 to 4 and as a result diagrams 1 to 8 are summarized. Mean values of k along with their standard deviations are listed in Table 5.

Table 5- Mean values and standard deviations of k (summary of data in Tables 1 to 4).

	Data from Table 1 $\alpha=22.5°$	Data from Table 2 $\alpha=45°$	Data from Table 3 $\alpha=22.5°$	Data from Table 4 $\alpha=45°$	Total average of k
Mean value of k for vessel	4.0	3.0	4.0	2.3	3.3
Standard deviation	0.91	0.57	1.1	1.1	
Mean value of k for nozzle	5.4	3.7	5.7	3.1	4.5
Standard deviation	2.1	0.9	2.8	2.0	

Total average of k for vessel is 3.3 and for nozzle is 4.5 which can be used in relation (2).

In order to estimate stress concentration factors more precisely, separate mean values of k for vessel, nozzle, angle of obliquity, and the quantity under consideration should be adopted to use in relation(2). This process to some extent will deteriorate the simplicity of the rule.

Instead of total average or separate values for k in Table 5, the linear relation (4) can be used provided that the factors A and B are determined before hand. To evaluate these parameters, mean values of k for various ratios of r/R and t/T correspondig to two different values of $\sqrt{(R/T)}$ are determined and then substituted in (4). The procedure is applied for $\alpha=22.5°$ and 45°. In this procedure the data in Tables 1 and 4 are summarized to a simple linear relation and diagrams 1 to 8 are replaced by a line. For diagrams 1 to 4 (Tables 1 and 3) the mean values are evaluated for $\sqrt{(R/T)}=7.07$ and $\sqrt{(R/T)}=25$. For diagrams 5 to 8 (Tables 2 and 4) this values are evaluated for $\sqrt{(R/T)}=7.07$ and $\sqrt{(R/T)}=20$. The values of A and B are listed in Table 6.

Table 6- Values of A and B to be used in relation (4)

		If K_1 and K_2 are calculated from (3a) $\alpha=22.5°$	$\alpha=45°$	If K_1 and K_2 are calculated from (3b) $\alpha=22.5°$	$\alpha=45°$	Averages of A&B
For vessel	A	0.07	0.07	0.08	0.11	0.08
	B	3.0	2.2	3.0	0.7	2.2
For nozzle	A	0.17	0.13	0.12	0.24	0.17
	B	3.0	2.0	3.7	0.1	2.2

The averages of A and B for vessel and nozzle are to be used separately in relation (4). The value of k then can be determined for every specific ratio of $\sqrt{(R/T)}$ to substitute in relation (2). If k is to be calculated more precisely, separate values of A and B must be used.

Discussion

From Tables 1 to 4 and Diagrams 1 to 8 it is observed that the values of k are so scattered that it is difficult to consider a single value for it.

variation of k for vessel when $\alpha=22.5°$. Figure 6 shows the similar diagram when $\alpha=45°$. Figures 4 and 8 indicate correspondig diagrams for nozzle at 22.5° and 45° respectively.

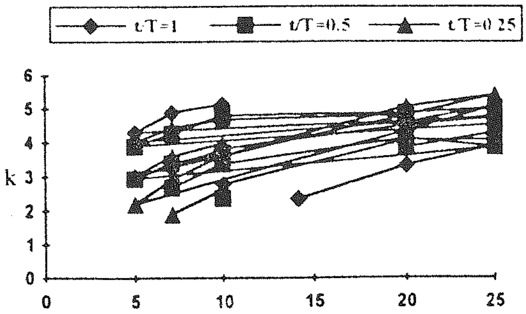

Figure 1-Data from Table 1 for the sphere

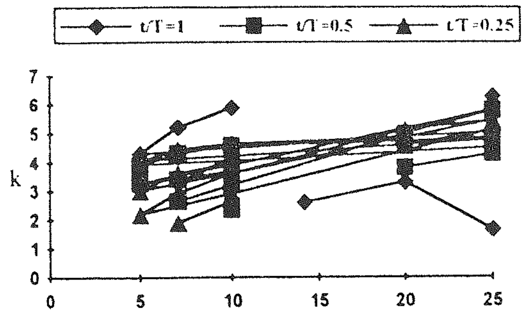

Figure 2-Data from Table 3 for the sphere

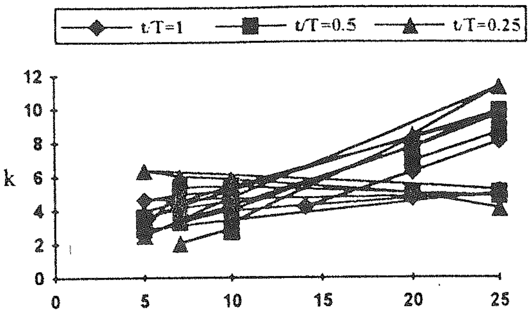

Figure 3-Data from Table 1 for the nozzle

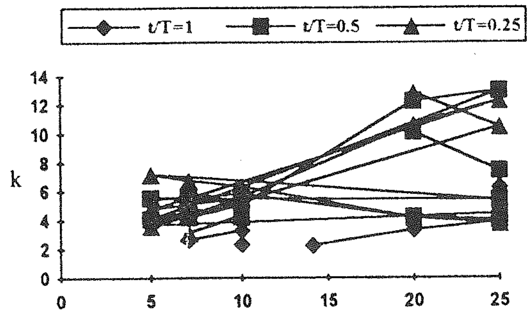

Figure 4-Data from Table 3 for the nozzle

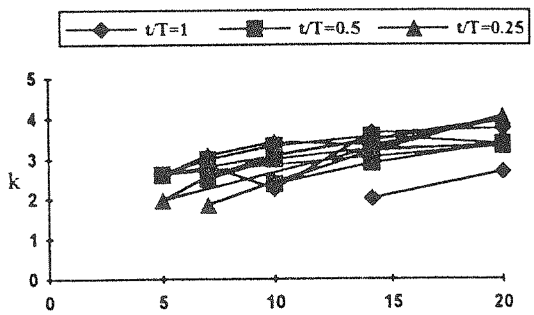

Figure 5-Data from Table 2 for the sphere

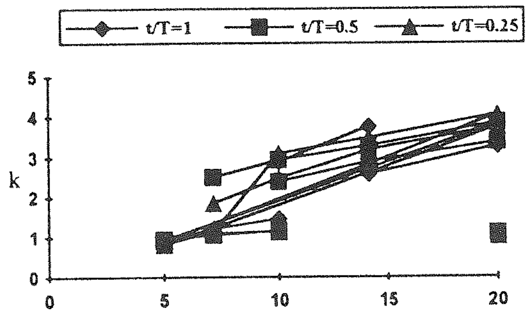

Figure 6-Data from Table 4 for the sphere

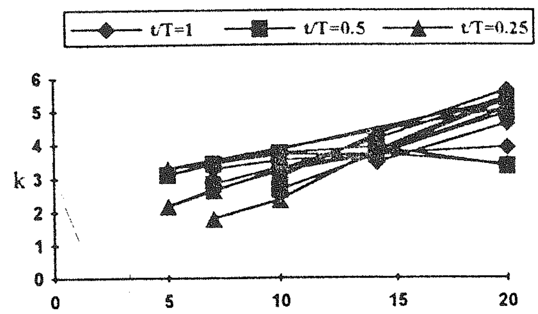

Figure 7-Data from Table 2 for the nozzle

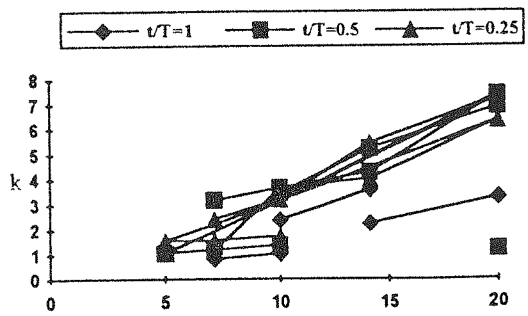

Figure 8-Data from Table 4 for the nozzle

Table 1- Values of $k=(K_2-K_1)/(K_1.Sin^2\ 22.5°)$ for spherical vessel and oblique nozzle at $\alpha=22.5°$. K_1 and K_2 are stress concentration factors based on maximum shear stresses in radial and oblique connection respectively.

r/R	√(R/T)	t/T=1		t/T=0.5		t/T=0.25	
		sphere	cylinder	sphere	cylinder	sphere	cylinder
1/20	7.07					1.9	2.05
	10			2.35	2.71	2.59	2.89
	14.14	2.36	4.23				
	20	3.35	6.25	3.87	6.9	4.63	8.45
	25	3.92	8.10	4.31	8.68	5.09	11.36
1/10	5					2.19	2.56
	7.07			2.66	3.3	2.9	3.37
	10	2.77	3.26	3.37	4.03	3.61	4.29
	20	4.1	8.26	4.14	7.76	5.07	7.7
	25	4.1	9.9	3.88	9.89	5.44	9.7
1/5	5			2.92	3.61	3.0	3.95
	7.07	3.23	3.59	3.39	4.11	3.59	4.51
	10	3.82	4.0	3.69	4.56	3.97	4.77
	20	4.53	4.7				
	25	4.76	4.87	4.56	5.05	4.79	5.24
1/2	5	4.3	4.64	3.89	2.89	4.0	6.39
	7.07	4.89	4.96	4.27	5.37	4.43	6.03
	10	5.12	5.15	4.79	5.45	4.68	5.85
	20			4.88	5.01	4.81	5.01
	25	5.03	4.85	5.03	4.9	5.44	4.12

Table 2- Values of $k=(K_2-K_1)/(K_1.Sin^2\ 45°)$ for spherical vessel and oblique nozzle at $\alpha=45°$. K_1 and K_2 are stress concentration factors based on maximum shear stresses in radial and oblique connection respectively.

r/R	√(R/T)	t/T=1		t/T=0.5		t/T=0.25	
		sphere	cylinder	sphere	cylinder	sphere	cylinder
1/20	7.07					1.86	1.80
	10			2.38	2.66	2.44	2.38
	14.14	2.02	3.48	2.9	3.70	3.18	3.84
	20	2.70	4.62	3.38	4.96	4.06	5.44
1/10	5					1.98	2.18
	7.07			2.50	2.84	2.58	2.66
	10	2.40	3.02	3.00	3.34	3.10	3.22
	14.14	3.20	4.30	3.26	3.72	3.48	3.70
	20	3.36	5.64	3.32	5.34	3.98	5.08
1/5	5			2.62	3.12	2.62	3.30
	7.07	2.84	3.28	3.00	3.44	3.10	3.46
	10	2.26	3.54	3.32	3.76	3.42	3.72
	14.14	3.64	3.72	3.56	3.92	3.34	3.74
	20	3.76	3.92	3.38	3.36	3.96	3.40

Table 3- Values of $k=(K_2-K_1)/(K_1.Sin^2\ 22.5°)$ for spherical vessel and oblique nozzle at $\alpha=22.5°$. K_1 and K_2 are stress concentration factors based on maximum principal stresses in radial and oblique connection respectively.

r/R	√(R/T)	t/T=1		t/T=0.5		t/T=0.25	
		sphere	cylinder	sphere	cylinder	sphere	cylinder
1/20	7.07					1.91	3.14
	10			2.38	4.22	2.66	4.37
	14.14	2.6	2.27				
	20	3.31	3.35	3.82	10.11	4.58	12.84
	25	1.67	3.89	4.30	7.44	5.12	10.45
1/10	5					2.19	3.62
	7.07			2.66	4.23	2.94	4.37
	10	2.73	2.32	3.21	5.05	3.62	5.60
	20			4.98	12.22	5.12	10.65
	25	5.60	6.28	5.80	12.97	5.46	12.29
1/5	5			3.28	4.10	3.0	4.78
	7.07	3.28	2.66	3.41	4.58	3.62	5.12
	10	4.17	3.28	3.69	4.99	3.96	5.53
	20						
	25	5.05	4.44	4.51	5.40	4.78	5.39
1/2	5	4.3	3.69	3.89	5.53	4.03	7.17
	7.07	5.19	4.64	4.30	5.60	4.44	6.76
	10	5.87	4.99	4.58	5.80	4.64	6.35
	20			4.71	4.23	4.85	4.17
	25	6.28	4.78	4.85	4.03	5.46	3.76

Table 4- Values of $k=(K_2-K_1)/(K_1.Sin^2\ 45°)$ for spherical vessel and oblique nozzle at $\alpha=45°$. K_1 and K_2 are stress concentration factors based on maximum principal stresses in radial and oblique connection respectively.

r/R	√(R/T)	t/T=1		t/T=0.5		t/T=0.25	
		sphere	cylinder	sphere	cylinder	sphere	cylinder
1/20	7.07					1.86	2.44
	10			2.40	3.46	2.48	3.24
	14.14	2.58	2.24	2.90	5.22	3.18	5.44
	20	3.28	3.30	3.38	6.90	3.76	7.28
1/10	5					0.84	1.08
	7.07			2.50	3.18	1.14	1.20
	10	2.90	2.38	2.94	3.68	3.10	3.66
	14.14	3.76	3.60	3.28	4.28	3.48	4.04
	20			3.86	7.38	4.06	6.36
1/5	5			0.94	1.06	0.92	1.56
	7.07	1.22	0.84	1.06	1.20	1.10	1.58
	10	1.46	1.04	1.16	1.38	1.14	1.74
	14.14						
	20			1.10	1.22	1.04	1.24

concentration factors directly and linear interpolation is allowed within the prescribed ranges of parameters.

Method of Investigation

If the view point dominant in subsection NB-33388.2 of Section III of ASME Code is to expand and an approximate relation applicable to another range of geometrical parameters of spherical vessel-oblique cylindrical nozzle structure is to be derived, the relation(1) is rewritten as bellow:

$$K_2 = K_1(1+k.\sin^2\alpha) \quad (2)$$

Factor k is to be determined in this paper. For this purpose the theoretical stress concentration factors obtained in the analysis of the structure by finite element method which are reported in [11] an [12] are used. In these reports it is assumed that the thin shell definition is valid simultaneously for vessel and nozzle and the intersection is sharp and there is no reinforcement material. The nozzle is considered to be flush and its obliquity angle α doesnot exceed 45°. The stress concentration factor K is defined in terms of membrane stress in the unpenetrated vessel according to one of the following relations:

$$K = 2\tau_{max}/(PR/(2T)) \quad (3a)$$

$$K = \sigma_{max}/(PR/(2T)) \quad (3b)$$

τ_{max} and σ_{max} are maximum shear and principal stresses in the structure respectively. R is the radius and T is the thickness of the vessel and P is internal pressure.

The data in [11] are prepared for radial connection and in [12] are for oblique nozzle at 22.5° and 45°. Other ranges of geometrical parameters are $625 \geq R/T \geq 25$, $0.5 \geq r/R \geq 0.05$, and $1 \geq t/T \geq 0.25$. r is the radius and t is the thickness of the nozzle. The values of K_1 and K_2 are substituted in (2) and the values of k which depend on geometrical ratios, stress index under consideration, and angle of obliquity are calculated. The average of these values are determined for vessel and nozzle separately. In cases where the requirements set by the Code are not satisfied the average value of k is used in relation (2) to estimate the stress concentration factors for oblique connection in terms of corresponding quantities radial connection.

A more accurate though complicated method is to consider a linear relation for k in terms of $\sqrt{(R/T)}$. By this method the values of k are determined with better approximation. The results are also presented in this work. The linear relation is expressed as bellow:

$$k = A.\sqrt{(R/T)}+B \quad (4)$$

A and B depend on $r/R, t/T, \alpha$, and the stress concentration factor under consideration but for the sake of simplicity they are considered to be constant. For two different values of $\sqrt{(R/T)}$ two averages for k are calculated and substituted in (4). Parameters A and B are determined for vessel and nozzle separately.

Use of statistical or numerical methods such as regression or curve fitting also can be employed in order to determine more accurate correlation between k and other geometrical ratios of the structure. The results may lead to more precise values for k but the simplicity of the method will be lost.

Results

The values of k which are obtained by direct substitution of data for K_1, K_2, and α into relation (2) are listed in Tables 1,2,3,and 4. The values are calculated for both vessel and nozzle. Variation of k with $\sqrt{(R/T)}$ and for different ratios of t/T, r/R, and the obliquity angles 22.5° and 45° are indicated in Figures 1 to 8. In Figure 1 values of k for sphere are indicated when α=22.5° While Figure 5 indicates the same quantity for sphere when α=45°. Figures 3 and 7 represent the similar diagrams for nozzle at 22.5° and 45° respectively. In all these four diagrams the stress concentration factors are assumed to be determined from (3a). In figures 2, 4, 6, and 8 relation (3b) is used in calculating the stress concentration factors. Figure 2 shows the nozzles (including oblique nozzle), arranged singly or in groups, in spherical, cylindrical, domed, and conical shells, provided that the conditions in Article 3.5.4.2 are fulfilled. The condition for a radial nozzle in this Article is specified to be $r/R \leq 0.5$ and for oblique nozzles in addition to this condition, the angle of obliquity, α must not exceed 50°. The design curves have been produced from consideration of the requirement to avoid incremental plastic strain during repeated pressure loading of the vessel. A distinction is drawn between openings or branches that are closely pitched and those that can be treated as isolated. For isolated openings and branches, a limited amount of plastic deformation in the most severely stressed region can safely be permitted during initial operating cycles. A residual stress distribution is established and subsequent structural response is entirely elastic. This **shakedown** behaviour is achieved by controlling the maximum stress (calculated on a linear elastic basis) in the region of the branch. For multiple openings or branches, the primary requirement is to limit the ligament stresses in the vessel. For designing branches in spherical shells, theoretical stress concentration factors, defined as the ratio of the maximum direct stress component to the circumferential stress in the unpierced shell, are used. They are given in [4] by Leckie and Penny and are derived from thin shell analysis. This means that very local effects arising for example from fillet weld details are not represented. Nevertheless the gross behaviour (deflections) of the structure is well predicted for the range considered. It is found that the theoretical SCF can to a good approximation, be obtained as a function of the branch to shell thickness ratio t/T and the parameter $\rho=(r/R)\sqrt{(R/T)}$. In this analysis, the allowable maximum, elastically calculated stress is set at 2.25 times the allowable stress in the unpierced shell. Hence a set of curves can be plotted giving the required branch to shell thickness ratio as a function of the $\rho=(r/R)\sqrt{(R/T)}$ parameter and the shell thickening.

Another simple method for estimating the SCF in a spherical pressure vessel with an oblique nozzle using the results in [4], is presented in [3]. The authors have expressed the shallow-shell equations in elliptic coordinates and have solved them in terms of Mathieu functions. Boundary conditions for the rigid insert and for the unreinforced hole are discussed in detail. Results for an unreinforced opening are compared with experiment and satisfactory agreement is obtained for smaller values of $\rho=(r/R)\sqrt{(R/T)}$. This method simply consists of calculating an **equivalent radius** for the oblique nozzle by multiplying the radius of nozzle by $\sec\alpha$ (α is the angle of obliquity of the nozzle), and then entering this radius in the curves for a radial nozzle, for suitable t/T, in [4]. By multiplying the SCF so determined by $\sec\alpha$, the SCF for an oblique intersection is calculated. On the basis of this study predictions of the stresses at the intersection of a non-radial nozzle in a pressure vessel have been made.

There are some other theoretical methods for predicting stresses in spherical vessels and oblique nozzles. For more information the reader should consult [5,6,7,8,9,10]. The requirements imposed on these methods which are mainly arised from shallow shell theory, limitations of radius and thickness ratios of the vessel and nozzle, small angles of obliquity, and neglecting the stresses in the nozzle in some of these methods, restrict their application as the effective method for designing the structure.

The finite element method is employed as a tool for analysis to determine stress concentration factors in a spherical vessel intersected by a radial or oblique nozzle and the results are reported in [11,12]. The wide range of geometrical ratios and relatively large angle of obliquity along with stress data for both vessel and nozzle has removed to some extent the limitations of other methods. These results which are valid for thin shells, provide stress

Simple Rules to Estimate Stress Concentration Factors for Oblique Nozzles

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Abstract

To estimate theoretical stress concentration factors for spherical vessels with a flush oblique nozzle from the corresponding data for a radial connection, a simple rule is suggested. The rule is derived using the theoretical data obtained from finite element method analysis of spherical vessels intersected by radial or oblique cylindrical nozzles. The existing data are substituted into a generalized form of a rule-of-thumb equation which is presented for oblique connections in Section III of ASME Code, and the correlation of results with geometrical parameters of the structure is investigated. The rule is used for vessels and nozzles made of thin shells with sharp intersections, no reinforcement material at the junction, and obliquity angles smaller than 45°. Therefore the restrictions imposed by Code are removed to some extent and the range of geometrical parameters of the structure for application of the rule-of-thumb is widened. Other methods are also presented when stress concentration factors must be estimated more accurately. The rule is a useful simple tool in rough estimation of theoretical stresses by pressure vessel designers.

1- Introduction

For designing the spherical pressure vessels with an oblique cylindrical nozzle there are simple experimental and theoretical rules in [1],[2],and [3]. In [1] the stress indices in a spherical vessel or head with oblique nozzle are related to the corresponding values of a radial nozzle by the simple relation :

$$K_2 = K_1(1+\sin^2\alpha) \qquad (1)$$

K_1 and K_2 are the stress indices for radial and non radial connection, respectively, and their values are prsented in [1]. α is the angle of the nozzle axis with the normal to the vessel at their intersection on the vessel. The term stress index is defined as the numerical ratio of the stress component under consideration to the computed membrane hoop stress in the unpenetrated vessel material.

The presented values for stress indices have severe limitations which are specified in [1]. The most important limitations that affect the application of relation (1) are:

-The existence of definite inner and outer corner radii,

-Definite amount of reinforcement,

-Restriction on values of r/R and R/T and (r/R)√(R/T),

-The indices are for sphere only.

-The fact that the higher stresses are sometimes in the nozzle is ignored,

-There is no guide line how to calculate the stress indices beyond these limitations.

Another method for designing openings and nozzles in vessels is described in Article 3.5.4 of BS 5500 ,[2]. The method is still based on more experimental requirements than in ASME Code,[1]. The basis of design charts is described in Appendix L of this standard. The application of requirements of this Article are valid for the design of circular and obround openings and