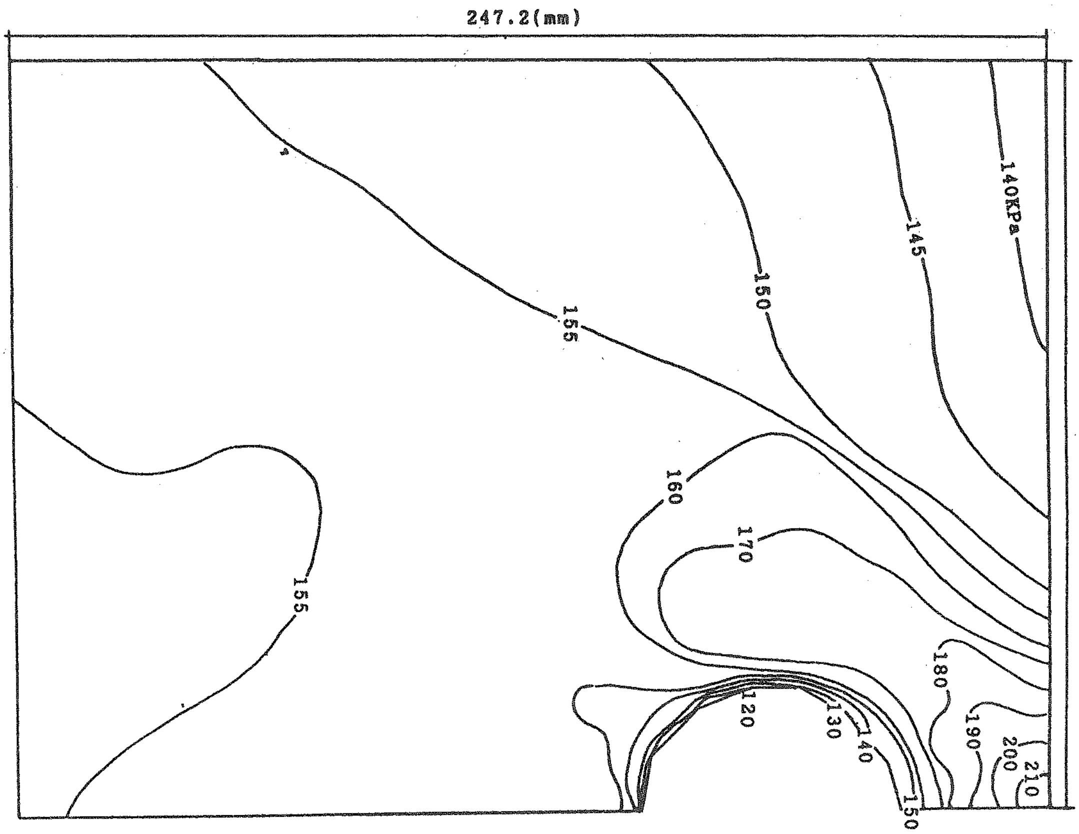

Fig.23- The distribution of the pore pressure under $\sigma_t - \sigma_s$ = 160 KPa for undraind condition.

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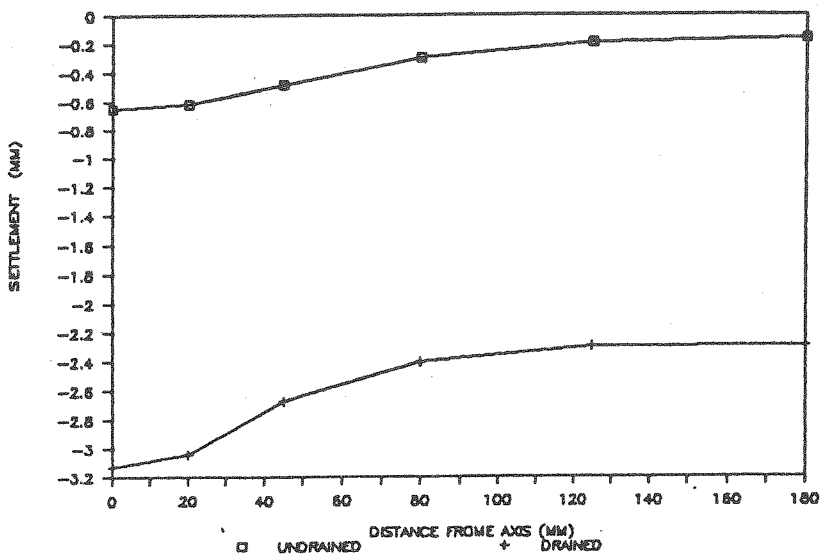

Fig.21- The ground settlements under $\sigma_t - \sigma_s$ = 100KPa
for drained and undrained conditions.

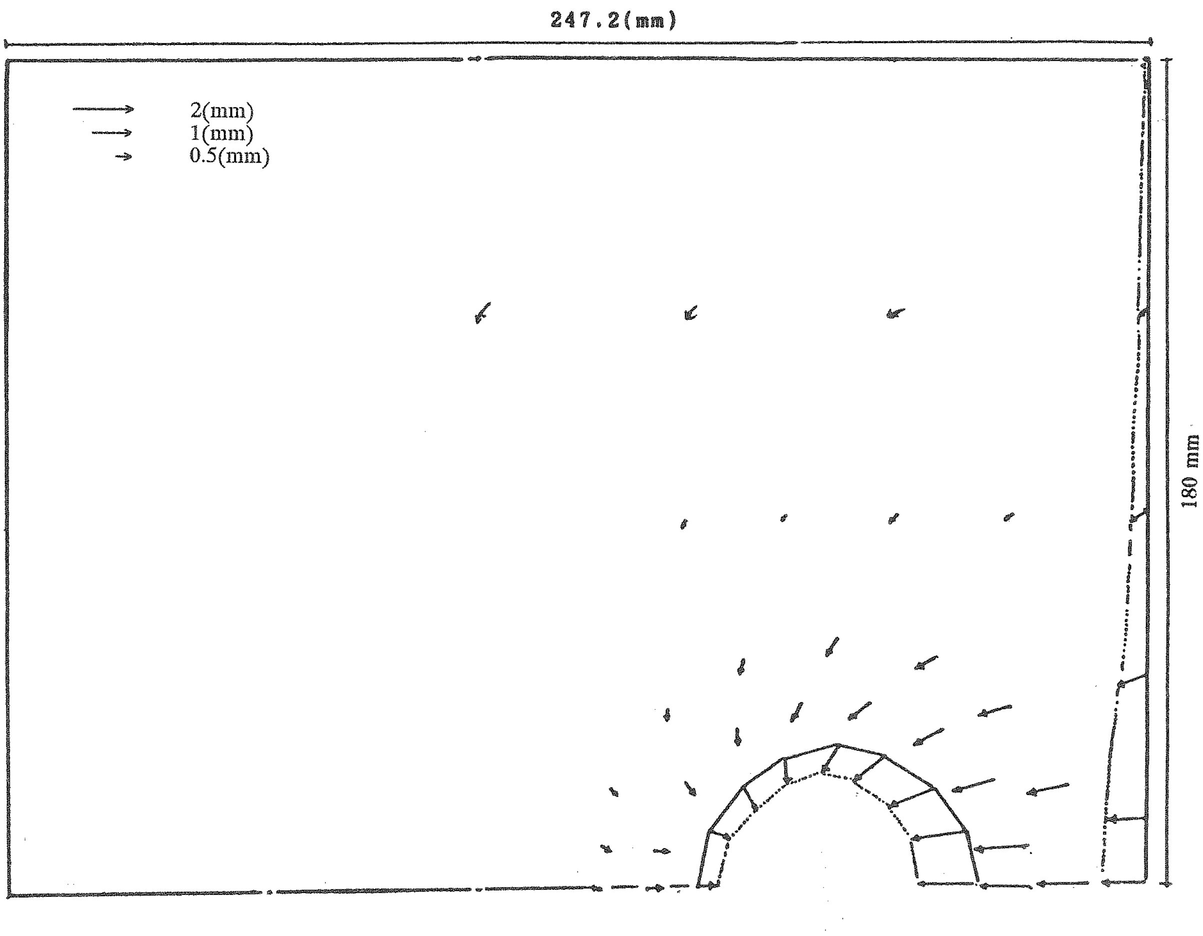

Fig.22- The distribution of vertical and horizontal displacements under $\sigma_t - \sigma_s$ = 160 KPa for undrained condition.

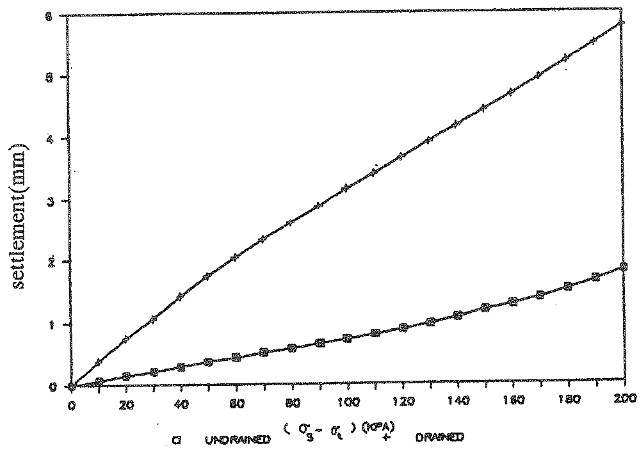

Fig.18- The ground settlements versus $\sigma_t-\sigma_s$ for drained and undrained conditions.

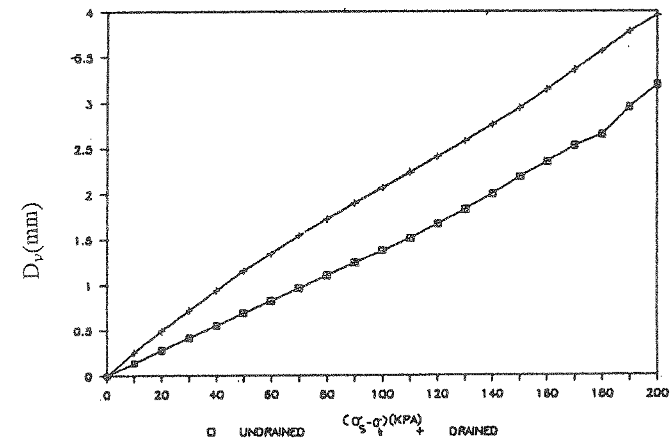

Fig.19- The variations of vertical diameter versus $\sigma_t-\sigma_s$ for drained and undrained conditions.

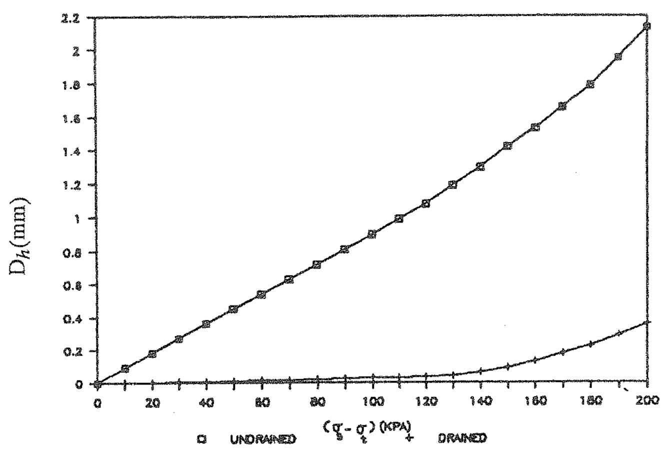

Fig.20- The variations of horizontal diameter versus $\sigma_t-\sigma_s$ for drained and undrained conditions.

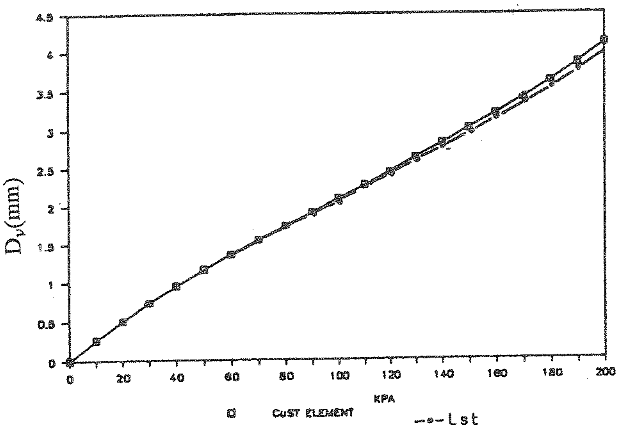

Fig.15- The variations of the vertical diamete using LST and C_uST elements.

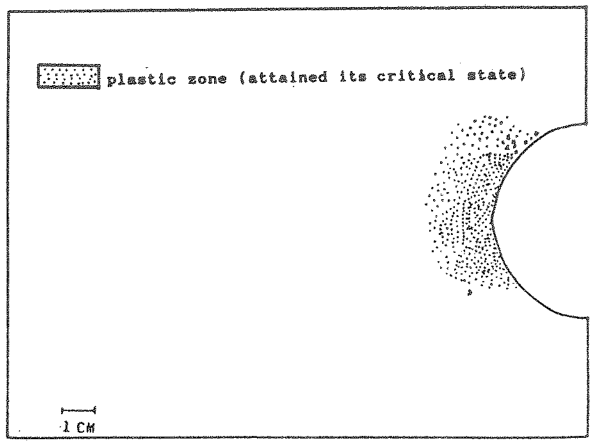

Fig.16- Zones reached to plastic state under $\sigma_t - \sigma_s$ = 200 KPa.

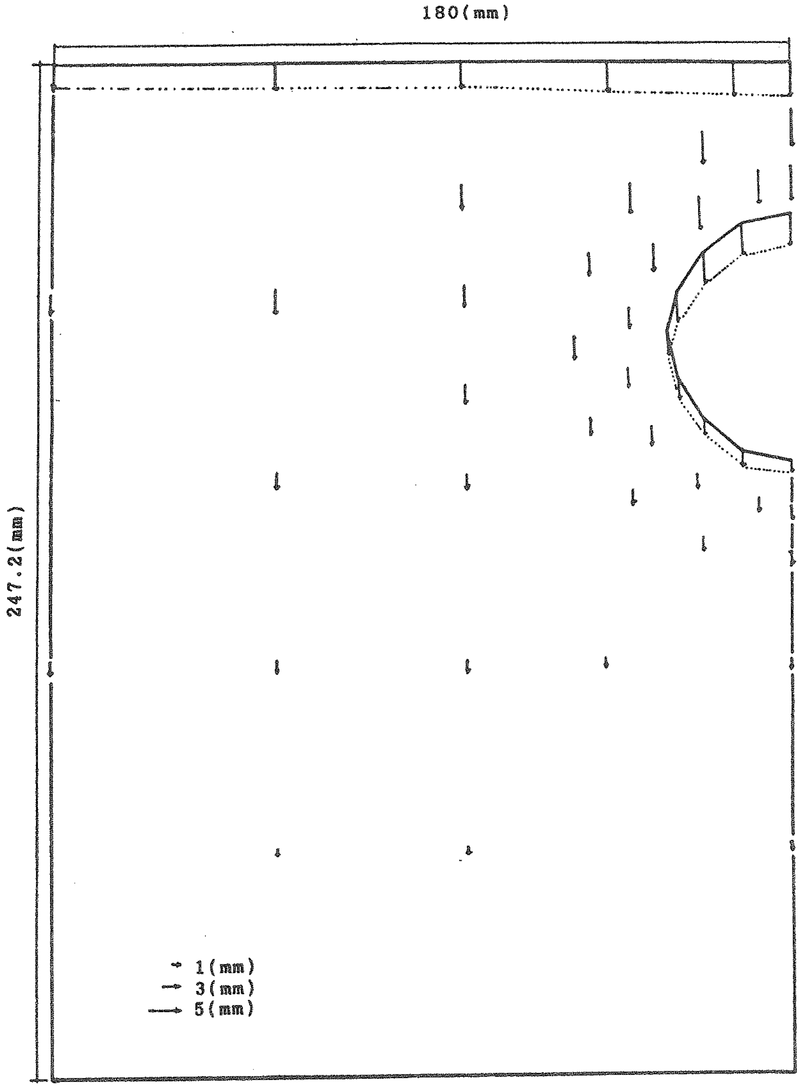

Fig.17- The directions and values of deformations around the tunnel for $\sigma_t - \sigma_s$ = 160 KPa.

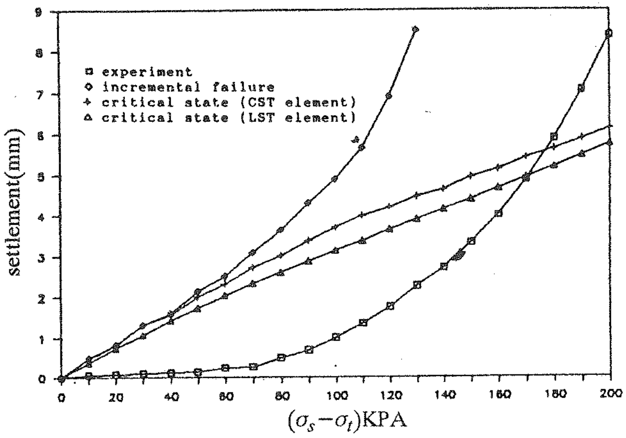

Fig.9- The variations of the maximum ground settlements versus $\sigma_t-\sigma_s$.

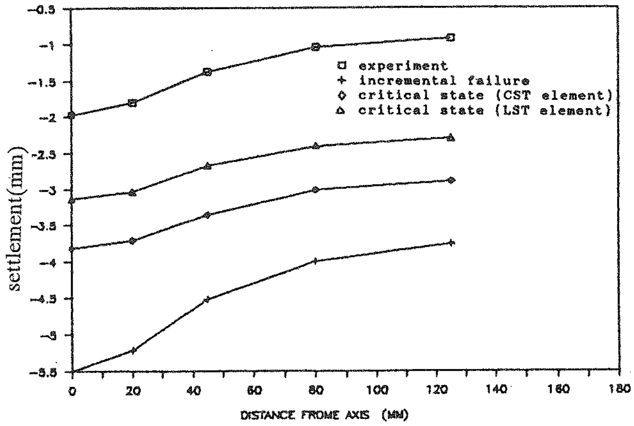

Fig.12- The ground settlements versus distances from tunnel axsis for $\sigma_t-\sigma_s$ = 100 KPa.

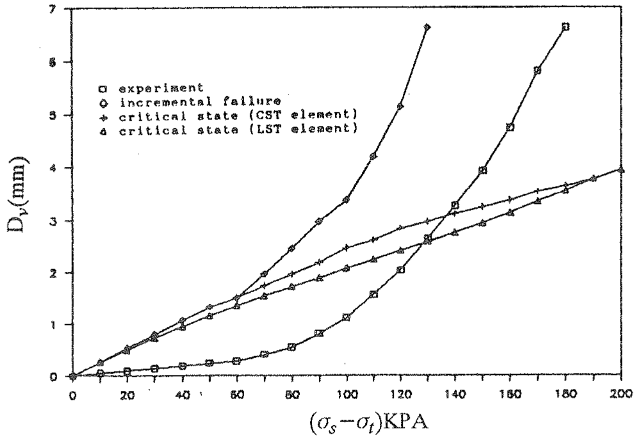

Fig.10- Variations of the tunnels' vertical diameter versus $\sigma_t-\sigma_s$.

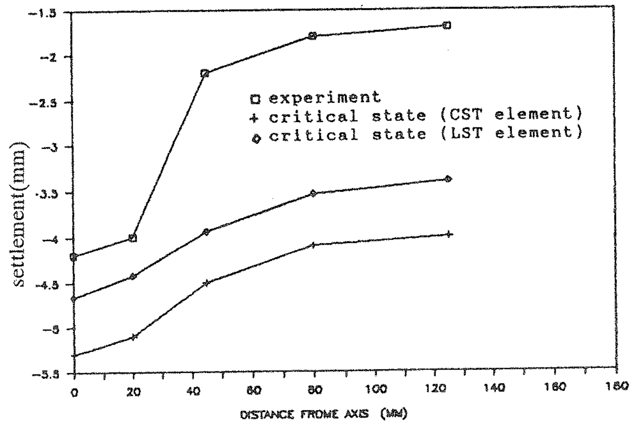

Fig.13- The ground settlements versus distances from tunnel axsis for $\sigma_t-\sigma_s$ = 160 KPa.

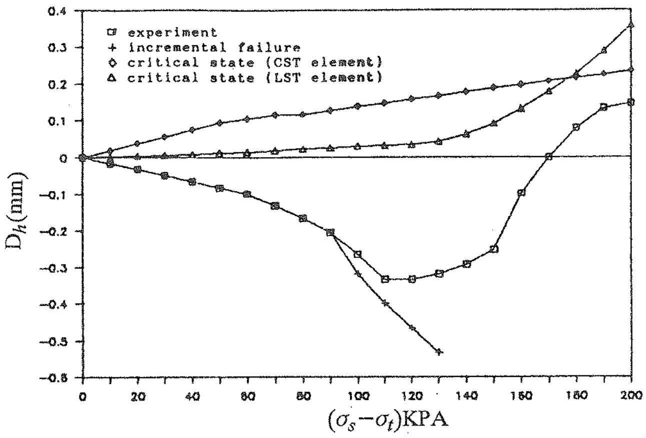

Fig.11- Variations of the tunnels' horizontal diameter versus $\sigma_t-\sigma_s$.

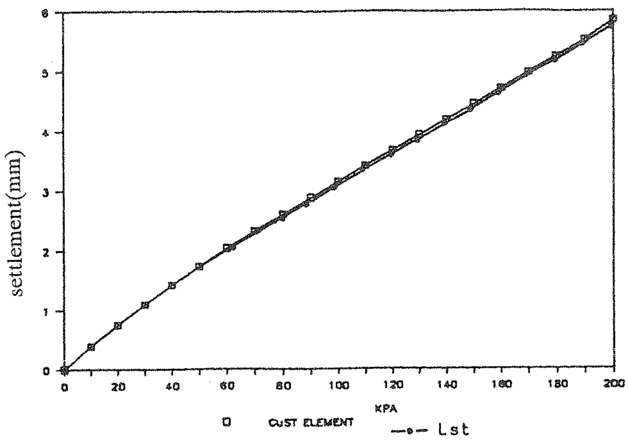

Fig.14- The ground settlements using LST and C_uST elements.

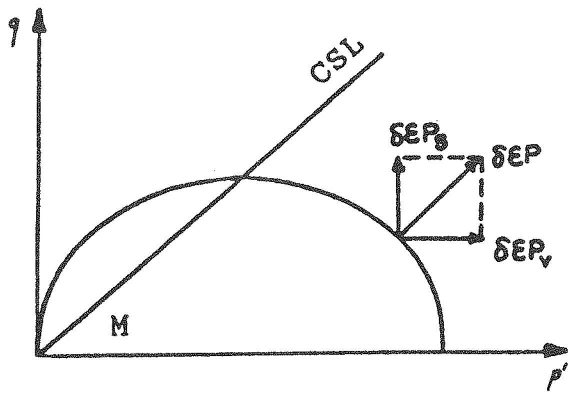

Fig.7- The yield surface in the critical state model.

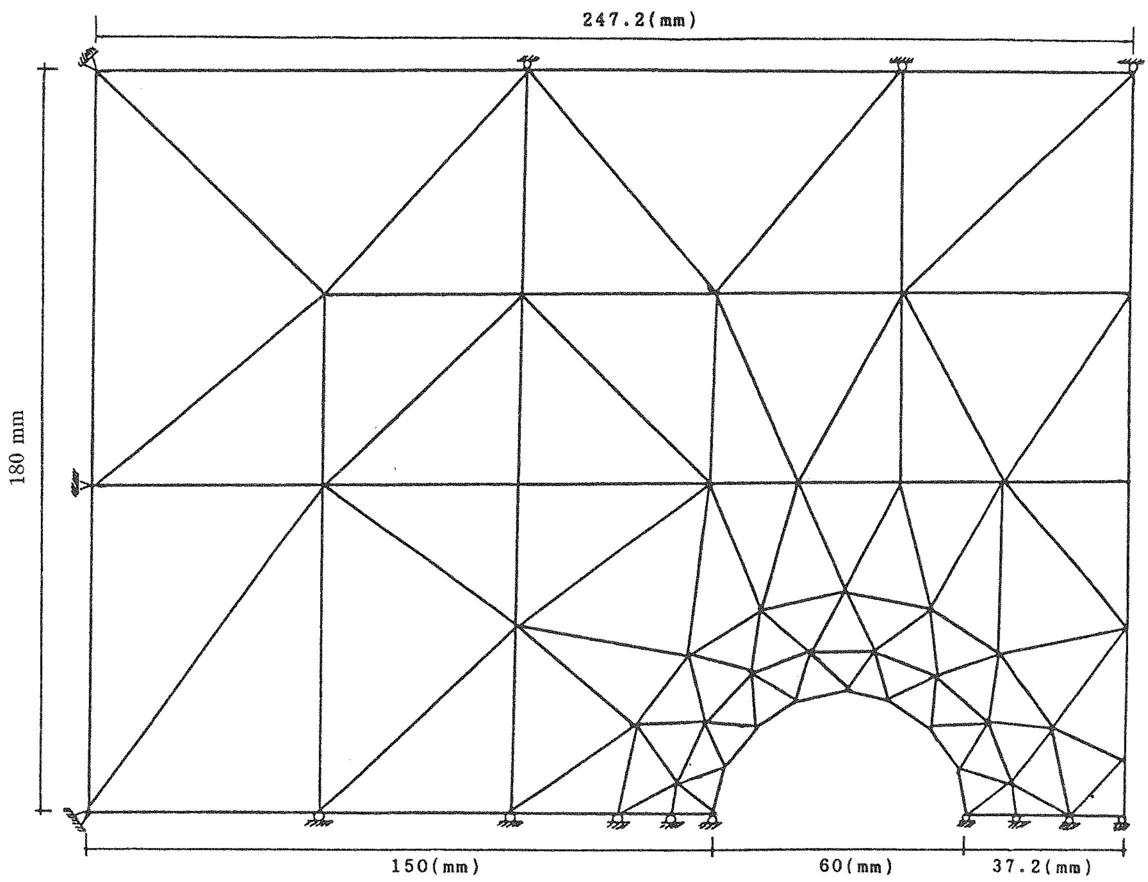

Fig.8- The mesh used to analyse the tunnel by the CRISP.

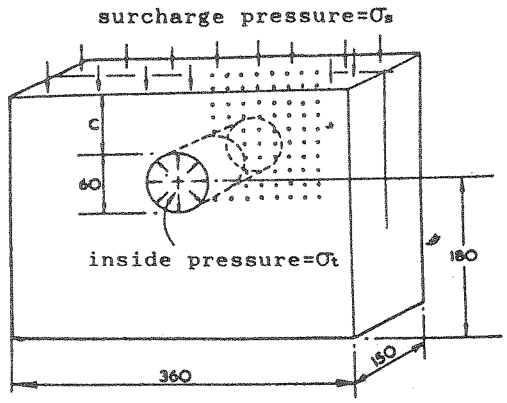

Fig.1- The experimental tunnel model developed in Cambridge University.

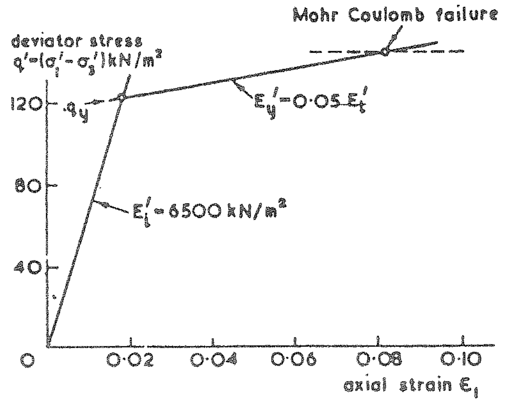

Fig.2- The bilinear stress-strain relationship used in the incremental failure method.

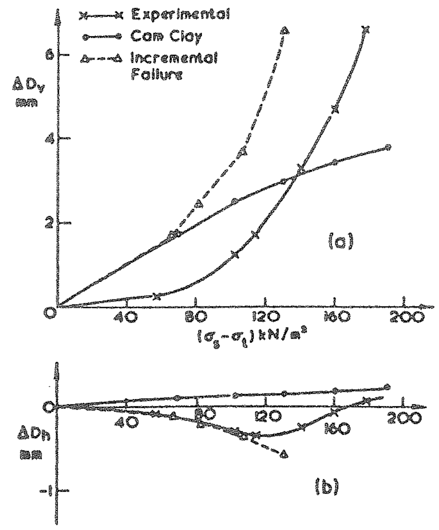

Fig.3- Variations of tunnels' diameters obtained from experimental and analytical methods.

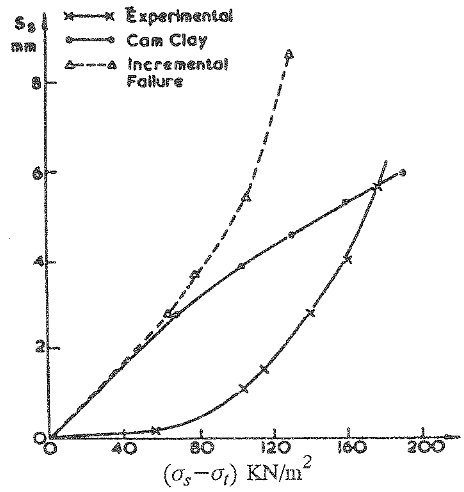

Fig.4- Variations of ground settlements obtained from experimental and analytical methods.

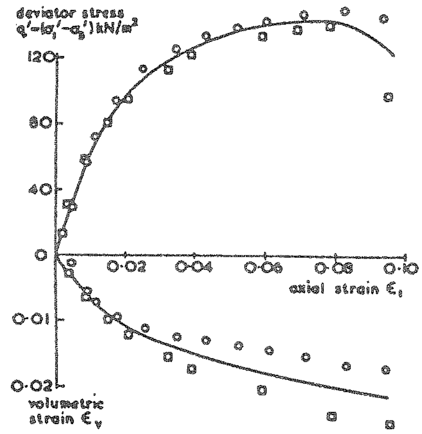

Fig.5- The stress-strain relationship obtained from triaxial tests.

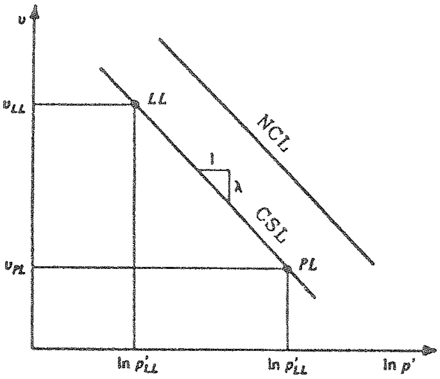

Fig.6- The critical state line for an isotropical soil sample.

elements. It is quite clear that there is no significant differences between the results of these two analyses.

Since the critical state model can predict the behaviour of the soil even before failure (compared with the Mohr - Coulumb. Von - Misses, Tresca and othres which are only able to define the failure condition), for a specific stress path, the effect of the hardening and softening of the soil can also be investigated. Therefore the states of stress and strain can be determined by applying the load in some increments and using the non - linear analysis. The expansion or contraction of the yield surface can be expressed when the stress path passes from the elastic range in the first yield surface, depending on the degree of consolidation of the soil.

In fig.16 zones reached to the plastic state as well as zones gained their critical state under 200 KPa surcharge are shown. The directions and values of deformations of different points around the tunnel, under 160 KPa surcharge are illustrated in fig.17.

6- Comparison of Drained and Undrained Analysis

The section of the tunnel with the same geometry and loading condition is analysed under undrained condition and the results are plotted together with the drained condition in fig.18. It can be seen that there is significant differences between the results of these two conditions. The variations of the vertical and horizontal diameters of the tunnel under these conditions are shown in figures 19 and 20. It is quite evident that the differences between the results of drained and undrained conditions for vertical diameter is smaller than horizontal diameter. It may be attributed to the heaving of the tunnels' floor during undrained condition.

The settlements of the ground surface above the tunnel and distribution of the vertical and horizontal displacements of the soil as well as the pore pressure, in undrained conditions, are illustrated in figures 21,22 and 23 respectively.

In all cases it is clear that the undrained behaviour of the tunnle is different from the drained conditions considerably. Therefore it is very important considering the undrained conditions while analysing the geotechnical structures in which the loading or unloading happen very fast. The situation which does not allow the soil behaves in drained condition even though the drain paths are provided in the soil.

7- Summary and Conclutions

In this paper the concept of the critical state soil mechanics is described briefly. Then a special software developed by the Cambridge University capable of using the critical state model, named CRISP (critical state program) is introduced and abilities of this package in analysing the behaviour of geotechnical structures are expressed.

An analytical study of a shallow tunnel similar to that developed and constructed by Atkinson et. al was carried out by finite elements method using CRISP, and the results are compared with the experimental data and other methods of analysis.

According to the analytical studies carrid out, the behaviour of shallow tunnels can be predicted relatively satisfactory by the critical state model, provided appropriate elements are selected while using finite element method. In this study the LST elements showed relatively good agreement with the experimental results.

Furthermore one of the significant factors affecting the shallow tunnels' behaviour, is the possibility of drainage of saturated soil layers. It can be concluded that the results of analysis by the critical state model for drained condition is quite different from those for undrained condtion.

rule as below:

$$\frac{\delta\varepsilon_v}{\delta\varepsilon_s} = \frac{M^2 - \eta^2}{2\eta} \qquad (10)$$

The parameters of λ,K, Γ, M, N, and vk are constants of the model that can be obtained from standard laboratory tests.

4- The Computer Software and Method of Modeling

It is possible to analyse a problem by the finite element method while the geometry of the problem, material properties, boundary conditions and the loading condition are known. In this study a special software developed by the soil group of the Cambridge University was used for analysing a shallow tunnel. This software which is based on the critical state soil mechanics, called CRISP (the critical state program) and make it possible to analyse geotechnical problems via finite elements. The facilities of the CRISP are as follows:

a) Type of analysis: Undrained, drained or fully - coupled consolidation analysis of two dimensional plane strain or axisymetric (with axisymetric loading) solid bodies.

b) Soil models: Anisotropic elasticity, inhomogeneous elasticity (properties varying with depth), critical state soil models (cam - clay, modified cam - clay).

c) Element types: Linear strain triangle and cubic strain triangle (with extra pore pressure degrees of freedome for a consolidation analysis).

d) Non - linear techniques: Incremental (tangent stiffness) approach. Options for updating nodal co - ordinates with progress of analysis. θ=1 for integration in time.

e) Boundary conditions: Element sides can be given prescribed incremental values of displacements or excess pore pressure.

Loading applied as nodal loads or pressure loading on element sides. Automatic calculation of loads simulating excavation or construction when elements are removed or added.

In order to analyse a shallow tunnel by the critical state model and compare the results with the previous works, a model tunnel similar to that developed by Atkinson et. all was used in this study. The boundary conditions and ranges of loadings are the same as the above mentioned experimental model.

The tunnel was analysed by the CRISP using the mesh shown in figure 8. The elements used in this mesh are linear strain triangle (LST) with seven nodes and cubic strain triangle (Cu ST) with 15 nodes. The surcharge pressure was applied to the system up to σ_s - σ_t = 200 KPa in 20 steps increments.

5- The Results of Studies

The results of analysis are shown in figures 9 to 13. In fig.9 the variations of maximum ground settlements versus (σ_s - σ_t) is shown. It is seen that deformations of the surface have been reduced about 20% for (σ_s - σ_t) up to the 110 KPa compared with the results of analysis using CST elements. In fig.10 variations of the vertical diameter of the tunnel versus (σ_s - σ_t) is illustrated. It can be seen that in the ranges of 90<(σ_s - σ_t)<140 the maximum differences happen between the results of analysis using LST and CST. The importance is that for the plastic ranges the LST curve tends to become close to the test results, while the CST curve changes in the opposite direction.

The variations of the horizontal diameter is shown in figure 11. In this case the difference between the results of two elements are considerable, but the LST curve changes from its linear form towards the experimental curve as the (σ_s - σ_t) increases.

The gruond settlements above the tunnel are shown in figures 12 and 13. Again it can be seen that the results of analysis using LST elements is closer to the test results than CST elements. Some analysis have been carried out using C_u ST elements as well. The results are shown in figures 14 and 15 together with the results of LST

Atkinson, Orr and Wrogh (1978) carried out a good piece of research in this regard. They made a small scale shallow tunnel in the laboratory and compared the results of some tests on this model with the results of finite element analysis using elastic and elasto - plastic incremental models. The experimental model developed in Cambridge University is shown in figure 1. In this study the soil layer, in plane strain conditions, was initially consolidated under 550 KPa surcharge pressure, and then was unloaded to 140 KPa. As a result the over consolidated ratio was about 4. After excavation the inside pressure of the tunnel was sustained to 140 KPa radially. Therefore the soil around the tunnel was subjected to an isotropic condition (K_0=1).

Some tests were carried out on the tunnel with C/D=0.62 in wich the failure occured due to increasing the surcharge pressure while the inside pressure of the tunnel (σ_t) was constant.

For analysing, the tunnel by the incremental failure method the Poissos' ratio considered to be constant during loading, while the stress - strain relationship was defined as a bilinear curve (fig.2) and the other parameters were as bellow :

E'_i=6500 KPa $\quad q_y$= 0.85 qf $\quad C'$ = 18.8 KPa

E'_y=0.5E'_i $\quad v'$ =0.2 $\quad \phi'$ =15.3

The experimental and analytical results (Using CST elements with constant strain) are shown in figures 3 and 4. It can be seen from the figures that the analytical model shows greater deformation than those measured in the laboratory. This means that the soil stiffness should be increased in the model which of course is not consistent with the stress - strain curve obtained from triaxial tests carried out on the soil samples (fig.5).

Differences between the results of experimental and analytical models may be attributed to the effect of some factors used in the theoratical model such as geometry and the type of the selected elements. These factors have been focussed and studied in this paper while using critical state model.

3- The Critical State Soil Mechanics

The main parameters in the critical state soil mechanics are defined as follows:

$P' = \frac{\sigma'_1+2\sigma'_3}{3}$ (1) the mean effective stress

$q' = \sigma_1 - \sigma_3$ (2) the effective deviator stress

v = 1 + e (3) the specific volume

The relationship between v and Inp' for normally consolidated soils in loading and unloading conditions is linear with slopes of λ and K respectively and can be expressesd by the following equations :

$$v = N - \lambda \ln p' \quad (4)$$

loading conditions (normally consolidated line, NCL).

$v = v_k$ - klnp' (5) unloading conditions

If an isotropically consolidated sample failes under axial loading in drained or undrained conditions the variations of v versus Inp' will be a new line with the same slope (λ) which is called the critical state line (CSL), (fig.6):

$v = \Gamma\text{-}\lambda \ln p'$ (6) the critical state line (CSL)

$q' = Mp'$ (7) the critical state line (CSL)

The equations (6) and (7) are representation of the critical state line in the (v - Inp') and (q' p') plots respectively. The parameter Γ is the specific volume of the soil for p'=1 KPa.

Considering the above basic consepts and using the theory of plasticity, the yield surface in the critical state model in the two dimensional space can be expressed as below:

$$q'^2 + M^2 p'^2 = p' pc' M^2 \quad (8)$$

wich represents an ellipse in (fig.7). For an arbitrary specific volume and from the equation of the normally consolidated line (NCL), which naturally p'_c is omitted, the equation of the state boundary surface in the critical state model will be as follows:

$$vk=\Gamma + (\lambda\text{-k}) \{1n(2) - 1n[1+(\eta/M)^2]\} \quad (9)$$

in wich $\eta = \frac{q'}{p'}$

The above yield surface is unified by the flow

The Analysis of Shallow Tunnels Using Critical State Model

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ABSTRACT

Predicting behaviour of shallow tunnels such as underground ways in saturated clays is of great importance. Thus many researchers have focussed on this subject. The availability of high capacity micro - computers, numerical methods and advanced behavioural soil models have made this problem to be analysed and solved much more easily.

In this paper a parametric analysis for a model tunnel in an over - consolidated clay has been carried out using critical state model in soil mechanics via finite element method. The influence of different factors on the results of analysis such as the type and the shape of the selected elements as well as the drainage conditions of the soil are investigated. The deformations of the tunnel and distribution of pore pressures as well as displacement of the soil around the tunnel are calculated and presented.

1- Introduction

It is essential to consider the behaviour of soils in order to determine the values of deformation and the stability of the earth around the tunnel. In the past, the soil media was considered as an elastic material and the deformations of the tunnel and the ground surface under desired loading conditions were calculated accordingly. Since the soil has non - linear elasto - plastic. Behaviour, the assumption of elastic behaviour lonely, can not lead to correct results in analysis and design of tunnels. This fact have been emphasized by many other researchers.

Today there are different models most of them try to predict the behaviour of soils under different loading conditions accurately. The elasto - plastic models based on the flow rule can show the soil behaviour more realistic than linear or non-linear elastic models. Using closed yield surface, the volumetric stresses and strains can be controlled by application of the appropriate hardening law. Also the loading - unloading and reloading behaviour of the soils can be shown easily in these models.

The critical state model is capable of predicting the soil behaviour particularly for normally consolidated and slightly over consolidated. Conditions the modified Cam-Clay model with the closed eleptical yield surface and other above mentioned characteristics are being used both in practices and research projects at present.

2- Review of Some Previous Studies

It is possible to determine the ground settlements during excavation for tunnels by the elasto - plastic model especially in soft clays.