Compressibility-Toughness Dominated Propagation Regime of a Fluid-Driven Fracture from a Borehole

Ali Lakirouhani[i], Ahmad Fahimifar[ii]

ABSTRACT

This paper is concerned with the problem of plane-strain hydraulic fracture growing transverse to a borehole in an impermeable elastic medium under condition of compressibility-toughness dominated propagation regime. Compressibility and near borehole effects are considered to explain the early unstable phase of the fracture growth observed in laboratory experiments conducted with low viscosity fluids. A solution is obtained in terms of dimensionless fluid net pressure, fracture length and borehole radius under conditions where it can be assumed that the fluid is inviscid. It is shown that the problem depends only on a dimensionless time and on a dimensionless borehole radius. With time, the solution evolves between two asymptotic regimes where the solution is self-similar. Compressibility effects control the solution at small time while the solution at large time is dominated by material toughness. An instability is identified in the problem after breakdown. The potential for unstable growth depends on the initial flaw length, fluid compressibility, volume of fluid, and the material elastic modulus. It is also seen that the deviatoric in-situ stress reduces the breakdown pressure.

KEYWORDS

Hydraulic Fracturing, Compressibility, Propagation, Plane Strain, Borehole.

1. INTRODUCTION

When conducting experiments designed to reproduce the process of hydraulic fracturing under laboratory conditions, it is generally desired for the fracture to grow in a controlled manner in a block without interacting with its outer boundary. Data sampling also demands for the fracture to propagate slowly enough for the acquisition system to collect relevant information. However, experiments with low viscosity fluids systematically exhibited an unstable crack growth step just after breakdown that was characterized by a very fast increase of fracture length and a sharp pressure decline. Numerical models available to describe the propagation of a fluid-driven crack rely on the assumptions that the fluid is incompressible and that the borehole can be treated as a point injection source [3,5,8]. Because compressibility effects are not considered, these models are in fact inappropriate to describe fracture breakdown and the early fracture growth.

In this paper, we consider the problem of a radial fracture transverse to a borehole with finite radius a, see Figure 1. The fracture is driven in an impermeable elastic material characterized by Young's modulus E, Poisson's ratio ν, and fracture toughness K_{Ic}. The fracture is driven by a compressible Newtonian fluid of viscosity μ, injected at a constant rate Q_0. The development proposed in this paper deals with the role of the fluid compressibility and the borehole finite radius upon the early stage of fracture propagation.

A solution is proposed in terms of the ratio of fracture length to borehole radius, and injection pressure for the simplified case of an inviscid fluid, for which the pressure gradient along the fracture is neglected.

The model is intended to reproduce the fracture response in terms of fracture length $l(t)$, width profile $w(x,t)$ and net pressure profile $p(x,t)$ where x is the radial coordinate with respect to the borehole center and t is the fluid injection time, the net pressure p is defined as the fluid pressure p_f minus any far-field compressive stress σ_3 acting perpendicular to the fracture plane.

Under above assumptions, the governing equations are formulated. The formulation makes use of the following effective material parameters:

$$E'=\frac{E}{1-\nu^2},\quad \mu'=12\mu,\quad K'=4\left(\frac{2}{\pi}\right)^{1/2}K_{Ic} \tag{1}$$

where E' is the plane strain elastic modulus, μ' is the effective viscosity, and K' is the effective toughness.

[i] Ph.D. Student, Amirkabir University of Technology
[ii] Faculty Member, Amirkabir University of Technology

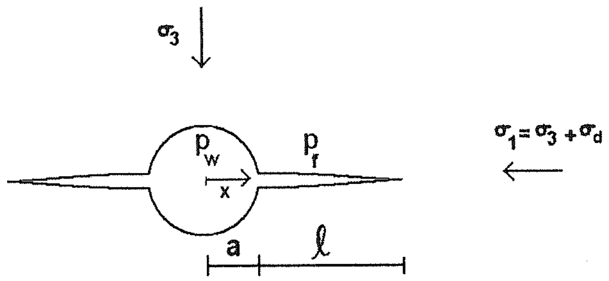

Figure 1: Problem definition.

2. GOVERNING EQUATIONS

2.1. Elasticity Equation

The crack opening w is related to the fluid pressure p_f by an integral equation [6]:

$$p_f(x,t)-\sigma_l(x)=\frac{E'}{4\pi}\int_a^{a+l}\frac{\partial w}{\partial \xi}\left[\frac{1}{x-\xi}+H(x,\xi)\right]d\xi \qquad (2)$$

where, $(\sigma_l(x)=\sigma_1 h_1(x)+\sigma_3 h_2(x)-p_w h_3(x))$ and p_w is borehole pressure. This equation can be obtained by superposition of climbing-edge dislocations, where the gradient $\partial w/\partial\xi$ is actually the dislocation density [2]. The kernel of integral has two terms, the first one, $(x-\xi)^{-1}$ is called *a simple Cauchy kernel* and appears in all plane crack problems. The second term, $H(x,\xi)$ is related to the borehole effect.

2.2. Lubrication equation

The flow of viscous fluid in the crack is governed by the Reynolds equation, according to lubrication theory [1]:

$$\frac{\partial w}{\partial t}=\frac{1}{\mu'}\frac{\partial}{\partial x}\left(w^3\frac{\partial p_f}{\partial x}\right) \qquad (3)$$

This non-linear differential equation is deduced from Poiseuille law

$$q=-\frac{w^3}{\mu'}\frac{\partial p_f}{\partial x} \qquad (4)$$

and the local continuity equation

$$\frac{\partial w}{\partial t}+\frac{\partial q}{\partial x}=0 \qquad (5)$$

where q denotes the flow rate.

2.3. Propagation criterion and boundary conditions

The problem formulation is completed by specifying a propagation criterion and the boundary conditions at the fracture inlet $(x=a)$ and at the tip $(x=a+l)$.

The boundary conditions at the crack tips $x=\pm(l+a)$ are given by a zero fracture opening and a zero flow conditions, i.e.,

$$w=0,\quad q=0,\qquad at\quad x=\pm(l+a) \qquad (6)$$

The assumption of no lag between the fluid fronts and the crack tips implies that there is no boundary condition for the fluid presure at these points.

The criterion of continuous quasi-static propagation of a fracture in mobile equilibrium [7],

$$K_I=K_{Ic} \qquad (7)$$

is expressed as the tip asymptote of the crack opening [9]

$$w=\frac{K'}{E'}((l+a)-x)^{1/2},\qquad (l+a)-x\langle\langle(l+a) \qquad (8)$$

where K_I is the mode-I (opening) stress intensity factor.

In the following, we use C_f to introduce system apparent compressibility, which accounts for the compliance of the injection system, borehole and compressibility of the volume of fluid upstream of the fracture inlet. If we consider that the system compressibility is constant, the boundary condition at the fracture inlet is given in terms of flow rate, including a component due to fluid compression:

$$q=\frac{Q_0}{2}-\frac{1}{2}C_fV_0\frac{\partial p_w}{\partial t}\qquad at\quad x=\pm a \qquad (9)$$

where V_o is the initial fluid volume in the system under zero net pressure.

The origin of time is fixed to the instant at which the net pressure in the initial flaw is equal to zero.

Given these boundary and initial conditions, the local continuity equation (5) can also be expressed in terms of a global continuity equation after integration both in time and space as below:

$$Q_0t-C_fV_0p=2\int_a^{a+l}w(x)\,dx \qquad (10)$$

where p is the net pressure.

3. SCALING AND SIMPLIFICATION

We seek an appropriate scaling, where each term of the governing equation consists of a dimensionless factor of order one and a dimensionless constant that closely approximates the term order. Building on a scaling originally introduced for hydraulic fractures with no lag [4], we introduce the scaled coordinate $\zeta=(x-a)/l$ and express the sought quantities $w(x,t)$, $p(x,t)$, $q(x,t)$, $l(t)$ as:

$$w=\varepsilon L\Omega,\qquad p=\varepsilon E'\Pi,\qquad q=Q_0\Psi,\qquad l=L\gamma \qquad (11)$$

where $\varepsilon(t)$ is a small parameter, $L(t)$ is a length scale and where $\Omega,\Pi,\Psi,$ and γ are dimensionless aperture, net pressure, flow rate and crack length, respectively.

If the far-field deviatoric stress is ignored, under this scaling (11) the main equations are transformed as follows

- *Elasticity equation*

$$\Pi=\frac{1}{4\pi}\frac{1}{\gamma}\int_0^1\frac{\partial\Omega}{\partial\zeta}H'\left(\zeta,\varsigma,\frac{\gamma}{G_a}\right)d\varsigma \qquad (12)$$

- *Lubrication equation*

$$\left(\frac{\dot{\varepsilon}t}{\varepsilon}+\frac{\dot{L}t}{L}\right)\Omega+\dot{\Omega}t-\zeta\left(\frac{\dot{L}t}{L}+\frac{\dot{\gamma}t}{\gamma}\right)\frac{\partial\Omega}{\partial\zeta}=\frac{1}{\gamma^2 G_m}\frac{\partial}{\partial\zeta}\left(\Omega^3\frac{\partial\Pi}{\partial\zeta}\right) \quad (13)$$

- *Propagation condition*

$$\Omega=G_k\gamma^{1/2}\sqrt{1-\zeta}, \qquad 1-\zeta\langle\langle 1 \quad (14)$$

- *Global volume balance*

$$\frac{1}{G_s}\gamma\int_0^1\Omega d\zeta=1-G_u\Pi \quad (15)$$

The five dimensionless groups G_a, G_m, G_k, G_s, G_u are defined as follows:

$$G_a=\frac{a}{L},\quad G_m=\frac{\mu'}{\varepsilon^3E't},\quad G_k=\frac{K'}{\varepsilon L^{1/2}E'}, \quad (16)$$
$$G_s=\frac{Q_0t}{\varepsilon L^2},\quad G_u=\frac{\varepsilon C_fV_0E'}{Q_0t}$$

The groups G_m and G_k are associated with energy dissipation processes while the groups G_s and G_u are related to storage of fluid and G_a is associated with the borehole radius effect. In fact, G_m represents viscosity, G_k toughness, G_s fluid storage in the fracture, and G_u fluid storage in the borehole due to compressibility effects.

Four specialized scaling types can then be constructed to capture the features of the solution for a specific propagation regime dominated by one group for the fluid storage and one group for the energy dissipation. This is done by setting either G_s to 1 or G_u to G_s^{-1} and either G_m or G_k to 1, so as to yield definite expressions for the small parameter $\varepsilon(t)$ and the length scale $L(t)$; the three other groups then become evolution parameters. The introduction of these different scalings enables us to study the particular conditions when one or both of the evolution parameters become zero without simplifying the equations.

3.1. Compressibility-toughness scaling ($\bar{K}$ -Scaling)

This scaling (denoted by the subscript $\bar{K}$) is obtained by setting $G_k=1$, and $G_u=G_s^{-1}$. The compressibility-toughness scaling is meant to describe a propagation regime controlled by the storage in the fracture of the fluid released by decompression. The energy balance is dominated by formation of new fracture surface while the energy dissipation in the viscous fluid flow can be neglected. In this form, the resultant scaling factors $\varepsilon_{\bar{k}}$ and $L_{\bar{k}}$ are given by

$$\varepsilon_{\bar{k}}=\left(\frac{K'^4}{E'^5C_fV_0}\right)^{1/4},\qquad L_{\bar{k}}=\left(C_fV_0E'\right)^{1/2} \quad (17)$$

And the three evolution parameters G_a, G_m and G_a correspond to a dimensionless borehole effect $A_{\bar{k}}$, dimensionless viscosity $M_{\bar{k}}$, and dimensionless fluid injected volume $S_{\bar{k}}$, given by

$$A_{\bar{k}}=\frac{a}{\left(C_fV_0E'\right)^{1/2}},\quad M_{\bar{k}}=\mu'\left(\frac{\left(C_fV_0\right)^{3/4}E'^{11/4}}{K'^3t}\right), \quad (18)$$
$$S_{\bar{k}}=Q_0t\left(\frac{E'}{K'^4\left(C_fV_0\right)^3}\right)^{1/4}$$

Hence, the field quantities are formally scaled as follows:

$$\begin{aligned}
l(t)&=L_{\bar{k}}\gamma_{\bar{k}}\left(A_{\bar{k}},M_{\bar{k}},S_{\bar{k}}\right)\\
w(x,t)&=\varepsilon_{\bar{k}}L_{\bar{k}}\Omega_{\bar{k}}\left(\zeta,A_{\bar{k}},M_{\bar{k}},S_{\bar{k}}\right)\\
p(x,t)&=\varepsilon_{\bar{k}}E'\Pi_{\bar{k}}\left(\zeta,A_{\bar{k}},M_{\bar{k}},S_{\bar{k}}\right)\\
q(x,t)&=Q_0\Psi_{\bar{k}}\left(\zeta,A_{\bar{k}},M_{\bar{k}},S_{\bar{k}}\right)
\end{aligned} \quad (19)$$

4. LIMITING CASE OF AN INVISCID FLUID

In the remaining part of this paper, we restrict our consideration to the case of a fracture driven by an inviscid fluid in a linearly elastic-brittle, impermeable material, taking into account upstream compressibility effects in the inlet boundary conditions.

4.1. Formulation for an inviscid fluid

Specialized expressions that are used to determine the evolution of the fracture can readily be deduced from the governing equations presented earlier. Since the fluid is assumed to be inviscid, the pressure in the fracture is uniform. It then follows that the model reduces to a set of three scalar equations; namely, an elastic expression for the fracture volume, the global fluid mass balance, and the propagation criterion.

It is in fact convenient to express the stress intensity factor K_I, the fracture volume V_f as their equivalent quantities (denoted by an asterisk) for the reference Griffith crack with the unit pressure

$$K_I=\left(pf_1(\beta)-\sigma_df_2(\beta)\right)K_{I*} \quad (20)$$
$$V_f=\left(pg_1(\beta)-\sigma_dg_2(\beta)\right)V_{f*} \quad (21)$$

where $\beta=l/a$ and $p=p_w-\sigma_3$ (net pressure). These dimensionless functions $f_i(\beta)$ and $g_i(\beta)$ (i=1, 2) are defined in such a way that their limits for $\beta=0$ is corresponded to the edge crack, and for $\beta=\infty$ to the Griffith crack. These functions can be found numerically [10]. Plots of these functions are also shown in Figure 2, where it can be seen that the effect of the wellbore becomes negligible when the crack length is larger than about 10 times the borehole radius.

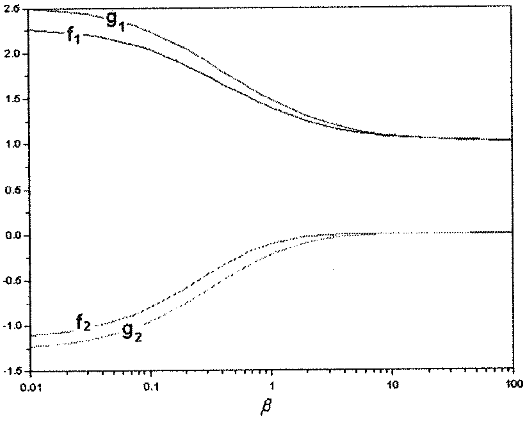

Figure 2: Dimensionless functions of $f_i(\beta)$ and $g_i(\beta)$ (i=1, 2).

Using the solution of a uniformly pressurized Griffith fracture in combination with (20) and (21) and propagation criterion (7), the following expressions can be deduced readily:

$$p=\frac{K'}{2^{5/2}l^{1/2}f_1(\beta)}+\sigma_d\frac{f_2(\beta)}{f_1(\beta)} \tag{22}$$

$$V_f=\left(pg_1(\beta)-\sigma_d g_2(\beta)\right)\frac{2\pi}{E'}l^2 \tag{23}$$

The above equations, together with the global volume balance

$$V_f=Q_0t-C_fV_0p \tag{24}$$

completely define the evolution of fracture, given the initial crack length l_0.

4.2. Solution in the $\overline{K}$-Scaling

The solution can conveniently be expressed in the $\overline{K}$-Scaling defined earlier. In this scaling, the solution, e.g., the fracture length $\gamma_{\bar{k}}(\tau;A_{\bar{k}},S_{\bar{d}})$ and the net pressure $\Pi_{\bar{k}}(\tau;A_{\bar{k}},S_{\bar{d}})$ depend only on dimensionless time τ, dimensionless borehole radius $A_{\bar{k}}$, and the dimensionless deviatoric far field stress $S_{\bar{d}}$, respectively as below:

$$\tau=\frac{t}{t_1},\qquad A_{\bar{k}}=\frac{a}{L_{\bar{k}}},\qquad S_{\bar{d}}=\frac{\sigma_d}{\varepsilon_{\bar{k}}E'} \tag{25}$$

where $t_1=\dfrac{\left(C_fV_0\right)^{3/4}K'}{E'^{1/4}Q_0}$.

Essentially, the zero viscosity model consists of the propagation criterion (22)

$$\Pi_{\bar{k}}=\frac{1}{2^{5/2}\gamma_{\bar{k}}^{1/2}f_1\left(\gamma_{\bar{k}}/A_{\bar{k}}\right)}+S_{\bar{d}}\frac{f_2\left(\gamma_{\bar{k}}/A_{\bar{k}}\right)}{f_1\left(\gamma_{\bar{k}}/A_{\bar{k}}\right)} \tag{26}$$

And an equation combining the global volume balance (24) and elasticity expression for the volume of a uniformly pressurized fracture (23)

$$\Pi_{\bar{k}}=\frac{\tau+2\pi\gamma_{\bar{k}}^2S_{\bar{d}}g_2\left(\gamma_{\bar{k}}/A_{\bar{k}}\right)}{1+2\pi\gamma_{\bar{k}}^2g_1\left(\gamma_{\bar{k}}/A_{\bar{k}}\right)} \tag{27}$$

where the normalized fracture length β has been expressed as

$$\beta=\frac{\gamma_{\bar{k}}}{A_{\bar{k}}} \tag{28}$$

Eliminating $\Pi_{\bar{k}}$ between (26) and (27) gives the time τ as a function of the fracture length $\gamma_{\bar{k}}$

$$\tau=\left(1+2\pi\gamma_{\bar{k}}^2g_1\left(\gamma_{\bar{k}}/A_{\bar{k}}\right)\right)\cdot\left(\frac{1}{2^{5/2}\gamma_{\bar{k}}^{1/2}f_1\left(\gamma_{\bar{k}}/A_{\bar{k}}\right)}+S_{\bar{d}}\frac{f_2\left(\gamma_{\bar{k}}/A_{\bar{k}}\right)}{f_1\left(\gamma_{\bar{k}}/A_{\bar{k}}\right)}\right)-2\pi\gamma_{\bar{k}}^2S_{\bar{d}}g_2\left(\gamma_{\bar{k}}/A_{\bar{k}}\right) \tag{29}$$

Then combining (27) and (29) yields an implicit relation between the dimensionless pressure $\Pi_{\bar{k}}$ and τ.

The large time asymptotic behavior of $\gamma_{\bar{k}}$ and $\Pi_{\bar{k}}$ are given by

$$\gamma_{\bar{k}}\sim\frac{2}{\pi^{2/3}}\tau^{2/3},\qquad \Pi_{\bar{k}}\sim\frac{\pi^{1/3}}{8}\tau^{-1/3}\qquad for\ \ \tau\rangle\rangle 1 \tag{30}$$

Figure 3 shows plots of the solution $\Pi_{\bar{k}}(\tau)$ for $S_{\bar{d}}=0$, $A_{\bar{k}}=1$, together with the large time asymptotic solution (dash-dot line). The solution is characterized by two branches, which indicate the existence of two possible solutions for the same time τ, and thus the existence of an instability. The upper branch gives the solution for a small crack with high pressure of the initial flaw, which the lower branch corresponds to a larger crack with a lower pressure.

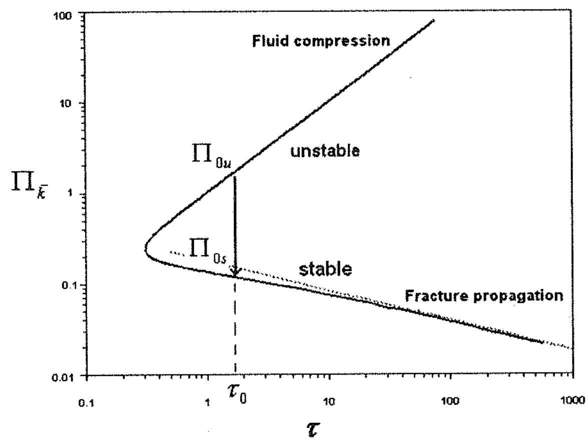

Figure 3: Two branches of the curve of the dimensionless pressure with respect to the time τ and the asymptotic trend for large values of τ.

4.3. Fracture breakdown and initial unstable growth

The origin of time is fixed to the instant at which the net pressure in the initial flaw is equal to zero. However, the system of governing equations is only applicable after breakdown, i.e., when the initial flaw starts to propagate. (Since the fluid is inviscid, fracture breakdown coincides with fracture initiation.) Thus the solution is meaningful only from the time of fracture breakdown, τ_0. The injection time required to reach initiation depends on the fluid injection rate, system compressibility, material toughness and initial flaw length.

Figure 4 shows the variation of $\gamma_{\bar{k}}(\tau)$ for $S_{\bar{d}}=0, A_{\bar{k}}=1$. The lower branch of $\gamma_{\bar{k}}(\tau)$ corresponds to upper branch of $\Pi_{\bar{k}}(\tau)$ when compressibility effect are significant (small initial flaws). Conversely, the upper branch of $\gamma_{\bar{k}}(\tau)$ corresponds to lower branch of $\Pi_{\bar{k}}(\tau)$, when compressibility effect are negligible (large initial flaws).

If compressibility effects are important, the pressure rises slowly and a large injection time is required to compress the fluid and reach the breakdown pressure Π_0. If initial notch is small, a higher pressure level is required to fulfill the propagation condition. Longer initial flaws are easier to propagate but more time is needed to fill the fracture and reach the breakdown pressure. The knowledge of τ_0 gives the position of the initial solution in Figure 3.

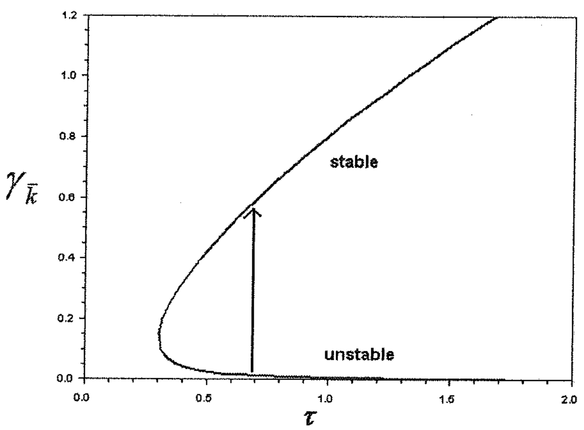

Figure 4: Dimensionless length of the crack versus τ.

Let us focus on the diagram of dimensionless pressure in Figure 3. If the initial solution $\Pi_0(\tau_0)$ is located on the upper branch, which corresponds to the toughness-compressibility regime, the solution jumps instantaneously from $\Pi_{0u}(\tau_0)$ to $\Pi_{0s}(\tau_0)$ as soon as the propagation condition is satisfied. The initial propagation phase is unstable. Conversely, if the initial solution $\Pi_0(\tau_0)$ is on the lower branch, the fracture propagation is always stable. A link can be established between the results formulated in Figures 3 and 4. The upper branch of $\Pi_{\bar{k}}(\tau)$ corresponds to the lower branch of $\gamma_{\bar{k}}(\tau)$, which indicates that small initial flaws or compressible systems both result in an unstable growth of the fracture after breakdown. The lower, stable branch of $\Pi_{\bar{k}}(\tau)$, corresponds to the upper branch of $\gamma_{\bar{k}}(\tau)$.

Figure 5 shows the dimensionless pressure for various dimensionless deviatoric far-field stresses. It can be seen that deviatoric stresses reduce breakdown pressure.

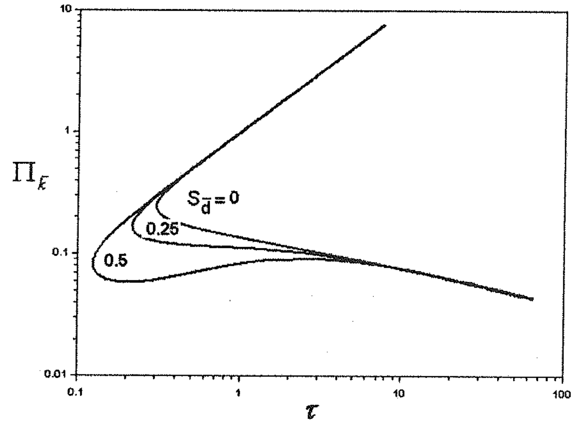

Figure 5: Dimensionless pressure versus τ for various dimensionless deviatoric stresses.

5. CONCLUCION

A model has been proposed for the propagation of two radial crack transverse to a borehole with a finite radius. This model considers effects of borehole and compressibility of the fluid. Scaling shows that problem depends on the dimensionless viscosity and two evolution parameters that are related to the compressibility and borehole radius. After scaling, the crack is assumed to be driven by an inviscid compressible fluid. The results presented in the previous section illustrate the model predictions of the initial unstable growth observed immediately after breakdown. During the initial unstable growth pressure decreases while the length of the crack increases. The stable propagation regime that follows the unstable growth is also reasonably well described by the model. The problem is solved for various deviatoric stresses and it is seen that deviatoric stresses decreases the breakdown pressure.

6. ACKNOWLEDGEMENT

The authors would like to thank the Division of Petroleum Resources in CSIRO, (Melbourne, Australia) for supporting and providing the facilities for completing this research.

7. REFERENCES

[1] Batchelor, G. (1967). An Introduction to fluid dynamics. Cambridge UK: Cambridge University Press.

[2] Bilby, B. and J. Eshelby (1968). Dislocation and the theory of fracture. In H. Leibowitz (Ed), Fracture, an Advanced Treatise, Volume I, Chapter 2, pp. 99-182. New York NY: Academic Press.

[3] Desroches, J. and M. Thiercelin, (1993). Modeling propagation and closure of micro-hydraulic fractures, Int. J. Roch Mech. Min. Sci. 30, 1231-1234.

[4] Detournay, E. (2004). Propagation regimes of fluid-driven fractures in impermeable rocks. Int J Geomech; 1-11.

[5] Garagash, D. (2004) Plane strain propagation of a hydraulic fracture: Large toughness solution. Engineering Fracture Mechanics.

[6] Hills, D., P. Kelly, D. Dai, and A. Korsunsky (1996). Solution of cracks problems. The Distributed dislocation technique, Volume 44 of Solid mechanics and applications. Dordrecht: Kluwer Academic Publ.

[7] Kanninen, M. and C. Popelar, (1985). Advanced fracture mechanics, Vol. 15 of the Oxford Engineering Science Series, Oxford University Press, Oxford, UK.

[8] Perkinş T. and L. Kern (1961). Widths of hydraulic fractures. J. Pet. Tech., Trans. AIME 222, 937-949.

[9] Rice, J. (1968). Mathematical analysis in the mechanics of fracture. In H. Liebowitz (Ed.), Fracture, an Advance Treatise, Volume II, Chapter 3, pp. 191-311. New York NY: Academic Press.

[10] Lakirouhani, A. (2007). Modeling initiation and propagation of hydraulic fracturing from a wellbore (PhD Thesis, Presented for defence).