4.Conclusion

The method of weighting to approximate equality constraints is commonly used in practice. It was therefore considered appropriate to make comparison study with other alternative methods. For this, four test case based on the 6-bus sample system were set up. The first included all measurements without equality constraints while the second treated three zero injection as measurement, the third case treated the three zero injections as a pseudo measurements and finally in fourth case the three zero injection measurements are considered as equality constraints. Results were subsequently compared to determine if the zero injection with equality constraints had any advantage over the others from the accuracy and convergence point of view. It was concluded that the convergence rate in the case of equality constraints wears low since the gain matrix becomes more dense. However the redundancy will be improved, hence the number of actual field measurements could be reduced.

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Table 4. Comparison of squared residual error with number of iterations.

	Number of iterations	Square of residuals error	
		Measured	Estimated
Case 1	3	0.020667	0.011693
Case 2	3	0.028438	0.001340
Case 3	3	0.028432	0.001353
Case 4	4	0.028400	0.001687

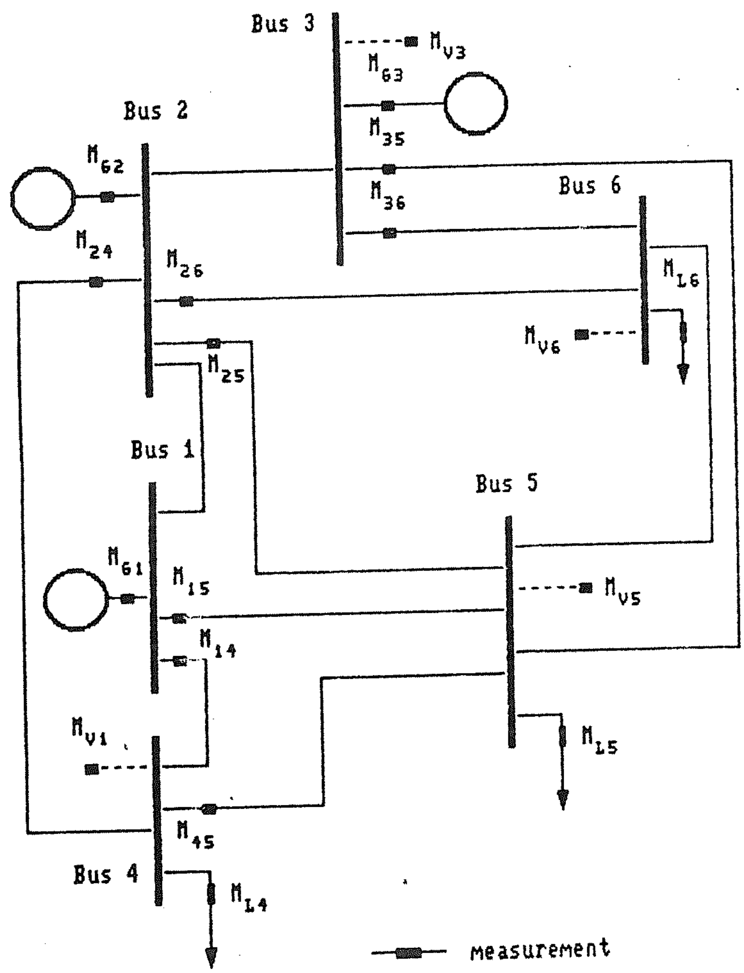

Figure 1. Six bus network with measurements [9].

Table 3. State Estimation Solution for Six - Bus System.

Measurement	Actual value			Measured value for Case 1			Estimated value for Case 1			Measured Value for Case 2,3,4			Estimated value for Case 2			Estimated Value for case 3			Estimated Value for Case 4		
	V	P	Q	V	P	Q	V	P	Q	V	P	Q	V	P	Q	V	P	Q	V	P	Q
MV1	1.0500						1.0478						1.0485			1.0485			1.0479		
MG1		1.0832	.2322		1.1309	.2740		1.1451	.2491		1.1309	.2348		1.0965	.2227		1.0961	.2227		1.0974	.2199
M12		.2905	-.1447					.3033	-.1425					.2941	-.1485		.2940	-.1485		.2926	-.1506
M14		.4366	.2278		.4479	.2327		.4524	.2340		.4749	.2668		.4412	.2225		.4410	.2225		.4431	.2220
M15		.3560	.1491		.3879	.1578		.3893	.1575		.3691	.1614		.3612	.1488		.3610	.1488		.3617	.1484
MV2	1.0500						1.0463						1.0490			1.0489			1.0489		
MG2		.5016	.8687		.5430	.8869		.5616	.8672		.5430	.9012		.5172	.8705		.5169	.8705		.5276	.8771
M21		-.2813	.1412					-.2934	.1404					-.2846	.1457		-.2844	.1457		-.2830	.1477
M23		.0298	-.1063					.0455	-.1051					.0385	-.1051		.0384	-.1051		.0372	-.1051
M24		.3335	.4965		.3626	.5280		.3397	.4950		.3482	.5292		.3330	.4927		.3329	.4927		.3396	.4950
M25		.1551	.1848		.1756	.2197		.1799	.1860		.2015	.1937		.1576	.1863		.1576	.1863		.1591	.1875
M26		.2643	.1525		.3112	.1710		.2899	.1509		.3056	.1827		.2726	.1509		.2724	.1509		.2747	.1520
MV3	1.0700						1.0654						1.0684			1.0683			1.0683		
MG3		.6003	.9880		.6107	1.0164		.6156	.9859		.6107	1.0174		.5842	.9839		.5839	.9839		.5954	.9879
M32		-.0294	.0746					-.0450	.0739					-.0380	.0736		-.0380	.0736		-.0368	.0736
M35		.1935	.2689		.2299	.3018		.2082	.2684		.2169	.3071		.1891	.2706		.1890	.2706		.1920	.2715
M36		.4362	.6446		.4390	.6635		.4525	.6436		.4910	.6740		.4332	.6397		.4328	.6396		.4402	.6428
MV4	.9863			1.0032			.9823			1.0511			.9857			.9856			.9850		
ML4		-.7004	-.7010			-.6976		-.7064	-.7031		-.6950*	-.6819		-.7026	-.6902		-.7024	-.6902		-.7112	-.6908
M41		-.4254	-.2036					-.4403	-.2064					-.4299	-.1979		-.4297	-.1979		-.4317	-.1970
M42		-.3171	-.4739					-.3230	-.4719					-.3167	-.4704		-.3166	-.4704		-.3230	-.4721
M45		.0420	-.0235		.0722	-.0101		.0570	-.0247		.0488	-.0150		.0440	-.0219		.0439	-.0219		.0435	-.0217
MV5	.9796			.9847			.9732			1.0004			.9779			.9778			.9773		
ML5		-.7003	-.7007			-.6894		-.7918	-.6962		-.6939*	-.6987		-.7023	-.7056		-.7020	-.7056		-.7066	-.7069
M51		-.3449	-.1381					-.3761	-.1385					-.3497	-.1366		-.3495	-.1366		-.3502	-.1361
M52		-.1495	-.1885					-.1734	-.1869					-.1518	-.1895		-.1518	-.1895		-.1533	-.1904
M53		-.1812	-.2685					-.1952	-.2662					-.1768	-.2702		-.1767	-.2702		-.1795	-.2707
M54		-.0417	-.0145					-.0563	-.0121					-.0436	-.0159		-.0436	-.0159		-.0431	-.0160
M56		.0169	-.0912		.0274	-.0734		.0091	-.0925		.0500	-.0746		.0197	-.0935		.0196	-.0934		.0195	-.0938
MV6	1.0015			1.0395			.9964			1.0152			1.0003			1.0002			.9998		
ML6		-.6998	-.6992			-.6839		-.7327	-.6905		-.6952*	-.6812		-.7077	-.6900		-.7072	-.6900		-.7164	-.6927
M62		-.2581	-.1611					-.2828	-.1567					-.2662	-.1587		-.2660	-.1587		-.2681	-.1596
M63		-.4255	-.6018					-.4415	-.5991					-.4226	-.5975		-.4223	-.5974		-.4295	-.5996
M65		-.0163	.0637					-.0085	.0653					-.0190	.0662		-.0189	.0662		-.0188	.0665

* These three are considered as zero injections and are added as measurements in case 2, pseudo - measurements in case 3 and equality constraints in case 4.

6-bus network in Table 3. As can be seen from the Table 3 all four test cases are observable and since the measurements are well distributed in the network, all the estimated quantities not only are close to the actual values they give better estimate of the system than the measurements. Table 4 shows the square residuals and number of required iteration for each test case. In test case four since the relative confidence of injections are higher with respect to other measurements the convergence rate degrades. Also, the test case four has low convergence since gain matrix ($A_k^T R^{-1} AK$) becomes more dense due to fill in resulting from matrix multiplication. However, in test case four, the redundancy will be improved, hence the number of actual field measurements could be reduced. Hence by this method the exact piece of information can be provided without metering installation cost. In addition, since no metering or telemetry is needed, it is not subject of metering error or telemetry failure.

Table 1. Network data for six bus system.

From	To	Impedance (p.u.) R	Impedance (p.u.) X	Suceptance/2
1	2	0.1000	0.2000	0.0200
1	4	0.0500	0.2000	0.0200
1	5	0.0800	0.3000	0.0300
2	3	0.0500	0.2500	0.0300
2	4	0.0500	0.1000	0.0100
2	5	0.1000	0.3000	0.0200
2	6	0.0700	0.2000	0.0250
3	5	0.1200	0.2600	0.0250
3	6	0.0200	0.1000	0.0100
4	5	0.2000	0.4000	0.0400
5	6	0.1000	0.3000	0.0300

Table 2. Six bus generator and load data.

Bus no.	Gen. (P.U.)	Voltage (P.U.)	P Load (P.U.)	Q Load (P.U.)
1	0.00	1.050	0.00	0.00
2	0.50	1.050	0.00	0.00
3	0.60	1.070	0.00	0.00
4	0.00	1.000	0.70	0.70
5	0.00	1.000	0.70	0.70
6	0.00	1.000	0.70	0.70

conditions for the unconstrained case, but here we will have to satisfy third condition such that

$$\nabla^2 L(X,\lambda)= \begin{vmatrix} A^TA & C^T \\ C & 0 \end{vmatrix} \quad (27)$$

Where $\nabla = L(X, \lambda)$ is Hessian matrix and must be positive semidefinite at the minimum. The nonlinear equations (24) and linear equation (26) may be solved for X by an iterative procedure; therefore at each iteration the following linearized equation is solved.

$$\begin{vmatrix} A^TA & C^T \\ C & 0 \end{vmatrix} \begin{vmatrix} X \\ \lambda \end{vmatrix} = \begin{vmatrix} A^Tb \\ d \end{vmatrix} \quad (28)$$

The above equation can be rewritten in terms of PSSE's variations such that:

$$\begin{vmatrix} H^TR^{-1}H & He^T \\ He & 0 \end{vmatrix} \begin{vmatrix} \Delta X \\ \lambda \end{vmatrix} = \begin{vmatrix} H^TR^{-1}\Delta Z \\ \Delta Ze \end{vmatrix} \quad (29)$$

Where $H=\partial h/\partial X$ and $He=\partial c/\partial X$ are Jacobian matrices, $\Delta Z=z-h(X)$ and $Ze=-C(X)$ are the measurement error vectors and R is the covariance matrix. In the computation of equation 29, both the state vector x and vector of multipliers λ in each iteration are updated. Also, the resultant system will be order of n+p as compared to n for method of weighting, but in gain matrix the order reduces from m+p into m, which will compensate for the increase size. The triangular factorization and sparsity are utilized in the computation; in order to achieve numerical stability and save computer memory requirements.

3.Numerical Example

The sample numerical example is the 6-buses, 11-lines system which is taken from Wood and Wollenberg [9] as shown in Figure 1. The network and generation data for the test system are shown in Tables 1 and 2. The metering locations, which have been selected by the author for the purpose of the paper are shown in Figure 1. The transmission network parameters are given in per unit, considering 100 MVA and 230 KV values. The number of state variables are 11, and the total number of measurable quantities varied between 30-33 in four test case. These four test cases are as follows:

Case 1: Complete measurements system with no zero injections.

Case 2: Complete measurements system, where three zero injections are added as measurements.

Case 3: Complete measurements systems, where three zero injections are added as pseudo-measurements.

Case 4; Complete measurements system, where three zero injections are added as equality constraints.

In test case 4 three new added measurements are considered to have very high weighting (with small variance), in order to process measurements with equality constraints. The results of specially developed computer program for the purposed algorithm are presented for the where

$A=R^{-1/2} H$ is m×m matrix of random number.

and

$b=R^{1-2} \Delta Z$ is m×1 residual vector.

Thus equation (10) can be written as

$$\min_X \{f(X) = 1/2(AX - b)^T (AX-b)\} \quad (10)$$

Now for a function f(x) on IR^n where f is twice continusly differentiable in a neighborhood of X, in order X to be a local minimum, the following conditions must hold:

$$g(X)= \nabla f(X) = -A^T b + A^T AX \quad (12)$$

$$g(X)= \nabla f(X) \mid x=x\hat{} =0 \quad (13)$$

$$X\hat{}= (A^T A)^{-1} A^T b \quad (14)$$

$$G(X) = \nabla^2 f(x)=A^T A \quad (15)$$

where G(X^) is positive definite. Thus, the solution X^ will be strict local minimum.

Now if function f(X) considered to be the same as unconstrained case and P to be a set of linear equality constraints such as

$$CX=d \quad (16)$$

then necessary conditions for a constrained minimum at X can be written as:

(i) $C\bar{X} - d = 0$ (17)

(ii) $g(\bar{X}) + C^T\lambda = 0$ (18)

or equivalently

$$Z^T g(\bar{X}) = 0 \quad (19)$$

(iii) $Z^T G(\bar{X})Z$ is positive semi-definite (20)

where Z is an nx(n-p) matrix whose columns form a basis for the null space of the constraints. The optimality conditions can be presented for the constrained least square problem in terms of a lagranigian. The problem is

$$\min_X \{f(X) = 1/2 \ (AX-b)^T (AX-b)\} \quad (21)$$

Subject to

$$CX - d = 0 \quad (22)$$

Where f(X) is the unconstrained least squares objective function. C is a P × n coefficient matrix for the constraints, and d is a p × 1 vector. The method of Lagrange multipliers solves the above constrained minimization problem by first defining the Lagrangian L(X,λ).

$$L(X, \lambda) = 1/2 \ (AX-b)^T (AX-b) + \lambda^T(CX-d) \quad (23)$$

Where the vector λ is the Lagrange multipliers. The estimated state vector X is the solution of equation 21, and must satisfy the following optimality conditions

$$\frac{\partial L(X,\lambda)}{\partial X} = \nabla L_x(X,\lambda)=A^T A\bar{X}-A^T b+C^T\lambda\hat{} \quad (24)$$

or $g(\bar{X}) + C^T\lambda = 0$ (25)

$$\frac{\partial L(X,\lambda)}{\partial \lambda} = \nabla L_\lambda(X,\lambda)=C\bar{X}-d=0 \quad (26)$$

Note that, these are the same as optimality

Lagrange multipliers [2], [6]. This method is gaining popularity in recent state estimation implementations. A third method uses direct elimination of variables using the equalities. The original objective function is reduced to a lower order function which can be solved by unconstrained methods [7]. Finally Hachtel's augmented matrix method with equality constraints have been applied to power system state estimation since zero injections are treated as equality constraints, the remaining equations do not have widely differing scales [8].

In this paper the general theory behind the equality constrained optimization problem is utilized by applying Lagrange multipliers to the equality constrained PSSE. The triangular factorization of the gain matrix along with the optimal ordering scheme is utilized in preserving sparsity. Finally a numerical test problem for the purposed method and specially developed computer program is presented.

2.PSSE With Equality Constraints

In power system state estimation, for an N-bus power system, there exist m measurements whose objective is the minimization of the weighted sum of squares of the measurement residuals. i.e.

$$\min_X f(X) = [h(X)-Z]^T R^{-2} [h(X)-Z] \quad (1)$$

where

Z : m×1 measurement vector.

h(X) : m×1 nonlinear vector function relating the measured quantities to the state variable.

X : n×1 true state vector.

R^{-2} : m×m diagonal weighting matrix.

The set of m equations relating the telemetered measurements and state variables can be expressed as:

$$z=h(X)+\eta \quad (2)$$

where η is the measurement error vector, and it is assumed to have zero mean and random variation, then

$$E\{\eta\}=0 \quad (3)$$

$$E\{\eta\eta^T\}=R \quad (4)$$

where R is m×m covariance matrix and E{.} is the expectation value. Applying a Taylor's series expansion to h(X) and defining the m×1 residual measurement vector as

$$\Delta z=z-h(X_0) \quad (5)$$

and n×1 state vector as

$$\Delta X = X-X_0 \quad (6)$$

the objective function can be written as

$$\min f(X) = \|R^{-1/2} H\Delta X- R^{-1/2}\Delta z\|_2^2 \quad (7)$$

where $\|.\|_2$ dentoes a 2-norm and H is n×n Jacobian matrix such that

$$H(X_0) = \left.\frac{\partial h(X)}{\partial X}\right|_{X=X_0} \quad (8)$$

Equation (7) can be written as standard linearized model of the least squares problem which is used at each iteration step in the solution, this is

$$\min_X \{f(X) = 1/2\|AX-b\|_2^2\} \quad (9)$$

POWER SYSTEM STATIC-STATE ESTIMATION WITH EQUALITY CONSTRAINTS

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ABSTRACT

within any electric power system network there are a number of buses which may have exact information regarding their real and/or reactive power or there is neither generation nor load. An advantage can be taken of these, known " zero injections " by formulating them as a set of equality constraints. In this paper Lagrange multipliers method is applied to equality constrained zero injections power system static state estimation. Finally test results for the four test cases are presented.

KEY WORDS: State Estimation, power System, Lagrangian multipliers method, Equality constraints

1. INTRODUCTION

Most power system state estimation (PSSE) programs process only noise corrupted measurement quantities to solve state variables. However, within any power system network there are a number of buses where there is neither generation nor load, or these buses may have exact information from network model. These measurements may be used in state estimation by assigning high weighting (i.e. small variance). However, the large disparity in the weights may cause the gain matrix to be ill-conditioned, thus degrading the convergence [1], i.e. it may take more iterations to converge, or, sometimes, fail to converge at all. An advantage can be taken of these, known " zero injection " by formulating them as a set of equality constraints [2]. Hence the overall redundancy will be increased without installing an additional metering. Very little information has been provided in the literature regarding the application of constrained least squares method to the problem of power system state estimation. In general an approximating equality constraints is used by applying an arbitrary large weighting factor to each constraint [3-5]. Another method, treat the zero injection as equality constraints by using