Fig(2)-Equivalent capacitances for CPW gap and interdigitated capacitor.

Fig(3)-Equivalent CPW gap capacitances ($\varepsilon_r=9.8$, $W1/h=.2$, $k=w/(w+2s)=.56$, $h=.635 \text{ mm}$).
**Table 1 - Comparison of computed and measured values of equivalent interdigitated capacitances in CPW.**

<table>
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<tr>
<th></th>
<th>εᵣ</th>
<th>hᵣ</th>
<th>Wᵣ</th>
<th>Sᵣ</th>
<th>lᵣ</th>
<th>t</th>
<th>Cₛ₁</th>
<th>Cₛ₂</th>
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<td>12.9</td>
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Cₛ is influenced by both length and number of the fingers while the shunt capacitances Cₛ₁ and Cₛ₂ depend mainly on the length of the fingers. The equivalent capacitances computed by this method are compared by the results given in reference [3] and tabulated in table 1, the agreement is very good.

**CONCLUSION**

The paper presents a direct calculation of lumped element capacitances for a series gap in the centre conductor with different width and interdigitated capacitor of a coplanar waveguide. The effects of substrate thickness, strip width and slot sizes are also considered. The results are compared with the results from finite difference method given in reference [3]. The agreement between two methods is very good.

**ACKNOWLEDGEMENT**

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**REFERENCES**


stant, the potential will be:

\[
V(x(i), z(j)) = \frac{1}{4 \pi \varepsilon 0} \int \oint \frac{\sigma(\xi(i), \eta(j))}{\sqrt{(x(\xi(i) - x(i))^2 + (z(\eta(j) - z(j)))^2 + h^2}} dx dz
\]

which can be evaluated analytically. The appropriate Green function \( D(i_1, j_1)(i_2, j_2) \) is found analytically with considering the images of each subsection as described in reference [4] using multiple image method. In this way the effects of substrate thickness \( h \) is considered. Then the total potential at the centre of subsection \((i_1, j_1)\) will be:

\[
V(i_1, j_1) = \sum_{i_2=1}^{M_1} \sum_{j_2=1}^{M_2} \sigma(i_2, j_2) D(i_1, j_1)(i_2, j_2)
\]

which can be written in a matrix form as: \([V] = [D][\sigma]\)

By this way the matrix relating charges on the subsections with their potentials is determined. Assuming zero potential for ground planes and +1 or -1 volt for the centre conductor, the unknown charge densities on the subsections are obtained by matrix inversion, by which the matrix capacitance is determined. By subtracting the line charge of the same piece of line, but without discontinuity, the excess charge due to the discontinuity (capacitance of the discontinuity) is obtained:

\[
C = \sum_{i_1=1}^{M_1} \sum_{j_1=1}^{M_2} \left( \sigma(i_1, j_1) - \sigma(i_1, j_0) \right) S(i_1, j_1)
\]

where \( M_1 \) and \( M_2 \) are the number of the subdivisions on the centre conductor in \( x \) and \( z \) directions respectively, \( S(i_1, j_1) \) is the area of the subsection number \((i_1, j_1)\) and \( \sigma(i_1, j_1) \) is the charge density on subsection \((i_1, j_1)\) when there is no discontinuity or the line length is infinite in \( z \) direction.

**GAP IN CENTRE CONDUCTOR**

The effect of this discontinuity which is shown in Fig. (2) can be considered as a series capacitance \( C_g \) and two shunt capacitances. The capacitance matrix for this discontinuity is a second order symmetrical matrix as:

\[
C = \begin{bmatrix}
C_{11} & C_{12} \\
C_{12} & C_{22}
\end{bmatrix}
\]

We compute the charge on one of the centre conductor \( (q_{1+}) \) by once putting the potential of both centre conductors at +1 volt and the ground planes at zero Volt (even mode). Again the charge on the same conductor \( (q_{1+}) \) is computed for one centre conductor at +1 volt and the other one at -1 volt and the ground planes at 0 volt (odd mode). Then the series gap capacitance \( (C_{g}) \) is obtained by subtracting \( q_{1+} \) from \( q_{1++} \), i.e.:

\[
C_{12} = C_g = \frac{q_{1+} - q_{1++}}{2}
\]

and the shunt capacitance is computed by adding these two charges and subtracting the line charge for infinite length \( (q_0) \) (i.e. without discontinuity).

\[
C_{11} = C_{12} + C_g = \frac{q_{1+} + q_{1+}}{2} - q_0
\]

\[
C_{22} = C_{12} + C_g = \frac{q_{1+} + q_{1+}}{2} - q_0
\]

The twofold symmetry of the structure is taken into account to reduce the size of the matrix by two, and hence by 8 the inversion effort with a Householder algorithm. Fig. (3) shows the results of computations for \( C_g \), \( C_{51} \) and \( C_{52} \) for CPW as a function of gap width for different centre conductor widths \( (W2/W1) \). The gap capacitance decreases and the shunt capacitance \( C_{51} \) increases with increasing gap width. M. Naghed and I. Wolff [3] used finite difference method to calculate CPW discontinuities. Fig. (3) compares their results with ours. The two methods match very well.

**INTERDIGITATED CAPACITORS**

The effect of this discontinuity can be considered as the model shown in Fig (2). The charge densities on subsections of a large waveguide rectangular section is computed as described above. The equivalent capacitances are computed for different number and length fingers. The coupling capacitance
Computation Of Capacitances For Gap Discontinuities And Interdigitated Capacitors In Coplanar Waveguide By Moment Method

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ABSTRACT:

The method of moment has been used for solving many electromagnetic problems, including computation of microstrip discontinuities [1,2]. In this paper, this method is used to calculate the lumped element capacitances of some discontinuity structures in coplanar waveguide (CPW). Structures considered here are: 1) gap in centre conductor and 2) interdigitated capacitors. The effects of the substrate thickness, permittivity and the physical dimensions are investigated. The calculation results are in good agreement with measurements and results of other methods.

INTRODUCTION

Coplanar waveguide (CPW) is a particular structure which consists of a centre line strip conductor and two semi-infinite grounded planes on both left and right sides. As it is known, the discontinuities with small geometrical dimensions with respect to the wavelength, can be approximated and modeled by lumped elements equivalent circuits. In CPW, the geometrical size of the components can be chosen as small as to fulfill the mentioned condition up to 25 GHz [3].

This paper considers the computation of capacitances for the following CPW structures: 1) series gaps in the centre conductor with different width on each side of gaps, 2) interdigitated capacitors, Fig.(1). The result is compared with the reference [3] to indicate the accuracy of the calculation.

PROCEDURE

To apply the method of moment a rectangular section of CPW is divided in M subsections. As the variation of the charge density near the edges and near the discontinuity is higher than at the centre, smaller subsections for the regions near the edges and discontinuities are chosen. Assuming that the charge density on each subsection is con-

![Diagram](image)

Fig.(1)- Geometry of CPW gap and interdigitated capacitor.