The Economic Lot and Delivery Scheduling Problem in Flexible Job Shops

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ABSTRACT

In this paper, we consider a typical supply chain where a single supplier produces multiple components on a flexible job shop (FJS) and delivers them directly to an assembly facility (AF). It is assumed that demand rates for these components are deterministic and constant over a finite planning horizon. The objective is to find a common cycle lot production and delivery schedule that minimizes the average of holding, setup, and transportation costs per time unit for the supply chain. This problem consists of a combinatorial part (machine assignment and sequencing sub-problems), and a continuous part (common cycle duration and scheduling sub-problems). To account for these two elements, a new mixed integer nonlinear program (MINLP) is developed which simultaneously determines machine allocation, sequencing, lot sizing and scheduling decisions. In order to reduce computational complexity, instead of solving this MINLP directly, we propose an efficient enumeration method to determine the optimal solution of the model. The performance of the proposed method is evaluated by some numerical experiments. Another applicable case (Lot streaming) is also studied and required modifications in the model formulation and the solution procedure are described. Moreover, a numerical example is presented to illustrate applicability of the proposed mathematical models and the solution methods.

KEYWORDS

Lot and Delivery Scheduling, Flexible Job Shop, Common Cycle, Supply Chain, Finite Horizon.

1. Introduction and Problem Definition

Increasing global competition has imposed tremendous pressure on all members of supply chains. Companies are responding to this pressure by optimizing their activities to better serve their customers. One of the main issues in this regard is efficient and effective management of material flow through a supply chain that is critical to its success [11].

In this paper, we consider a simple supply chain where a captive supplier produces multiple components on a flexible job shop (FJS), accumulates these components and delivers them directly to an assembly facility (AF). Such situations are common in the automobile industry, and may occur in other industries as well [3]-[4].

The supplier's production facility is a flexible job shop. A flexible job shop is one of the most usual production systems in manufacturing discrete parts that can be considered as an extension of two classical systems, namely the job shop and the parallel shop. It involves several work centers (production stages) where each stage has one or more identical parallel machines. Each component requires a sequence of operations in different stages based on its unique process route. Moreover, each component must be processed by at most one machine at each stage, but some components may skip some stages.

The customer (AF) uses the components at a fairly constant rate. This may be due to higher-level production smoothing, or various characteristics of the demand or manufacturing processes. For example, in the automobile

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industry, the customer is a final assembly facility that uses a fixed-pace assembly line. To facilitate both workload smoothing in the assembly line and making possibility of just-in-time (JIT) delivery of components from supplier to AF, the assembly line is scheduled so as to smooth the usage rate of each component [3].

Our concerned problem is the lot and delivery scheduling in such supply chain where all parameters (such as demand rates) are deterministic and constant over a given finite planning horizon. This problem involves a combinatorial part (assignment of components to machines at each work center and their sequencing on each machine at the supplier), and a continuous part (duration of production and delivery cycle or lot sizing and production starting time for each component at each stage). In other words, machine allocation and sequencing sub-problems are the combinatorial part of the problem and lot sizing and scheduling sub-problems are the continuous part of the problem. The objective is minimizing the average of transportation, setup and inventory holding costs per time unit without backlogging across the supply chain. To solve the problem, we assume a common cycle for producing all components and their delivery. Thus, during each cycle, one batch of each component is produced and one delivery at the end of the cycle for accumulated components is scheduled. Moreover, it is required that the planning horizon to be an integer multiple of the common cycle length. So, a new mixed zero-one nonlinear program is developed which its optimal solution determines simultaneously the optimal assignment of components to machines at each stage, the optimal component sequence for each machine, the optimal lot sizes and the optimal beginning times for each production run.

The problem considered here, is an extension of economic lot and delivery scheduling problem (ELDSP) to flexible job shop systems in finite horizon case. ELDSP is a NP-hard problem; therefore it is obvious that our more general problem is definitely NP-hard.

The original ELDSP (the single item case) was introduced by Hahm and Yano [2]. Then, they extended their previous work to multiple components case so that they considered a simple supply chain where a supplier produces multiple components on a single machine or production line, accumulates the components, and finally delivers them to an AF [3]-[4]. They provided an excellent review of models related to the ELDSP and developed two efficient heuristic algorithms for solving it. In the first algorithm [3], they used the common cycle for all components and assumed that the time between deliveries is equal to the duration of the common production cycle. In the second one that is a generalization of the former [4], they assumed that multiple deliveries within a global production cycle are allowed (the nested schedule case).

Other researchers introduced several extensions to original ELDSP. Khouja [5] considered the ELDSP for a supplier that uses a volume flexible production system where component quality depends on both lot sizes and unit production times and developed an algorithm for solving it. Vergara et al. [10] extended ELDSP to multiple-supplier, multi-component simple supply chain and proposed an evolutionary algorithm (EA) to obtain an optimal, or near optimal, synchronized delivery cycle time and suppliers' component sequences.

In all above works, it is assumed that the production system of supplier (or each supplier) is a single production line or machine. Moreover, it is assumed that the planning horizon is infinite. There are several reasons for infinite horizon assumption. First, constructing a mathematical model for infinite case is easier. Further, this assumption makes feasible solution space larger and consequently may lead to better solutions. However, this assumption considerably reduces the usefulness of the proposed contributions, because in practice, planning horizons are always finite and rarely longer than 12 months. Further, in most cases, the schedules obtained by infinite horizon assumption could not be repeated an integer number of times during the finite planning horizon chosen in practice. Thus practitioners usually adjust such schedules to meet this condition, which may lead to a non-negligible increase in total cost [7].

Literature review in finite horizon case reveals that there are only four contributions from Ouenniche et al. [6]-[7] and Torabi et al. [8]-[9]. In [6]-[7], the production scheduling problem in job shops is studied under constant demand rates over a finite planning horizon either using the common cycle approach [6] or the multiple cycle approach [7], to obtain a cyclic schedule. The authors developed an optimal solution method in common cycle case and an efficient heuristic method to obtain a near optimal solution in multiple cycle case. It is noted that these two works are extensions for ELSP problem where demands (deliveries) are continuous and optimization issue is focused on a supplier with a job shop production system, but not on the supply network. Torabi et al. extended the common cycle economic lot scheduling problem (ELSP) to flexible job shops in finite horizon case and developed an optimal enumeration method to obtain optimal solution of this problem [8]. Moreover, they considered the common cycle economic lot and delivery scheduling problem (ELDSP) in flexible flow lines in finite horizon case and developed an efficient hybrid genetic algorithm to obtain optimal or near-optimal solutions for this problem [9].

However, to the best of our knowledge, no contributions are reported to economic lot and delivery scheduling problem in such supply chains where the supplier's production system is a flexible job shop. Thus, in this paper, a new mathematical model and efficient solution method is developed for this problem.

The outline of this paper is as follows. In section 2, problem formulation as well as necessary conditions to have a feasible solution are presented. In section 3, an enumeration method to obtain optimal solution is developed. Another applicable case of original problem (Lot streaming) as well as required modifications in the model formulation and the solution method are studied in section 4. In order to validation of the proposed solution method, numerical experiments are done and the corresponding results are presented in section 5. A numerical example is presented in section 6 that its solution is determined at the two studied cases. Finally, section 7 is devoted to conclusions and some recommendations for future studies.

2. Problem Formulation

The following assumptions are considered in the problem formulation:

- There is a captive supplier that produces several components for a single customer (AF) and makes direct deliveries to that customer;
- Supplier's production system is a flexible job shop which consists of several work centers, where each stage has one or more parallel machines that are identical in all parameters such as production rates and setup times (costs);
- Each component has a predetermined unique process route (based on its route sheet) that must be adhered to it fully; that is, no alternate routings are allowed;
- Each component must be processed at most by one machine at each stage; that is, no sharing is allowed for components among the machines at each stage. This assumption is optimal policy when there are parallel identical machines at each stage and also has practical advantages, since it does not require duplication of tooling and follows the group technology idea [1];
- The lots of each component are of equal sizes at different stages;
- Machines of different stages are continuously available and each machine can only process one component at a time;
- The system is deterministic, i.e., all parameters such as demand and production rates, transportation cost, and setup times (costs) are deterministic and constant over a finite planning horizon;
- The common cycle approach is used as production policy, i.e., there is a single delivery from the supplier to AF per cycle, during which one batch of each component is produced;
- Backlogging is not allowed;
- The supplier incurs sequence independent setup times and costs;
- Setup times and costs for the AF are negligible, which is reflective of many assembly environments;
- Delivery lead time is negligible and it is assumed zero;
- Production sequence for each machine at each stage is unique and is determined by mathematical model;
- Both the supplier and the assembler incur linear inventory holding cost on final components;
- The supplier incurs linear inventory holding costs on semi-finished components;
- Preemption is not allowed; that is, at a given stage, once the processing of a lot starts, it must be completed without interruption;
- Lot streaming is not allowed; that is, sub-batches of each component are not transferred to the next stage until the entire lot is processed at the current stage;
- There are unlimited buffers between successive stages, hence in process inventories are allowed, i.e., components may wait for their next operations;
- Total capacity of different stages are sufficient to meet the demands; thus there exists at least one feasible lot and delivery schedule;
- Integer number of cycles *(F)* are repeated until the planning horizon is covered;
- Zero switch rule is used. This means that production of each component at each cycle begins when its inventory level reaches zero.

Moreover, the notations used for the problem modeling are defined as follows:

Parameters:

n	number of components
m	number of work centers (stages)
i, u	component indices
j	stage index
m_i	number of required operations (or stages) for component i
M_j	number of parallel machines at stage j
n_j	number of components to be processed at stage j
M_{kj}	k-*th* machine at stage j
$p(j)$	set of components to be processed at stage j
$\mu(i,.)$	route of component i (indicated by an ordered sub-set of stages)
$\mu(i,r)$	r-*th* stage on the route of component i
d_i	demand rate of end component i
$p_{i,\mu(i,r)}$	production rate of component i at r-*th* stage of its route
$t_{i,\mu(i,r)}$	processing time for a lot of component i at r-*th* stage of its route ($t_{i,\mu(i,r)} = d_i \cdot T / p_{i,\mu(i,r)}$)
$s_{i,\mu(i,r)}$	sequence-independent setup time of component i at r-*th* stage of its route
sc_i	total setup costs of component i over all stages
$h_{i,\mu(i,r)}$	holding cost per unit of component i per time unit between r-*th* and $(r+1)$-*th* stages of its route
h_i	holding cost per unit of final component i per time unit (both at the supplier and at the AF)
A	transportation cost per delivery
H	planning horizon length
M	a large real number

Decision variables:

σ_j	production sequence vector at stage j,
σ_{kj}	production sequence vector at machine M_{kj}

J_{kj} the set of components which are assigned to machine M_{kj}

n_{kj} the number of components which are assigned to machine M_{kj}

T common production and delivery cycle length (time interval between setups and deliveries)

Q_i production lot size of component i at different stages ($Q_i = d_i \,.\, T$)

F number of production cycles over the planning horizon

$b_{i,\mu(i,r)}$ process beginning time of component i at r-th stage of its route (after setups)

$$z_{ilj} = \begin{cases} 1 & \textit{if product } i \textit{ is assigned to } l^{th} \textit{ position in } \sigma_j \ \ (j \mid M_j = 1) \\ 0 & \textit{otherwise} \end{cases}$$

$$x_{ilkj} = \begin{cases} 1 & \textit{if product } i \textit{ is assigned to } l^{th} \textit{ position in } \sigma_{kj} \ \ (j \mid M_j > 1) \\ 0 & \textit{otherwise} \end{cases}$$

Since after processing each component at each stage, there would be a value added for the component, thus, values of h_{ij} parameters will be non-decreasing, that is: $h_{i,\mu(i,r)} < h_i \,.\; h_{i,\mu(i,r-1)} \le h_{i,\mu(i,r)}$; $i=1,\dots,n$, $r = 2,\dots,m_i-1$.

Moreover, variables z_{ilj} are sequencing sub-problem variables at stages with only one machine and variables x_{ilkj} are both sequencing and machine assignment sub-problem variables at stages with more than one machine.

The problem can be formulated as a mixed zero-one nonlinear program (problem P). As mentioned earlier, to formulate this problem, we assume a common cycle for all products (T) and choose a cycle time such that the finite horizon H is an integer multiple of T. this assumption allows constructing production schedules that are easy to implement and generally preferred in real-life situations. Hereafter, we first describe the mathematical model of Problem P and then in section 3, we propose a procedure for its solution.

The objective of Problem P is to minimize the average of transportation, setup, work-in-process and end component inventory holding costs per time unit for the supply chain. Two terms in the objective function are evident: the average setup cost per time unit is $\sum_i sc_i / T$, and the average delivery cost per time unit is A/T. The inventory holding costs are somewhat more complicated. Inventory holding costs are incurred at both the supplier and the assembler. Figure 1 shows the inventory level of final component i in one cycle at the assembly facility. Therefore, the average inventory of component i per unit time at the assembly facility is: $\frac{1}{T}\left\{(d_i T)\frac{T}{2}\right\} = \frac{d_i T}{2}$, and then the average holding cost per unit time at the assembly facility would be:

$$\sum_{i=1}^{n} d_i h_i T / 2 \,.$$

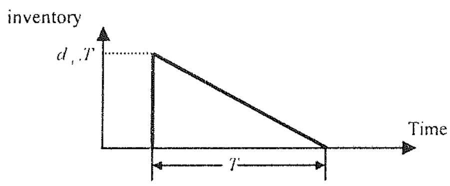

Figure 1 Inventory level at the assembler in one cycle.

Two types of inventory are considered for the supplier: work-in-process inventory and finished component inventory. Figure 2(a) and Figure 2(b) show the evolution of work-in-process inventory of component i between two successive stages $\mu(i,r-1)$ and $\mu(i,r)$, and the inventory level of final component i, respectively.

From Figure 2(a), it is obvious that the average work-in-process inventory of component i between two successive stages $\mu(i,r-1)$ and $\mu(i,r)$ per unit time is:

$$I_{i,\mu(i,r-1)} = \frac{1}{T}\left\{\frac{d_i T}{2}\cdot\frac{d_i T}{p_{i,\mu(i,r-1)}} + d_i T\left(b_{i,\mu(i,r)} - b_{i,\mu(i,r-1)} - \frac{d_i T}{p_{i,\mu(i,r-1)}}\right) + \frac{d_i T}{2}\cdot\frac{d_i T}{p_{i,\mu(i,r)}}\right\} = d_i \left(b_{i,\mu(i,r)} + \frac{d_i T}{2 p_{i,\mu(i,r)}} - b_{i,\mu(i,r-1)} - \frac{d_i T}{2 p_{i,\mu(i,r-1)}}\right).$$

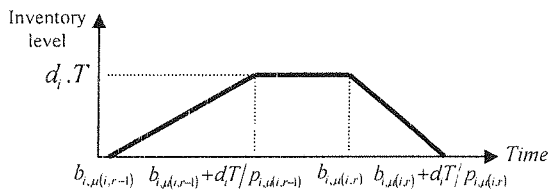

Figure 2(a) WIP inventory between stages $\mu(i,r-1)$ and $\mu(i,r)$.

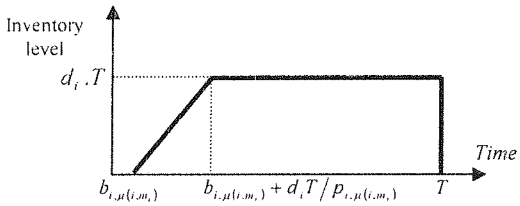

Figure 2(b) Finished component inventory.

Therefore, the total work-in-process inventory holding cost for all components per unit time at the supplier is:

$$TC_{WIP} = \sum_{i=1}^{n}\sum_{r=2}^{m_i} h_{i,\mu(i,r-1)} \,.\, d_i \,.\left(b_{i,\mu(i,r)} + \frac{d_i T}{2 p_{i,\mu(i,r)}} - b_{i,\mu(i,r-1)} - \frac{d_i T}{2 p_{i,\mu(i,r-1)}}\right).$$

Also, from Figure 2(b), we can see that the average inventory of final component i per unit time is:

$$I_{i,\mu(i,m_i)} = \frac{1}{T}\left\{ \frac{d_i T}{2} \cdot \frac{d_i T}{p_{i,\mu(i,m_i)}} + d_i T\left(T - b_{i,\mu(i,m_i)} - \frac{d_i T}{p_{i,\mu(i,m_i)}} \right) \right\}$$

$$= d_i \left(1 - \frac{d_i}{2 p_{i,\mu(i,m_i)}} \right) . T - d_i . b_{i,\mu(i,m_i)} .$$

Thus, the total inventory holding cost for all final components per unit time at the supplier is:

$$TC_{FPI} = \sum_{i=1}^{n} h_i . d_i \left(1 - \frac{d_i}{2 p_{i,\mu(i,m_i)}} \right) . T - \sum_{i=1}^{n} h_i . d_i . b_{i,\mu(i,m_i)} .$$

Therefore, the total cost per unit time (i.e. objective function of Problem P) would be:

$$TC = \sum_{i=1}^{n} \frac{A_i}{T} + \sum_{i=1}^{n} \left[h_i . \frac{d_i}{2} \left(1 - \frac{d_i}{p_{i,\mu(i,m_i)}} \right) + \frac{d_i^2}{2} \sum_{r=2}^{m_i} h_{i,\mu(i,r-1)} \left(\frac{1}{p_{i,\mu(i,r)}} - \frac{1}{p_{i,\mu(i,r-1)}} \right) \right] . T - \sum_{i=1}^{n} h_i . d_i . b_{i,\mu(i,m_i)}$$

$$+ \sum_{i=1}^{n} \sum_{r=2}^{m_i} h_{i,\mu(i,r-1)} . d_i \left(b_{i,\mu(i,r)} - b_{i,\mu(i,r-1)} \right)$$

Given the objective function and logical relationships between variables of Problem P (that some of them are extractable from inventory evolution curves), a new mixed nonlinear model is developed to obtain optimal solution of the problem that is presented in Figure 3.

Problem P has several constraints. Constraints (2) state that no component can be processed before it is completed at previous stage. Constraints (3) show that at each stage with one machine ($M_j=1$), no component can be processed before the completion of its predecessor in the related production sequence (σ_j). Constraints (4) are similar to Constraints (3), but they apply to stages with more than one machine ($M_j>1$). Constraints (5) and (6) state that each component has a unique position in the sequence of stages with only one machine. Also, Constraints (7) to (9) are applied to stages with parallel machines. Constraints (7) state that each component has a unique position in the sequence of one of the machines at these stages and Constraints (8) show that on each position of each machine at these stages, there is at most one component, because at each machine such as M_{kj}, it may be assigned less than n_j components to this machine. Constraints (9) stipulate that, one component can be positioned at one position of machine M_{kj}; if another component is to be positioned at previous position of this machine. Constraints (10) imply that at each stage with only one machine, processing the first component in the related sequence cannot start before setting up the corresponding machine. Also, Constraints (11) show that if component i is the first component in the sequence related to one of the machines in stage j (Mj>1), its processing can not start before setting up the corresponding machine. Constraints (12) assure that the obtained schedule is cyclic and state that the processing completion time of each component at final stage is less than or equal to cycle time. Constraint (13) imply that the common cycle is such that the planning horizon is an integer multiple of T. Constraints (14) shows that F is an integer greater than one, and finally, Constraints (15) are the non-negativity Constraints.

Moreover, since some time must be left for setups at each stage, the necessary conditions to have feasible solutions for the problem can be written as follows:

$$\sum_{i \in p(j)} \left(\frac{d_i}{p_{ij}} + \frac{s_{ij}}{T} \right) \le M_j \ ; \ \forall j = 1,\ldots, m.$$

But the value of variable T is not determined so far, thus we can redefine necessary conditions as follows:

At each stage j (j=1,..., m), the products are sorted in a non-increasing order of d_i/p_{ij} values. The term d_i/p_{ij} represents the fraction of one machine at stage j required by product i. Then according to this order, each product is assigned to the first available machine. At the end. if the following conditions are satisfied, then there would be at least one feasible schedule.

$$Min_k \left(1 - \sum_{i \in J_{kj}} \frac{d_i}{p_{ij}} \right) > 0 \ ; \ \forall j = 1,\ldots, m.$$

3. Solution Procedure

Problem P is a mixed zero-one nonlinear program. Non-linearity of this model is due to slightly nonlinear term $(A+\sum sc_i)/T$ in objective function (respect to T) and nonlinear constraint (13) in the constraints sets. Since it will be difficult to solve this mixed nonlinear model directly, we propose an enumeration method with an iterative process for solving it to optimality.

Let T* and Z* denote the optimal common cycle and the corresponding total cost per time unit. respectively. Moreover, let ZF denotes the objective function value of Problem P for a given value of F. Then this Problem can be solved using the following iterative procedure:

Initialization step. Let F=1, and solve the resulting mixed zero-one linear program. Set: Z* =Z1, T* =H

Iterative step. Increase F by 1 and solve the corresponding mixed zero-one linear program for this new value of F. If this model has no feasible solution, stop; else, if ZF < Z* then set Z* = ZF and T* =H/F and go to the next iteration.

Basically, this procedure enumerates all feasible values of F and for each value of F, it solves a mixed linear model to optimality. Thus, this procedure produces the optimal solution of problem P.

To solve these mixed linear models, we can use one of the large-scale mixed integer optimization tools such as CPLEX and LINGO. However, within our computational study, we used the LINGO 6.0 solver from LINDO systems, Inc.

4. Numerical Experiments

In this section, in order to evaluate the performance of the proposed solution method, we indicate how the computational time increases as the size of the test problems increase. The test problems are randomly generated so that all parameters are drawn from discrete uniform distributions that are presented in Table 1. Moreover, for each problem instance, the necessary conditions are checked in order to make sure that these test problems are suitable for our experiments.

The process route of each product is also randomly generated without any skipping of stages by products. Four sets of test problems with different sizes have been considered (see Table 2), and five problems for each set are randomly generated.

For each set of test problems, a LINGO model has been generated using LINGO 6.0 modeling language, and all of the test problems are solved on a personal computer with an Intel Pentium 4 processor running at 3.2 GHz.

Table 3 represents the average CPU time required to obtain an optimal solution for each set of test problems. It is noted that for 8×5 problems, it was not possible to find an optimal solution within a reasonable CPU time.

Computational results indicate that the proposed solution method can obtain an optimal solution for small-sized and moderately medium-sized problems within a reasonable time. But it can not obtain an optimal solution for medium and large size problems within a reasonable time because solution time grows exponentially with the size of the problem.

Therefore, a more efficient heuristic method should be developed to obtain a near-optimal schedule for medium and large size problems within a reasonable CPU time.

5. Lot Streaming

In this section, we consider lot streaming case that is a generalization of Problem *P*. Lot streaming is the process of splitting a lot into a number of portions, often called transfer sublots (or batches) so that successive operations can be overlapped in a multi-stage production system. A major benefit of lot streaming is the reduction in the manufacturing lead time (MLT) and thereby provides an opportunity to the considerably reduction in work-in-process inventories (WIP) at the supplier and corresponding holding costs. The required modifications in this case, can be examined in the following sub- cases:

5.1. $p_{i,\mu(i,r-1)} \ge p_{i,\mu(i,r)}$

In this sub-case, the production rate for product i at stage $\mu(i,r-1)$ is greater than at stage $\mu(i,r)$. Therefore, the processing start time of this product at stage $\mu(i,r)$ must be at the time that the first batch is transferred from stage $\mu(i,r-1)$ to stage $\mu(i,r)$. Then Constraints (2) must be substituted with the following constraints:

$$b_{i,\mu(i,r)} \ge b_{i,\mu(i,r-1)} + \frac{a_{i,\mu(i,r-1)}}{p_{i,\mu(i,r-1)}} + \tau_{i,\mu(i,r-1)} \quad , \forall i = 1,\ldots,n\ ,\ r = 2,\ldots,m_i \qquad (16).$$

Where $a_{i,\mu(i,r-1)}$ is the transfer batch size of product i from stage $\mu(i,r-1)$ to stage $\mu(i,r)$ (determined based on existing unit load), and $\tau_{i,\mu(i,r-1)}$ is the transfer time for one batch of product i from stage $\mu(i,r-1)$ to stage $\mu(i,r)$.

The evolution of inventory of product i between two successive stages $\mu(i,r-1)$ and $\mu(i,r)$ in this sub-case is shown in Figure 4(a). Therefore, we will have:

$$I_{i,\mu(i,r-1)} = d_i\left(b_{i,\mu(i,r)} - b_{i,\mu(i,r-1)}\right)\left(2 - \frac{p_{i,\mu(i,r)}}{p_{i,\mu(i,r-1)}}\right) + \frac{1}{2}d_i^2\left(\frac{1}{p_{i,\mu(i,r)}} - \frac{1}{p_{i,\mu(i,r-1)}}\right).T \qquad (17).$$

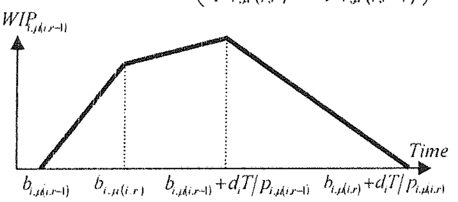

Figure 4(a). WIP inventory between stages $\mu(i,r-1)$ and $\mu(i,r)$ $(p_{i,\mu(i,r-1)} \ge p_{i,\mu(i,r)})$

5.2. $p_{i,\mu(i,r)} \ge p_{i,\mu(i,r-1)}$

In this sub-case, the production rate of product i at stage $\mu(i,r)$ is greater than at stage $\mu(i,r-1)$. Therefore, the processing start time on last batch of this product at stage $\mu(i,r)$ must be at the time that the processing of entire lot of product i at stage $\mu(i,r-1)$ is completed and last batch of this product is transferred from stage $\mu(i,r-1)$ to stage $\mu(i,r)$. Then, instead of Constraints (2) we will have the following constraints:

$$b_{i,\mu(i,r)} + \frac{d_i.T}{p_{i,\mu(i,r)}} - \frac{a_{i,\mu(i,r-1)}}{p_{i,\mu(i,r)}} \ge b_{i,\mu(i,r-1)} + \frac{d_i.T}{p_{i,\mu(i,r-1)}} + \tau_{i,\mu(i,r-1)} \ ;\forall\, i\,,\ r = 2,\ldots,m_i \qquad (18).$$

The evolution of inventory of product i between two successive stages $\mu(i,r-1)$ and $\mu(i,r)$ in this sub-case is shown in Figure 4(b). In this sub-case, $I_{i,\mu(i,r-1)}$ will be similar to (17) and thus, the sum of work-in-process inventory holding cost per unit time when Lot streaming is allowed, will be :

$$TC_{WIP} = \sum_{i=1}^{n}\sum_{r=2}^{m_i} h_{i,\mu(i,r-1)}.d_i\left[\left(b_{i,\mu(i,r)} - b_{i,\mu(i,r-1)}\right)\left(2 - \frac{p_{i,\mu(i,r)}}{p_{i,\mu(i,r-1)}}\right) + \frac{1}{2}d_i\left(\frac{1}{p_{i,\mu(i,r)}} - \frac{1}{p_{i,\mu(i,r-1)}}\right).T\right].$$

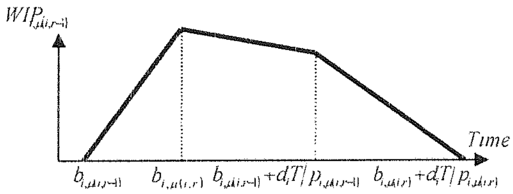

Figure 4(b). WIP inventory between stages $\mu(i,r-1)$ and $\mu(i,r)$ $(p_{i,\mu(i,r)} \ge p_{i,\mu(i,r-1)})$

Therefore, the objective function in this case can be written as equation 19. In this case, we again deal with a mixed nonlinear model and we can apply the enumeration method for solving this model to optimality.

Problem P:

$$Min \quad Z = \frac{A + \sum_{i=1}^{n} sc_i}{T} + \sum_{i=1}^{n} \left[h_i \cdot \frac{d_i}{2}\left(3 - \frac{d_i}{p_{i,\mu(i,m_i)}}\right) + \frac{d_i^2}{2} \sum_{r=2}^{m_i} h_{i,\mu(i,r-1)} \left(\frac{1}{p_{i,\mu(i,r)}} - \frac{1}{p_{i,\mu(i,r-1)}} \right) \right] \cdot T$$

$$(1) \qquad + \sum_{i=1}^{n} \sum_{r=2}^{m_i} h_{i,\mu(i,r-1)} \cdot d_i \left(b_{i,\mu(i,r)} - b_{i,\mu(i,r-1)} \right) - \sum_{i=1}^{n} h_i \cdot d_i \cdot b_{i,\mu(i,m_i)}$$

Subject to:

$$(2) \quad b_{i,\mu(i,r-1)} + \frac{d_i \cdot T}{p_{i,\mu(i,r-1)}} \le b_{i,\mu(i,r)} \; ; \; i = 1, \ldots, n, \; r = 2, \ldots, m_i$$

$$(3) \quad b_{ij} + \frac{d_i \cdot T}{p_{ij}} + s_{uj} - b_{uj} \le M \left(2 - Z_{ilj} - Z_{u,l+1,j}\right) ; \; j : M_j = 1, \; i, u \in p(j), \; u \ne i, \; l < n_j$$

$$(4) \quad b_{ij} + \frac{d_i \cdot T}{p_{ij}} + s_{uj} - b_{uj} \le M \left(2 - x_{ilkj} - x_{u,l+1,kj}\right) ;$$
$$j : M_j > 1, \; k = 1, \ldots, M_j, \; i, u \in p(j), \; u \ne i, \; l < n_j$$

$$(5) \quad \sum_{l=1}^{n_j} z_{ilj} = 1 \; ; \; j : M_j = 1, \; i \in p(j)$$

$$(6) \quad \sum_{i \in p(j)} z_{ilj} = 1 \; ; \; j : M_j = 1, \; l = 1, \ldots, n_j$$

$$(7) \quad \sum_{k=1}^{M_j} \sum_{l=1}^{n_j} x_{ilkj} = 1 \; ; \; j : M_j > 1, \; i \in p(j)$$

$$(8) \quad \sum_{i \in p(j)} x_{ilkj} \le 1 \; ; \; j : M_j > 1, \; k = 1, \ldots, M_j, \; l = 1, \ldots, n_j$$

$$(9) \quad \sum_{i \in p(j)} x_{i,l+1,kj} \le \sum_{i \in p(j)} x_{ilkj} \; ; \; j : M_j > 1, \; k = 1, \ldots, M_j, \; l < n_j$$

$$(10) \quad b_{ij} \ge s_{ij} \cdot z_{i1j} \; ; \; j : M_j = 1, \; i \in p(j)$$

$$(11) \quad b_{ij} \ge s_{ij} \cdot \sum_{k=1}^{M_j} x_{i1kj} \; ; \; j : M_j > 1, \; i \in p(j)$$

$$(12) \quad b_{i,\mu(i,m_i)} + \frac{d_i \cdot T}{p_{i,\mu(i,m_i)}} \le T \; ; \; i = 1, \ldots, n$$

$$(13) \quad F \cdot T = H$$

$$(14) \quad F \ge 1 \text{ and integer}$$

$$(15) \quad T \ge 0, \; b_{ij} \ge 0 \; ; \forall i, j, \quad z_{ilj}, \; x_{ilkj} \in \{0, 1\} \; ; \forall i, l, k, j.$$

Figure 3. A common cycle model for Problem P.

Table 1

Uniform distributions used for the parameters

Parameter	d_i	p_{ij}	s_{ij}	h_{ij}	A_i
Distribution	*U (100, 1000)*	*U (1000, 10000)*	*U (0.01, 0.25)*	*U (1, 20)*	*U (100, 4000)*

Table 2

Structure of the test problems

Problem set number	Number of products	Number of stages	Number of machines at each stage	Problem size: Number of integer var.	Problem size: Number of continuous var.	Problem size: Number of constraints
1	*5*	*2*	*1,2*	*76*	*11*	*295*
2	*5*	*5*	*1,2,1,2,1*	*176*	*26*	*688*
3	*5*	*10*	*1,2,1,2,1,2,1,2,1,2*	*376*	*51*	*1467*
4	*8*	*5*	*1,2,1,2,1*	*449*	*41*	*2950*

Table 3

Average CPU times (in minutes) for the test problems

Problem set number	Number of CPU test problems	Average time
1	*5*	*87*
2	*5*	*179*
3	*5*	*295*
4	*5*	*N. A.*

$$TC = \frac{A + \sum_{i=1}^{n} sc_i}{T} + \sum_{i=1}^{n} \left[h_i \cdot \frac{d_i}{2} \left(3 - \frac{d_i}{p_{i,\mu(i,m_i)}} \right) + \frac{d_i^2}{2} \sum_{r=2}^{m_i} h_{i,\mu(i,r-1)} \left(\frac{1}{p_{i,\mu(i,r)}} - \frac{1}{p_{i,\mu(i,r-1)}} \right) \right] .T + \sum_{i=1}^{n} \sum_{r=2}^{m_i} h_{i,\mu(i,r-1)} .d_i \left(b_{i,\mu(i,r)} - b_{i,\mu(i,r-1)} \right) \left(2 - \frac{p_{i,\mu(i,r)}}{p_{i,\mu(i,r-1)}} \right) - \sum_{i=1}^{n} h_i .d_i .b_{i,\mu(i,m_i)} . \quad (19)$$

6. Numerical Example

In this section, a numerical example is presented to illustrate applicability of mathematical model and its solution method. The supplier's production system involves two work centers. In first work center there is one machine ($M_1=1$) and second work center has two parallel identical machines ($M_2=2$). Also, the time unit is assumed one week and planning horizon length is equal to one year or 52 weeks ($H=52$). Table 1 presents other required data for this example.

Table 4

Required data for the example

i	$\mu(i,r)$	d_i	$p_{i,\mu(i,r)}$	$s_{i,\mu(i,r)}$	$a_{i,\mu(i,r-1)}$	sc_i	$h_{i,\mu(i,r)}$
1	1	41	3600	0.09	20	360	4
	2	0	2500	0.03		0	7
2	2	60	4700	0.15	30	160	3
	1	0	2500	0.04		0	5
3	1	70	5800	0.03	25	130	6
	2	0	3000	0.16		0	9
4	2	57	3800	0.05	30	350	7
	1	0	2700	0.14		0	9
5	1	48	3000	0.08	20	140	5
	2	0	2000	0.01		0	8

Moreover, the transfer times of batches between successive stages are negligible and assumed to be zero, and transportation cost per delivery is $A=10000$. The corresponding mathematical models of this example at both allowing lot streaming (with subscript 1) and without it (with subscript 2) are solved using enumeration method and optimal values are computed as follows:

$Z_1 = 40323.7, \quad F_1 = 42, \quad T_1 = 1.238,$

$Z_2 = 35861.4, \quad F_2 = 37, \quad T_2 = 1.405.$

It is observed that the total costs per time unit at lot streaming case can be decreased by 11%.

7. Conclusion

In this paper, we have considered the common cycle approach to solve the economic lot and delivery scheduling problem in a simple supply chain where the supplier's production system is a flexible job shop. First, we developed a new mixed zero-one nonlinear model to solve the problem to optimality. Then, to avoid solving the complex mixed nonlinear program directly, we have suggested an efficient enumeration method to determine its optimal solution. For validation of the proposed solution method, some numerical experiments are carried out. Another applicable case of Problem P (Lot streaming) is also presented and required modifications in the model formulation and the solution procedure are described. Moreover, through a numerical example, applicability of this formulation and its solution method is shown.

However, applying the proposed solution method to determine optimal solution in medium and large size problems requires solving several large-scale mixed zero-one programs that need high computational efforts. Therefore, further research to develop efficient heuristic methods is on our research line.

Moreover, through a numerical example, applicability of this formulation and its solution method is shown. However, applying the proposed solution method to determine optimal solution in medium and large size problems requires solving several large-scale mixed zero-one programs that needs high computational efforts. Therefore, further research to develop efficient heuristic methods is on our research line. Moreover, there are different directions for future studies. Among them, the following topics are recommended:

- Modeling problem P using the basic period (multiple cycle) approach. In this case, it is possible that different components have different production cycle times and global cycle time is an integer multiple of individual cycle times. This approach usually produces better solutions than common cycle approach;
- Allowing multiple deliveries per each cycle (using of nested schedules);

- Considering non-identical parallel machines at each stage of supplier's production system.

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