mulate the overall system that includes the arm and actuator, the friction force was incorporated into the system dynamics. These effects have not been considered previously by most researchers in the field. To validate simulation results, a flexible model is compared with rigid model in a case study. The simulated case study for a planar two-link flexible joint manipulator has illustrated the effectiveness of the formulation. It is found that a way alleviating the vibrations is to increase damping coefficients.

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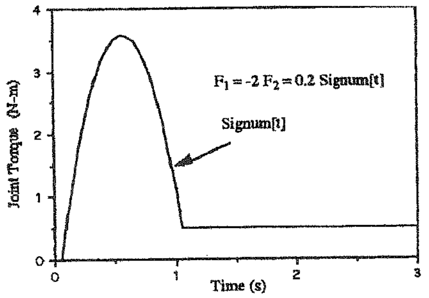

Fig. 4. A typical torque curve applied at joint 1 and 2

Table I Numerical values for simulation

Parameter	Value	Unit
Spring Constant	$K_1 = k_2 = 5.5$	N/m
Area Moment of Inertia	$I_1 = I_2 = 9.9 \times 10^{-12}$	m^4
Link Length	$L_1 = L_2 = 1.05$	m
Link Linear Mass Density	$\mu_1 = \mu_2 = 0.405$	kg/m
Actuator Constants	$k_t = 0.63$ & $K_2 = 0.18$	N-m & N-m /rad respectively
Stall Torque	$\tau_S = 0.63$	N-m
No Load Speed	$w_0 = 3.5$	rad/sec
Initial Joint Angles	$q_1(0) = 0$ & $q_2(0) = 90$	degree
Mass of End Effector	$m_e = 0.1$	kg

Solving the above equations as described in the previous section, we can find generalised coordinate in the presence of friction. Figure 5 shows profiles of the joint one and two without effect of frictional forces for the model. For an actuator torque pattern given in Fig. 5, the responses of the joint angle in the presence of frictional forces are shown in Fig. 5. Figure 5 shows the responses of the system with small damping coefficients; $v_1 = v_2 = 0.01$. Figure 5 is to investigate the effect of increased damping. Thus, the increased damping is effective to suppress the vibrations in comparison with Figure 3. The computation algorithm was implemented on a Quadra Macintosh/700 computer.

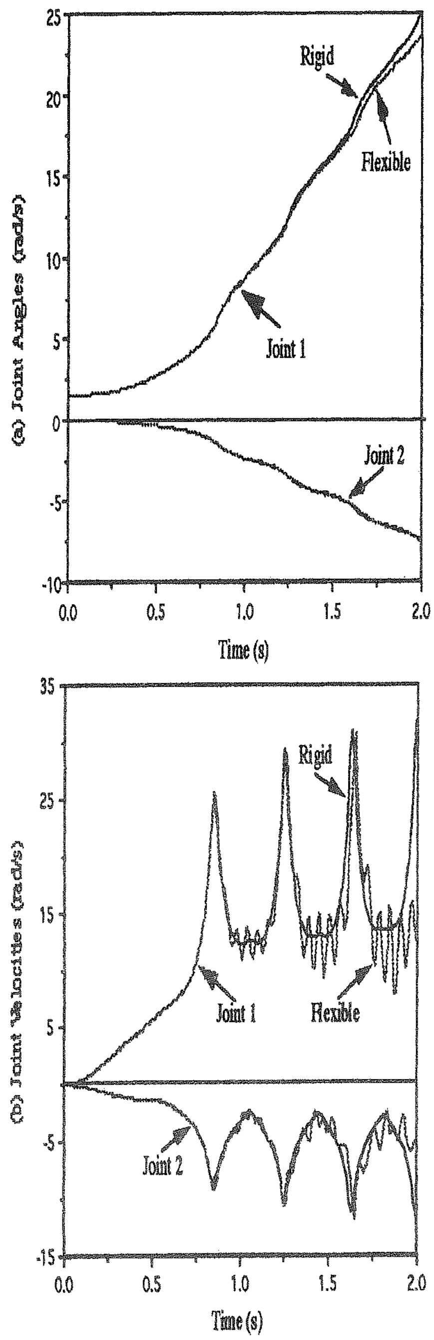

Fig. 5. Effect of friction on both rigid and flexible joint manipulator ($v_1 = v_2 = .01$)

6 - Conclusion

In this paper, the effect of friction on the solution of the dynamics problem is examined for flexible joint manipulators. To for-

$$D_{11}\ddot{q}_1 + D_{12}\ddot{q}_2 - C\dot{q}_2^2 - 2C\dot{q}_1\dot{q}_2 + K(q_1 - q_3) + b_1 = 0 \quad (12)$$

$$D_{22}\ddot{q}_2 + D_{12}\ddot{q}_1 + C\dot{q}_1^2 + K(q_2 - q_4) + b_2 = 0 \quad (13)$$

$$J\ddot{q}_3 + K(q_3 - q_1) = \tau_1 \quad (14)$$

$$J\ddot{q}_4 + K(q_4 - q_2) = \tau_2 \quad (15)$$

where

$$D_{11} = m_1 l_{c1}^2 + I_1 + m_2\left[l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos(q_2)\right] + I_2 + m_L\left[l_1^2 + l_2^2 + 2l_1 l_2\cos(q_2)\right] + I_L \quad (16)$$

$$D_{22} = m_2 l_{c2}^2 + I_2 + m_L l_2^2 + I_L \quad (17)$$

$$D_{12} = m_2 l_1 l_{c2}\cos(q_2) + m_2 l_{c2}^2 + I_2 + m_L l_1 l_2\cos(q_2) + m_L l_2^2 + I_L \quad (18)$$

$$C = m_2 l_1 l_{c2}\sin(q_2) + m_L l_1 l_2\sin(q_2) \quad (19)$$

$$b_1 = v_1\dot{q}_1 + k_1\,\mathrm{sgn}(\dot{q}_1) \quad (20)$$

$$b_2 = v_2\dot{q}_2 + k_2\,\mathrm{sgn}(\dot{q}_2) \quad (21)$$

The numerical values used in the simulation are listed in Table I.

5.2. Results and Discussions

This work used MATHEMATICA® for symbolic derivation, numerical solution [12- 14]. It was chosen mainly because of its versatile symbolic manipulation capabilities, such as, symbolic simplification of polynomials and rational expressions, linearization of trigonometric functions, automated evaluation of the relative significance of terms and subsequently, neglecting the less significant terms, and symbolic integration and differentiation [15]. It was used also because of the Macintosh® platform it runs on, its user friendliness and its integrated graphics environment. It can also communicate at a high level with other programs using the Mathlink communication standard.

In order to initially check the validity of the model, the following test was performed. Figure 3 shows the response of the flexible joint arm when the torque commands shown in Fig. 4. are applied. To verify the model, the results are also compared with that of rigid links shown in Fig. 3. The responses of the system with large spring constant are in good agreement with the motion of an rigid joint manipulator as shown in Fig. 3. As shown in Fig. 4, the flexible joint results in a large oscillation due to joint flexibility while the rigid joint model does not yield a large oscillation.

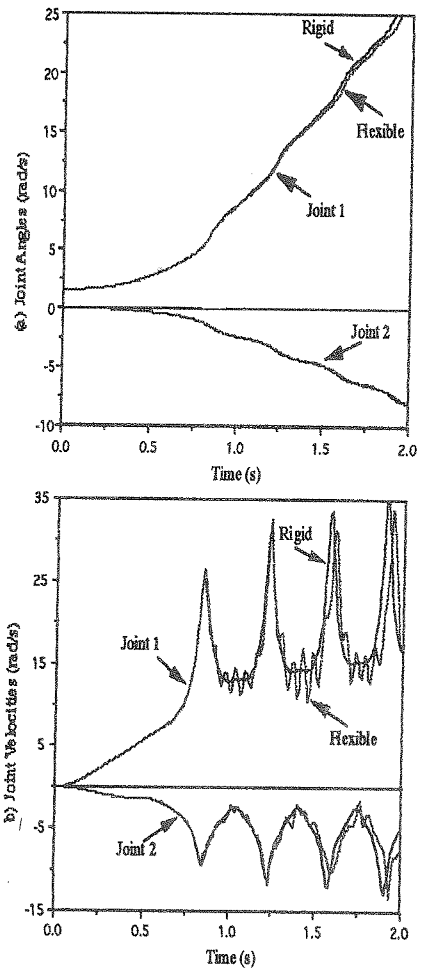

Fig. 3. Comparison of the joint responses of flexible joint manipulator with rigid case

4. Dynamic Model of Robots With Joint Elasticity

The elasticity at the ith joint can be modelled as a linear torsional spring having the spring constant K_i as shown in Fig.2 [11]. $q_1 = \{q_1, q_3, ...\}$, as the link angle and $q_2 = \{q_2, q_4, ...\}$ as the rotor angles are defined for the multi-link flexible joint manipulator. The dynamic equations of motion including friction are obtained using a Lagrangian approach as follows:

$$D(q_1)\ddot{q}_1 + c(q_1, \dot{q}_1)\dot{q}_1 + G(q_1) + K(q_1 - q_2) + b(\dot{q}_1) = 0 \tag{3}$$

$$J\ddot{q}_2 + K(q_2 - q_1) = \tau \tag{4}$$

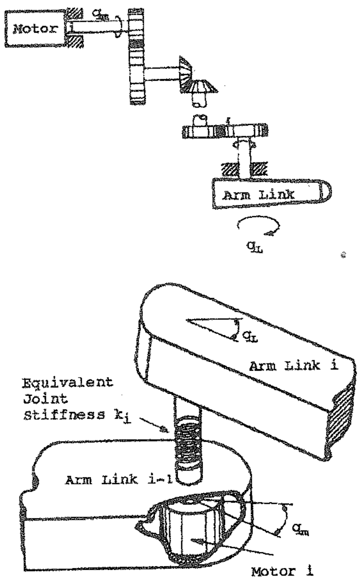

Fig. 2. Schematics of a flexible joint model
a) a representative motor-link transmission mechanism
b) a flexible joint represented as an equivalent spring

where $D(q_1)$ is the inertia matrix for the associated rigid system, $C(q_1, \dot{q}_1)$ is the vector of damping, coriolis and centripetal forces, $G(q_1)$ is the vector of forces due to gravity, k=diag $[k_1, ..., k_n]$ is a diagonal matrix of restoring force constants modelling the joint elasticity, $b(\dot{q}_1)$ is the vector of friction terms, J = diag $[J_1, ..., J_n]$ is diagonal matrix representing rotor inertia, and τ is the generalised force delivered by the actuator.

5. Simulation Results and Discussions

The direct dynamics problem is to solve joint positions, velocities, and accelerations for a given actuator torque/force. Efficient solution of this problem is necessary for model-based control. By using direct dynamic, the velocities and accelerations of the links are successively computed.

5.1. Simulation Conditions

A simulation study was carried out to further investigate the validity and effectiveness of the frictional forces presented above. computing the effect of frictional forces is presented for a two-link flexible joint manipulator. Only joint flexibilities are considered while link compliances are neglected. It is assumed that end effector deflection is primarily caused by joint deflection or oscillation at higher speeds, and the elasticity at the joint can be modelled as a linear torsional spring. The following system of equations can be assembled

i) Kinematic Equations

$$J_{r11}\ddot{q}_1 + J_{r12}\ddot{q}_2 = R_{t1} \tag{6}$$

$$J_{r21}\ddot{q}_1 + J_{r22}\ddot{q}_2 = R_{t2} \tag{7}$$

where the expressions of rigid body Jacobian J_r are given.

$$J_{r11} = -l_1 \sin(q_1) - l_2 \sin(q_1 + q_2) \tag{8}$$

$$J_{r12} = -l_2 \sin(q_1 + q_2) \tag{9}$$

$$J_{r21} = -l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) \tag{10}$$

$$J_{r22} = l_2 \cos(q_1 + q_2) \tag{11}$$

ii) Dynamic equations based on Euler-Lagrange method are magnitude as well as lubricant viscosity and flow [9]. Thus friction modelling and parameter identification are not unattainable goals.

Three kinds of frictional effects are important in joint operation such as viscous friction, static Coulomb friction, and dynamic Coulomb friction. Since rotary joints are the kind most commonly used in assembly robots, these effects are discussed with friction induced torques, rather than forces.

3. Modelling of Friction Forces

Some researchers and manufacturers have developed theoretical and experimental friction models for machine elements and complete systems as well as typical values of the model parameters. The values of these parameters provide only rough estimates of the behaviour of a particular system. While it is possible to use them to identify the dominant sources of friction in a system, the actual parameter values should be identified.

The effect of friction can be modelled as a generalised force applied to the joints of the arm. Friction has a significant effect on robot arm dynamics and three distinct components of friction using the following frictional force acting on the kth joint can be examined [10]:

$$b_k(\dot{q}_l) = b_k^v \dot{q}_k + \mathrm{sgn}(\dot{q}_k)\left[b_k^d + (b_k^s - b_k^d)\exp\frac{-|\dot{q}_k|}{\varepsilon}\right] \quad 1 \le k \le n \tag{1}$$

where, b^v_k, b^s_k, b^d_k are the coefficients of viscous friction, static friction and dynamic friction, respectively. The first term in Eq (1) represents viscous friction, while the second and third terms represent dynamic friction and static friction or Coulomb friction, respectively. ε is a small positive parameter and the signum function is defined as follows:

$$\mathrm{sgn}(x) = \begin{cases} +1 & 0 < x \\ 0 & 0 = x \\ -1 & x < 0 \end{cases} \tag{2}$$

From Eq (1), it is evident that as the velocity approaches zero, the frictional force approaches $\pm b^s_k$. As a result, b^s_k can be interpreted as the torque force required to overcome friction when motion is initiated. It is clear that viscous friction is a linear function of $\dot{q}_l$ whereas dynamic and static friction are discontinuous nonlinear functions. The model of frictional forces acting on the kth joint is summarised in graphical form in Fig. 1.

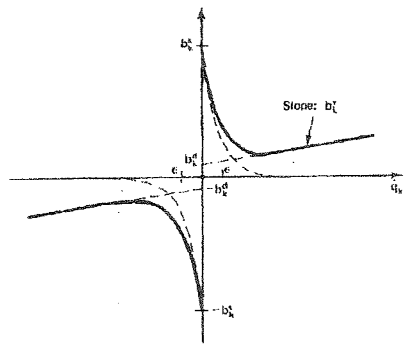

Fig. 1. Friction model for mechanical manipulator acting on joint k

For a well-designed robotic arm, particularly a direct drive arm, the coefficients of friction will be small. Note that Eq (1) is only a rough model of the frictional forces. A more comprehensive representation of friction might employ a term of the form b $(q_l, \dot{q}_l)$ which depends on joint position as well as joint velocity. This would be necessary, for example, if the shafts about which the links rotate or translate were not perfectly round and uniform. The frictional forces influence the dynamic equation and oppose the motion generated by the actuators. It is also possible to add a third term to the generalised force vector, the joint torques induced by an end of arm force and moment vector. However, to keep the dynamic model simple, it is assumed that the arm is moving freely in its workspace and the carrying payload is implicitly included in the physical description of the last link. Under these circumstances, the end of arm force and moment vector is equal to zero.

k_i stiffness of joint i

L_i length of link i

m_i total mass of the beam i

m_e mass of end effector (planar case)

n number of links

q_1 joint variable of the jth joint asso ciated with link

q_2 joint variable of the jth joint asso ciated with joint flexibility

μ_i link density

τ is the generalised force delivered by the actuator

ν_i is damping coefficient

1. Introduction

In many cases, the elasticity of a robot structure considerably influences the precision of operational task execution and needs to be taken into account in the dynamic analysis of industrial robots. While the elastic compliance is mainly concentrated in joints and harmonic drives [1]. Therefore, there is growing interest in the area of modelling of flexible joint manipulator.

One major problem associated with many commercial manipulators is the use of gears, which amplify the limited torque capabilities of most electric motors. Gears create joint friction and backlash which provide nonlinearities. These nonlinear effects are due to preloading, tooth, misalignment, and gear eccentricity. They are extremely difficult to be modelled, although parametric models of friction have been attempted [2]. It is more appropriate to minimise backlash and friction by mechanical tuning techniques rather than modelling [3].

To realise fully the potential of flexible manipulators, new and more complete techniques for predicting their behaviour must be developed. Of the methods developed to date, some of the most remarkable are those of [4-8]. One limitation of these studies is that they are only applicable to robots with revolute joints by neglecting friction forces. Some efforts have been made to develop techniques for dealing with friction force in robots. However, these either have been general formulatoins, which have not addressed the detailed problems of implementation, or have focused on a particular type of robot containing only a single joint. The lack of attention given to elastic manipulators with friction force is primarily due to eliminating gearing in which joint friction is minimised.

This study examines the friction problem for elastic joint manipulators. Recent experimental work [4] provides convincing evidence that joint elasticity is the dominant source of compliance in most current manipulator designs. In addition to joint elasticity, the problem of friction forces in a wide range of configurations calls for accurate modelling of the significant nonlinearities. The purpose in this paper is to demonstrate the effect of friction forces such as viscous damping and coulomb forces on the formulation and numerical solution of elastic manipulators. To illustrate these concepts, a detailed study of a two-link manipulator with joint elasticity is presented. To evaluate the performance of the proposed method simulation test is carried out with joint flexibility including frictional forces.

2. The Effect of Friction on the Model

Friction plays an important role in power transmission elements such as gears and screws as well as in bearings, seals, hydraulic components, and electric motors. Friction function in each case is a complex phenomenon. For example, friction in rollig-element bearings is a function of bearing size, type, and design. Additional factors include speed, load type, and

The Effect of Friction on the Formulation and Numerical Solution of Elastic Joint Manipulators

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Abstract

The purpose of this investigation is to study the effect of friction on the formulation and numerical solution of elastic joint mechanical manipulators. To this end, friction forces that act on a flexible manipulator as the results of the finite rotation are categorised into two classes. These are the Coulomb friction and viscous friction. The effect of the Coulomb friction of a driven system is recognised in the presence of contact between parts. While the viscous friction forces of such systems are understood from the drag imposed on moving parts by the lubricants used on them. The behaviour of the friction when they are subjected to driving constraints (specified torque) is examined. Simulation tests are performed and the results show that the friction in a two-joint SCARA type robot in which both joints are flexible is quite important.

Nomenclature

b_k^v, b_k^s, b_k^d	are the coefficients of viscous friction, static friction and dynamic friction
$b(\dot{q}_1)$	is the vector of friction terms
$C(q_1, \dot{q}_1)$	is the vector of damping, Coriolis and centripetal forces
$D(q_1)$	is the inertia matrix for the associated rigid system
I_i	is a area moment of inertia of element i
J	is diagonal matrix representing rotor inertia
$G(q_1)$	is the vector of forces due to gravity
K	a diagonal matrix of joint stiffness