say up to 1 year, closely tracks monthly flows and may be used in modelling dam operation with due regards to storage levels and yield from the reservoir.

ACKNOWLEDGEMENTS

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the sum of squares of the first k residual autocorrelations multiplied by the number of observations for the period of time-series study. The Q values distributed approximately chi-square with the degree of freedom equal to K minus the number of estimated parameters. Table 1 shows the results are not significant at the 0.05 level.

Table 1. portmanteau test.

K	Q	degrees of freedom	level of significance
6	8.16	4	0.272
12	12.59	10	0.408
18	18.59	16	0.426
24	23.65	22	0.478

CONCLUSION

Equation 8 was used to forecast monthly flows in the river for the 12 - month period beginning in May 1984. The model forecasts with 95% confidence interval for the period of May 1984 - April 1985 are shown in Fig. 5. Most of the flows are within 5% of the actual flows.

The supremacy of ARIMA modelling for short-term forecasts lies in the simplicity of structure and proper incorporation of lag structure and seasonality; attempt has therefore been made to make a parsimonious selection of the parameters without seriously affecting the accuracy of the results. Although the model of equation 8 understimates 1984 annual flows by about 7%. Nontheless the forecasts are very reasonable and accurately replicate the 1984 cyclical pattern as depicted in Fig. 5. Short-term forecasting

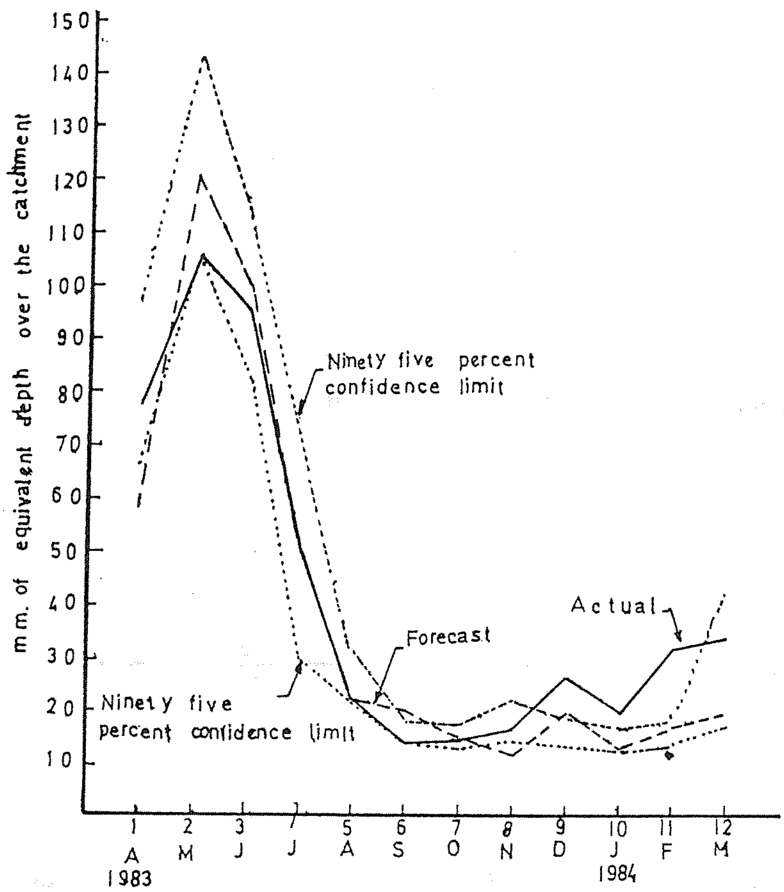

Fig. 5 Comparison of actual and forcasted monthly flows for 1983-84 in river Karaj

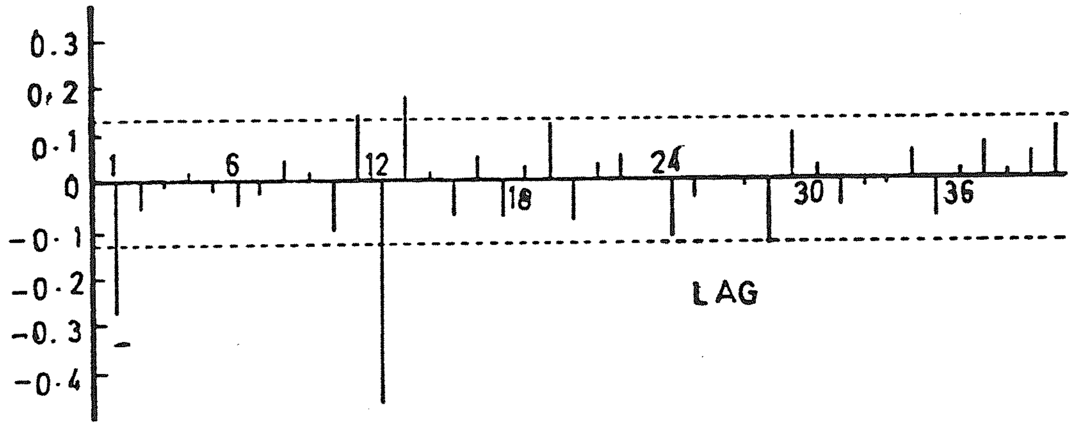

Fig. 3a Autocorrelation function of regular and 12-months seasonally differenced logarithm of monthly flows in Karaj

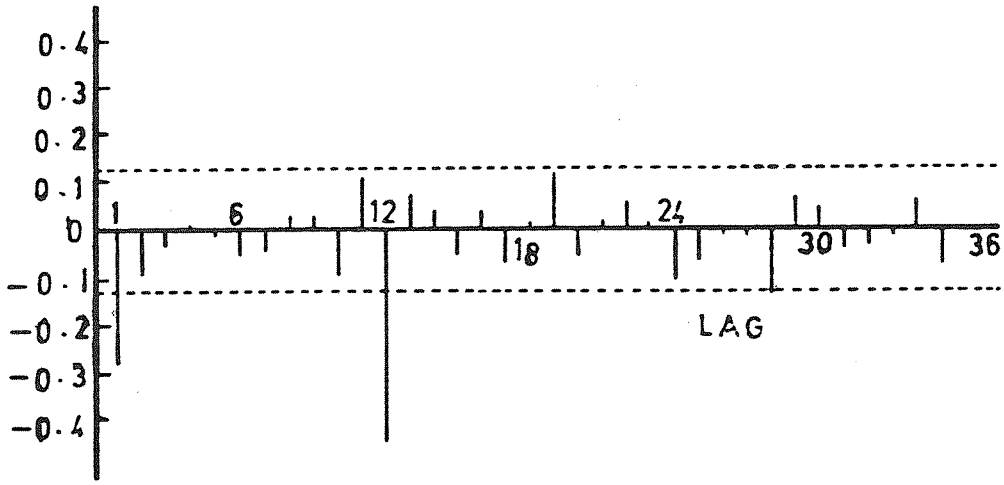

Fig. 3b Partial autocorrelation function of regular and 12-month seasonally differenced logarithm of monthly flows in Karaj 1963-83

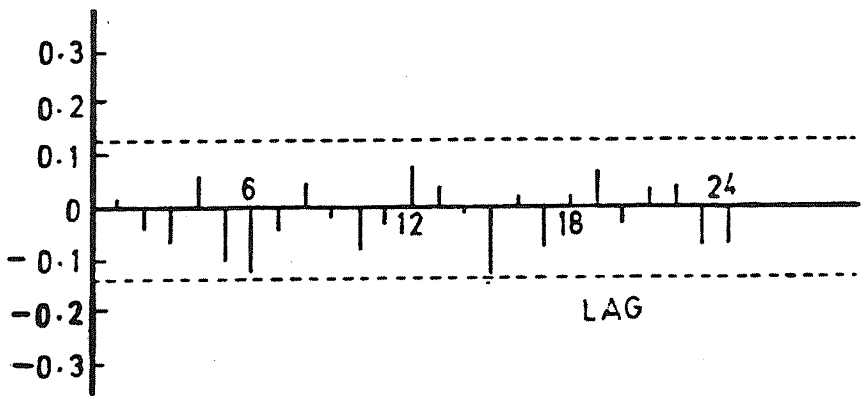

Fig. 4 Residual autocorrelation function of 1963-83 arima model of monthly flows in Karaj

selves a new model should be identified.

The ACFs of the logarithm of monthly flows for the period of 1963-1984 are shown in Fig. 2 dashed lines represent the 95% confidence interval. As Fig. 2. shows, the ACFs fail to die out and confirm a non-stationary time-series as is the case with the ACFs of the raw monthly flows in the river. Stationarity, however, was ensured by adding two types of differencing namely regular and seasonal to the logarithm of monthly flows. PACFs of the dirrerenced flows are shown in Fig. 3. The PACFs slowly die out and show decaying pattern of spikes at lags 12, 24 and 36. This suggests a candidate MA seasonal model; which is confirmed by the negative spikes at lags, 1 and 12 in the ACFs.

Although a general model of (1, 1, 1)(1, 1, 1) or (0, 1, 1) (1, 1, 1) could be tried on the data a parsimonious selection of the parameters was made using a (0, 1, 1)(0, 1, 1) model. When the residuals from this model was examined they showed no significant spike or pattern in the correlogram. The selected model has the following form:

$$\Delta \Delta_{12} \log_{10} X_t = (1 - 0.31B)(1 - 0.83B^{12}) q_t \ldots (8)$$

The residual ACFs of the model are shown in Fig. 4 and are inside the range of 95% confidence interval and therefore not significant. To check whether the entire residual autocorrelation is different from what could be expected of white noise the portmanteau test was performed (box and jenkins 1976). Table 1. summarizes the test conclusions. The Q statistics is

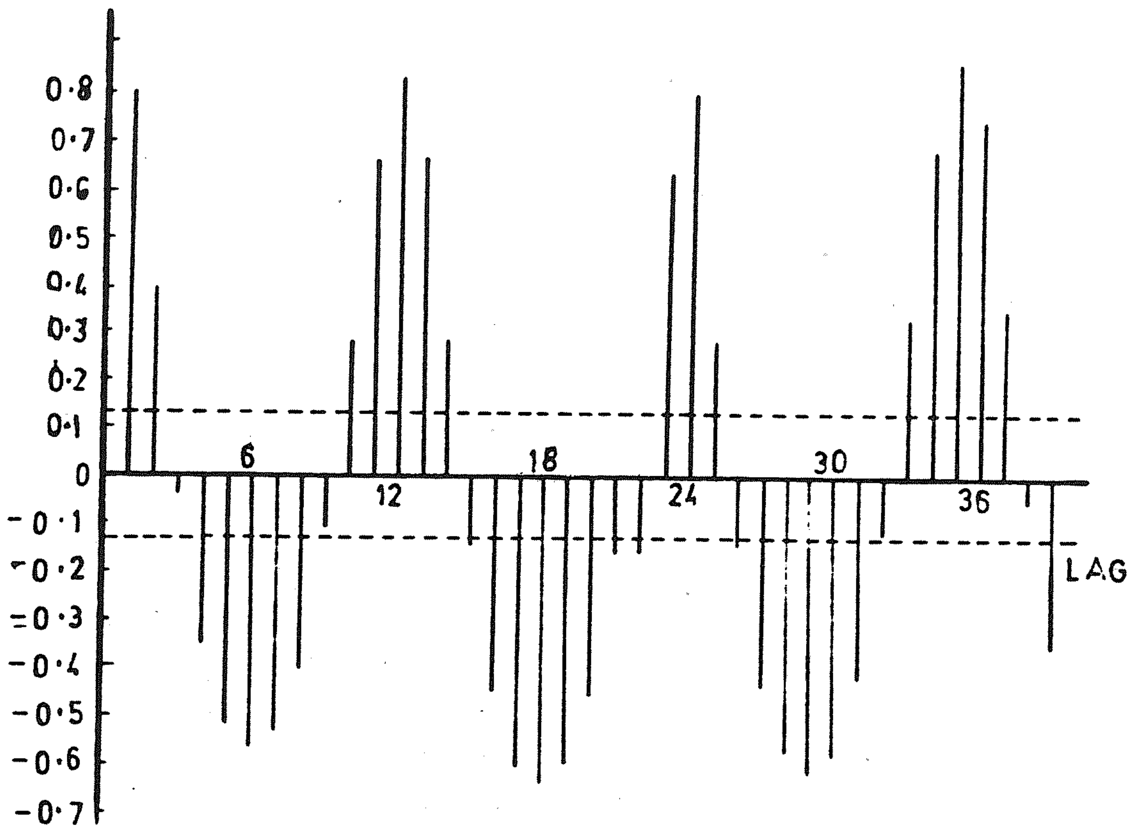

Fig. 2 Autocorrelation function of logarithm of monthly flows in Karaj 1963 - 83

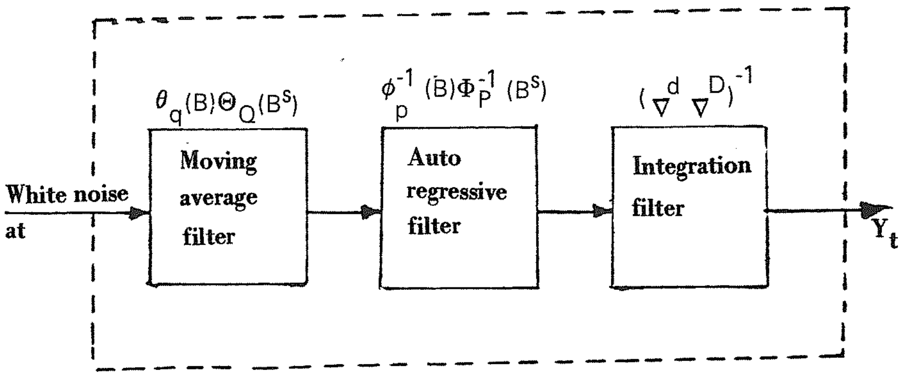

Fig. 1 Block diagram of the arima model

a moving average filter, has input a_t and transfer function $\theta_q(B)\ \theta_Q\ (B^s)$. The second filter, an autoregressive filter, has the output of first filter as input, and a transfer function of $\phi_p^{-1}\ (B)\ \Phi_P^{-1}\ (B^s)$. Finally the third filter, an integration filter, has the output of second filter as input and has a transfer function of $(\nabla^d \nabla^D)^{-1}$.

A useful notation to describe the order of various components in multiplicative arima model of equation 1 is given by (p, d, q) $(P, D, Q)^S$ and corresponds to the orders of the regular and seasonal factors, respectively. By representing a time-series in terms of a multiplicative model it is often possible to reduce the number of paramaters to be estimated.

Moldel Building

The selection of a model for any time-series data from the family of multiplicative autoregressive integrated moving average is in large part a matter of judgement. Nonetheless, a generally accepted model building strategy includes iterative identification, estimation, and diagnosis stages (levenbach and hay; 1980). The identification of a model may be accomplished on the basis of:

Prior knowledge of the data pattern,

Evaluation of the plotted series,

Evaluation of the sample autocorrelation coefficients

Evaluation of the sample partial autocorrelation Coefficients (PACFS).

Two steps are usually undertaken in this process. First the level of differencing, d and D, will be derminded. Second with the information of ACFs and PACFs, the order of autoregressive and moving average components will be verified. Nonstationary time series may be detected if ACFs fail rapidly to die out. The proper differencing may be detected by evaluating ACFs. The differencing which exhibit the smallest ACF values for increasing lags will be the superior ones. Once the order of differencing d and D are determined, the second step involves selecting the candidate ARIMA model. Inspection of ACFs is used to single outmoving average components, q and Q; and PACFs are used to identify autoregressive components p and P. Once a tentative model is identified, its parameters are estimated and tested for statistical significance. In addition, parameter estimates must meet the stationarity-invertibility requirements. If either criterion is not met, a new model should be identified and its parameters estimated and tested. After successful estimation and testing, the model should be diagnosed. To pass diagnosis, the autocorrelation of the residuals, RACFs from the estimated model should be sufficiently small and should resemble white noise. If the residuals remain significantly correlated among themuseful for short-term forecasting say up to one year, when it is expected that the underlying factors determining the level of the variable of interest in the past, herein monthly river flows, will behave the same in the near future.

This paper presents a multiplicative IMA model for forecasting monthly flows in Karaj river at the Amirkabir dam reservoir using data from 1963 to 1984. Water authorities, reservior operating personnel and water use policy makers may use the methodology of this study to enhance forecasting endeavors.

Model Structure

The ARIMA model and the method for assessing its parameters, as presented in the following section were primarily developed by box and jenkins (1976) as a means of predicting and controlling a time-series. The model has the general form of:

$$\phi_p (B)\, \Phi_p (B^s)\, \Delta^d \Delta^D Y_t = \theta_q(B)\, \Theta_Q(B^s)\, a_t \qquad (1)$$

Where Φ_p is a regular polynominal operator of order p; B is a regular backshift operator pertinent to the index of Y_t and a_t; i.e; $BY_t = Y_{t-1}$ and $B^2a_t = a_{t-2}$; Φ_p is a seasonal backshift operator peritinent to the index of Y_t and a_t; i.e; $B^sY_t = Y_{t-s}$ and $B^{2s}a_t = a_{t-2s}$; s is the period of sesonality; i.e; for periodicity of 12 months s = 12; ∇^d is a regular differencing of order d. i.e; d = 2 follows $\nabla^2 Y_t = (1\text{-}B)^2\, Y_t = (1\text{-}2B + B^2)\, Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$; ∇^D is a seasonal differencing of order D; i.e; D = 2 follows $\nabla^2\, Y_t = (1 - B^s)^2\, Y_t = (1 - 2B^s + B^{2s})\, Y_t = Y_t - 2Y_{t-s} + Y_{t-2s}$; Y_t is an index of monthly flows in the river; θ_q is a regular polynominal operator of order q, θ_Q is a seasonal polynominal operator of order Q, and a_t is the white noise variable for moth t, independent and normally distributed with mean = zero and varience σ^2. For equation 1, the polynominal operators of ϕ and ϕ, θ and Θ with order of p, **P**, q and Q respectively mean that:

$$\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots\ldots - \phi_p B^p \qquad (2)$$

where $\phi_1, \phi_2 \ldots, \phi_p$ are paramters; i.e; $\phi_p(B)\, Y_t = a_t$ is

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots\ldots + \phi_p Y_{t-p} + a_t \qquad (3)$$

and is a regular auto-regressive model of order P indicated by **AR(P)**. In this model, Y_t is expressed as weighted average of past Y_s plus random noise a_t.

$$\Phi_p(B^s) = (1 - \Phi_s B^s - \Phi_{2s} B^{2s} - \ldots\ldots - \Phi_{ps} B^{ps}) \qquad (4)$$

where $\Phi_s, \Phi_{2s}, \ldots \Phi_{ps}$ are parameters; i.e; $\Phi_p(B^s)\, Y_t = a_t$ is

$$Y_t = \Phi_s Y_{t-s} + \Phi_{2s} Y_{t-2s} + \ldots\ldots + \Phi_{ps} Y_{t-ps} + a_t \qquad (5)$$

and is a seasonal autoregressive model of order P indicated by AR(P). In this model, Y_t is expressed as a weighted average of p past seasonal Y_s plus random noise a_t.

$$\theta_q(B) = (1 - \theta_1 B - \theta_2 B^2 - \ldots\ldots - \theta_q B^q) \qquad (6)$$

Where $\theta_1, \theta_2, \ldots, \theta_q$ are parameters, i.e; $Y_t = \theta_q(B)\, a_t$ is

$$Y_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots\ldots - \theta_q a_{t-q} \qquad (7)$$

And is a regular moving average of order q indicated by MA(q). In this model, Y_t is expressed as a weighted average of q prior seasonal observed noise plus current noise a_t.

The nostationary process Y_t, once it has become stationary by differencing and deternding, may be viewed as being generated from the independent random variable a_t, which is filtered using transfer function $|\, \theta_q (B)\, \Theta_Q (B^s)\, \phi_p^{-1} (B)\, \Phi_p^{-1} (B^s)\, |$ and constrained by the bounds of stationarity and invertibility (box and jenkins 1976). The block diagram of Fig. 1 indicates the filtering process of a_t. The first filter,

Extending the Record of the Karaj

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ABSTRACT:

This paper focuses on short-range modelling and forecasting of the monthly flows in River Karaj at the AmirKabir DAM reservoir. The 1963-83 time-series data suggest a multiplicative, two parameter, integrated moving average (IMA) model to replicate monthly flows in the river. The identified model required a regular and 12-month seasonal differencing in logarithm of monthly flows and has a moving average components with LAG 1 and 12, respectively.

Model predictions for 1984 were very reasonable when compared with actual flows. Previous investigation using fourier series for cyclical pattern were erroneous because the residual flows above the most significant harmonics proved to be nonstationary. Simplicity of the arima model; the reliability of their predictions make it attractive compared with the "combined" models for monthly flows in Karaj.

KEYWORDS: *Water resources: time-series analysis; forecasting; mathematical modelling; systems analysis.*

INTRODUCTION

Karaj with a catchment area of 855 Km^2 at the Amirkabir DAM is a major source of water and electricity supply to Tehran. With increase in population there is a heavier demand for water from this major source of supply. Indeed failures to meet the demand for water and electricity has already been experinenced in recent years. Future flows should therefore be needed in reservoir operation, planning and management of the water consumption from the dam.

A class of models proven to be particularly well suited to short-term forecasting is that referred to as **ARIMA**, autoregressive integrated moving average, (box and jenkins 1976). such models are particularly