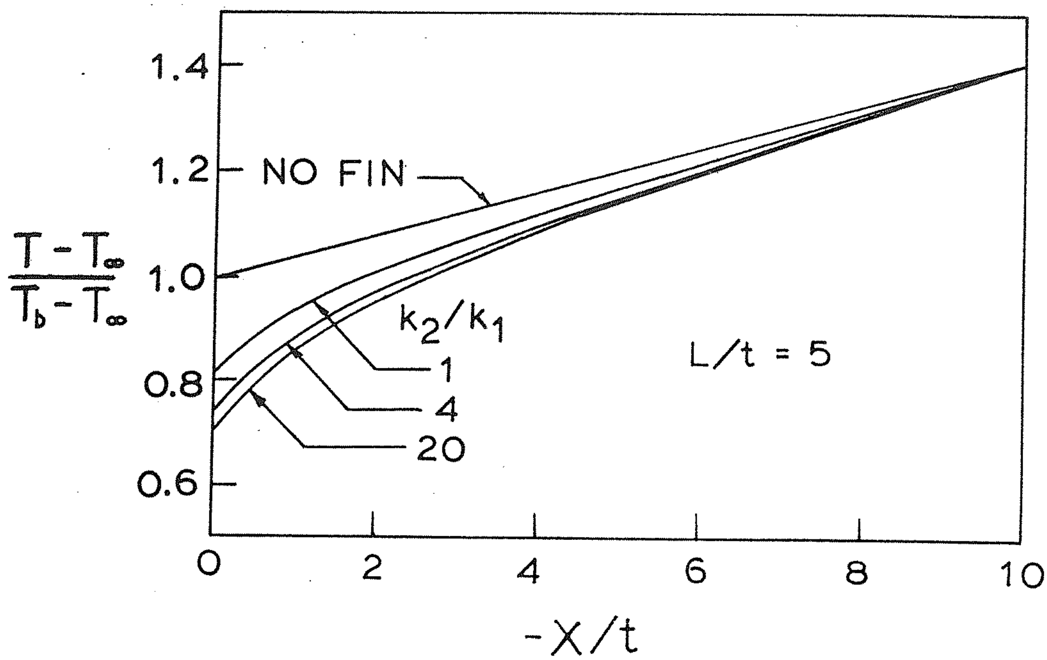

Fig. 3b Distribution of wall temperature along its axis of symmetry ($Y = 0$) for $L/t = 5$ and $K_2/K_1 = 1,4,20$.

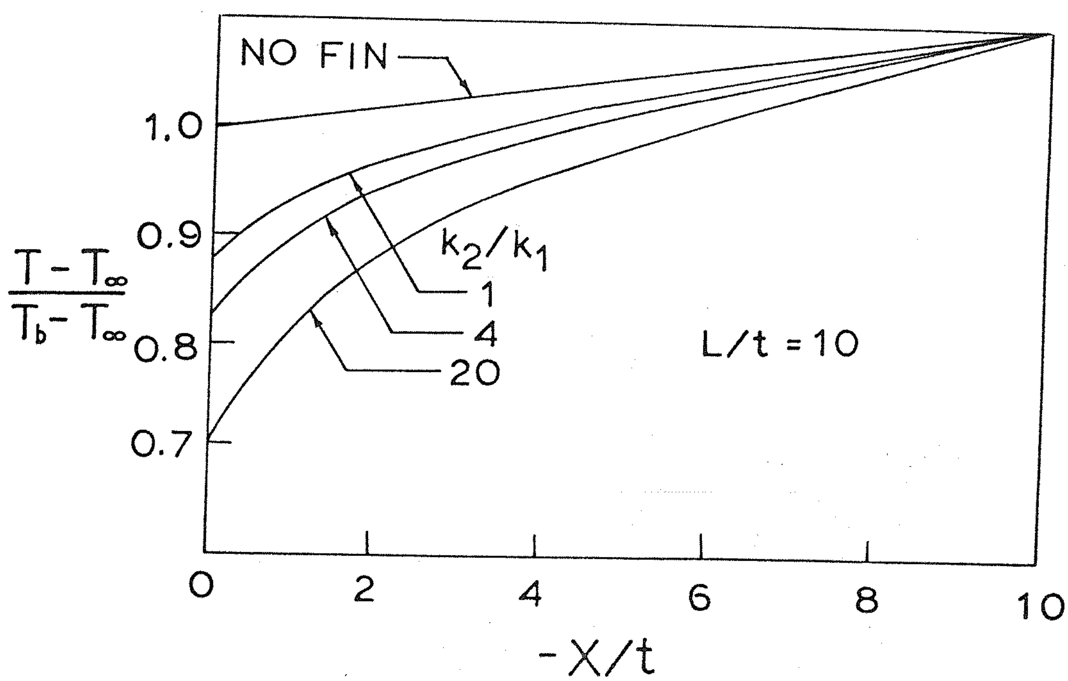

Fig. 3c Distribution of wall temperature along its axis of symmetry ($Y = 0$) for $L/t = 10$ and $K_2/K_1 = 1,4,20$

Table 4: The maximum error in temperature $(T-T_\infty)/(T_b - T_\infty)$, and rate of heat transfer,O,Caused by the one-dimensional analysis of fins.

L/t	K2/K1	Max. error in Temp.,%	Location of Max. error (X/t,Y/t)	Efficiency 1D	Efficiency 2D	Error in Q,%
2	1	32.8	(0.5,0.5)	2.67	2.60	2.7
2	4	53.6	(0.5-1.5,0.5)	4.01	3.39	18.3
2	20	66.3	(0,0),(1-2,0.5)	4.76	3.78	25.9
5	1	22.4	(0,0)	6.47	6.74	-4.0
5	4	34.9	(0,0)	9.21	9.03	2.0
5	20	40.6	(0,0),(4-5,0),(3-5,0.25),(1.5-5,0.5)	10.58	10.78	-1.9
10	1	13.7	(0,0)	12.76	15.16	-15.8
10	4	23.3	(10,0.5)	17.84	21.94	-18.7
10	20	49.4	(10,0-0.5)	20.26	37.58	-46.1

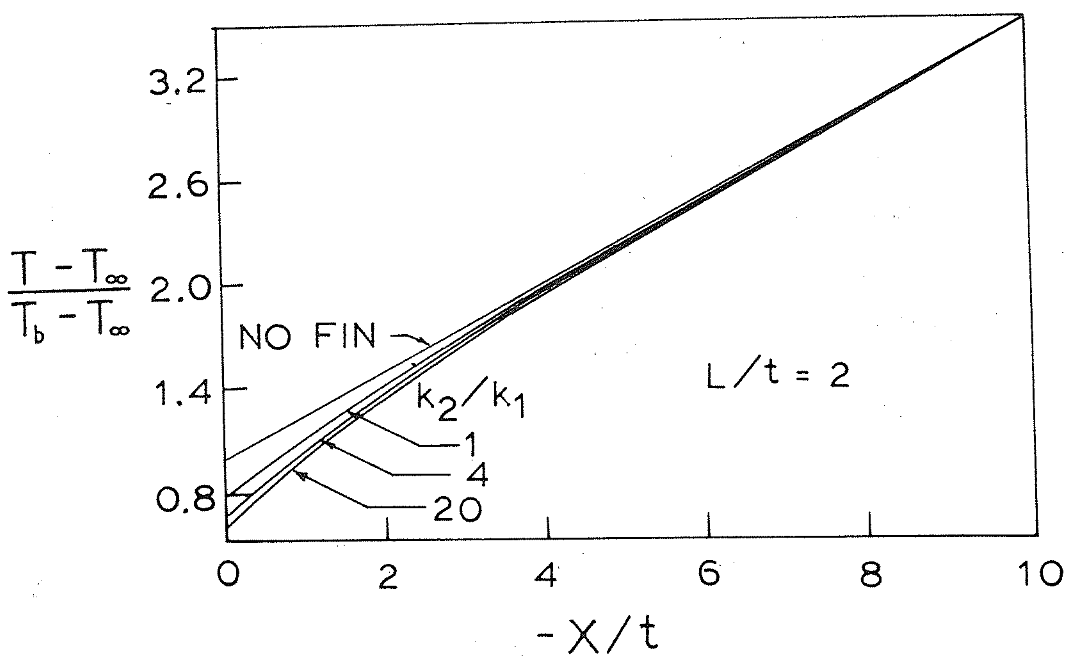

Fig.3a Distribution of wall temperature along its axis of symmetry ($Y = 0$) for $L/t = 2$ and $K_2/K_1 = 1,4,20$.

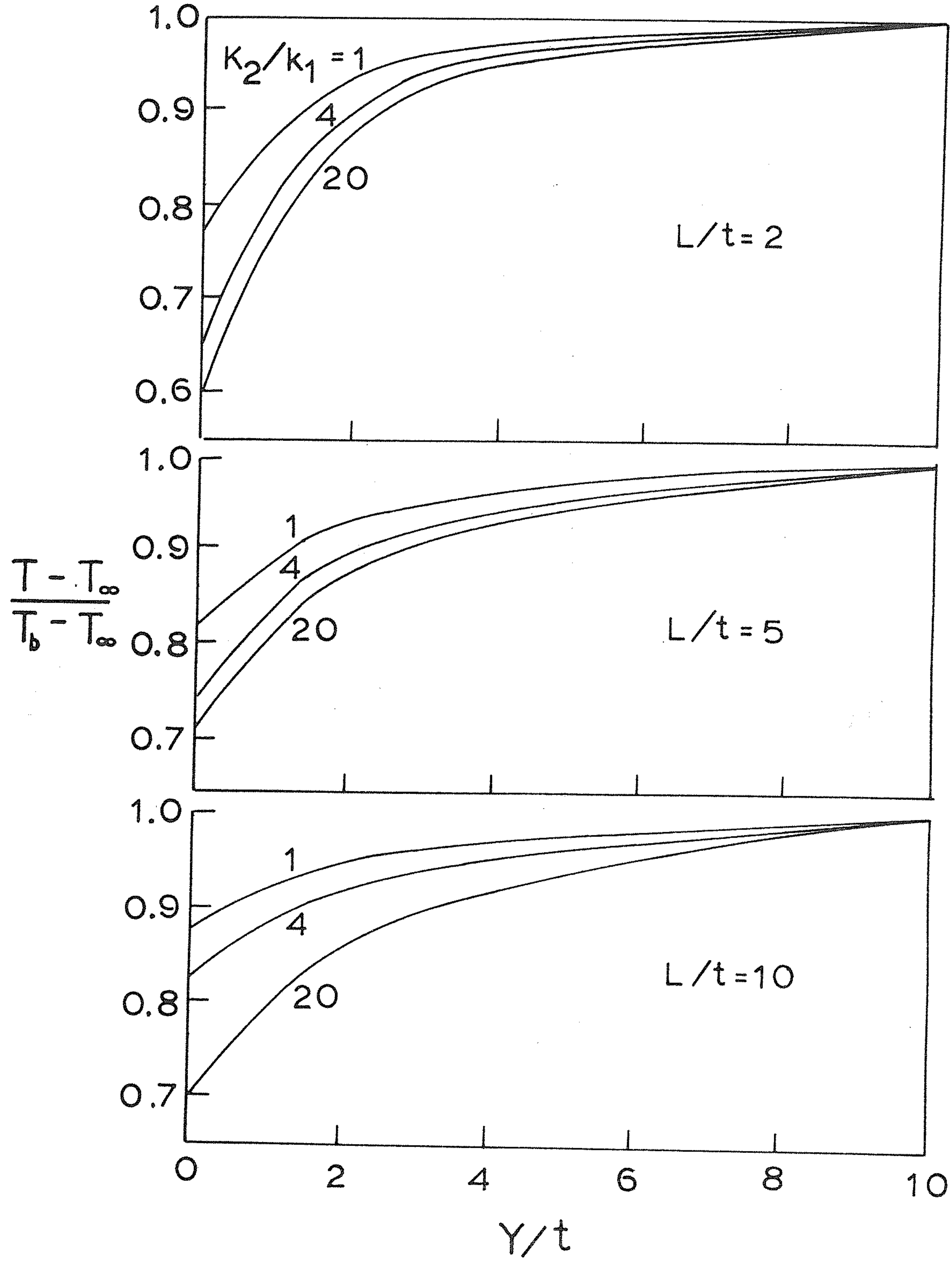

Fig.2 Distribution of wall surface temperature (X = 0).

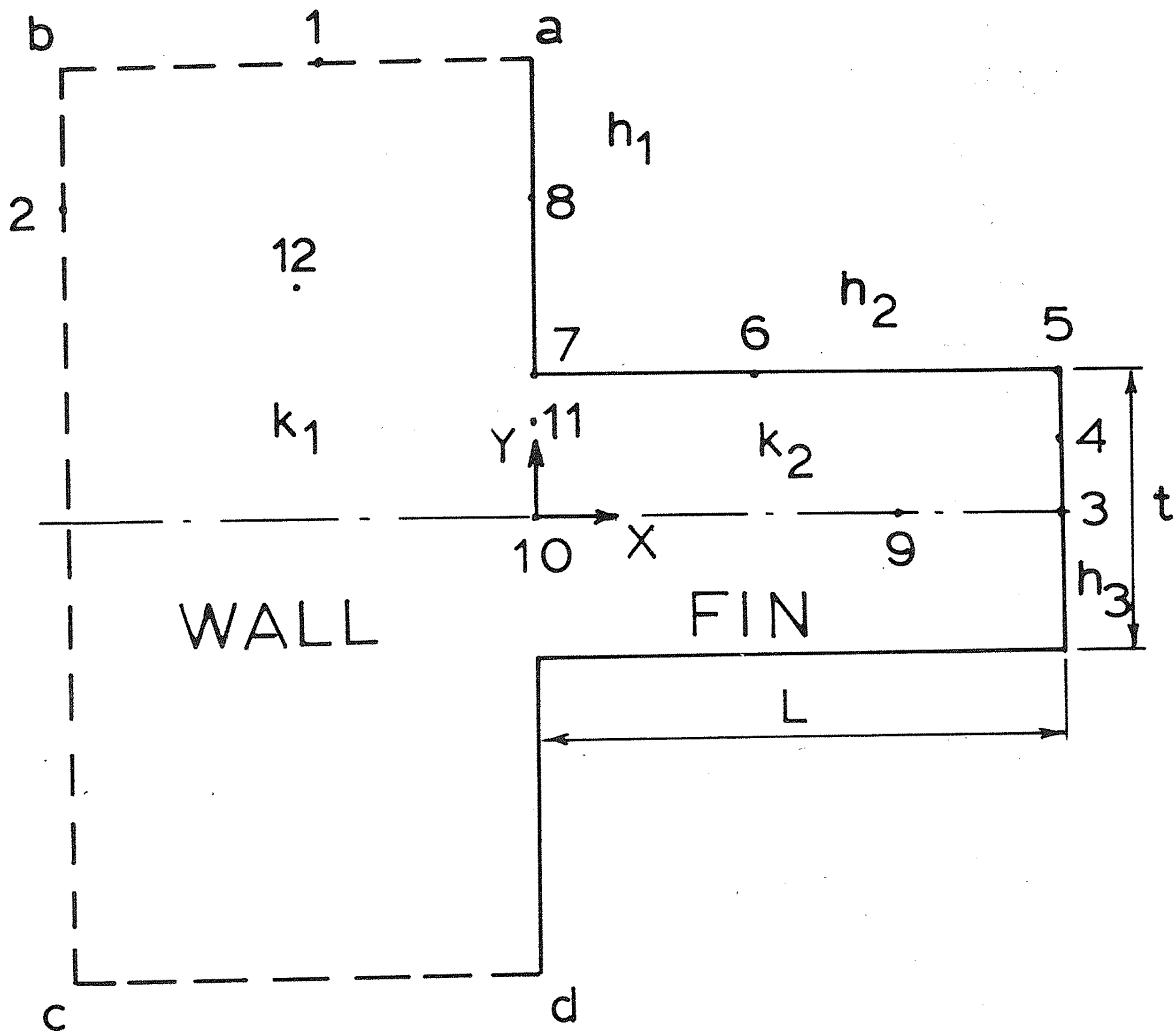

Fig.1 Schematic of the fin-wall arrangement.

Table 3: Fin temperature distribution $(T-T_\infty)/(T_b - T_\infty)$, and the percent error of the one-dimensional analysis for L/t and K2/K1 = 1,4,20.

L/t = 10

Y/t	0.00		0.50	
	Temp.	1-D error, %	Temp.	1-D error, %
X/t	K2/K1 = 1			
0	0.880	13.7	0.891	12.2
1	0.782	12.9	0.780	13.1
2	0.694	12.6	0.693	12.9
3	0.621	12.3	0.619	12.6
4	0.559	12.0	0.558	12.2
5	0.509	11.6	0.508	11.9
6	0.468	11.3	0.467	11.6
7	0.437	11.0	0.436	11.3
10	0.391	10.6	0.390	10.9
	K2/K1 = 4			
0	0.826	21.0	0.834	19.9
1	0.791	21.0	0.791	21.1
2	0.758	21.4	0.758	21.5
3	0.729	21.8	0.729	21.8
4	0.704	22.1	0.704	21.2
5	0.683	22.4	0.682	22.5
6	0.665	22.7	0.664	22.8
7	0.651	22.9	0.650	23.0
10	0.630	23.2	0.630	23.3
	K2/K1 = 20			
0	0.703	42.3	0.706	41.7
1	0.691	43.3	0.691	43.4
2	0.679	44.5	0.679	44.5
3	0.669	45.6	0.669	45.6
4	0.660	46.5	0.660	46.5
5	0.653	47.4	0.653	47.4
6	0.646	48.1	0.646	48.1
7	0.641	48.6	0.641	48.7
10	0.634	49.4	0.634	49.4

Table 2: Fin temperature distribution $(T-T_\infty)/(T_b - T_\infty)$, and the percent error of the one-dimensional analysis for $L/t = 5$ and $K2/K1 = 1,1,20$.

L/t = 5

Y/t	0.00		0.25		0.50	
	Temp.	1-D error, %	Temp.	1-D error, %	Temp.	1-D error, %
X/t			K2/K1 = 1			
0.0	0.817	22.4	0.820	21.9	0.835	19.7
0.5	0.727	21.1	0.725	21.3	0.721	22.1
1.0	0.644	20.8	0.643	21.1	0.638	22.0
1.5	0.574	20.6	0.572	20.9	0.568	21.8
2.0	0.514	20.3	0.513	20.6	0.509	21.5
2.5	0.465	20.1	0.464	20.4	0.460	21.3
3.0	0.425	19.9	0.424	20.2	0.421	21.1
4.0	0.369	19.5	0.368	19.8	0.365	20.7
5.0	0.342	19.4	0.341	19.7	0.338	20.6
			K2/K1 = 4			
0.0	0.741	34.9	0.743	34.6	0.753	32.8
0.5	0.712	34.3	0.712	34.3	0.711	34.5
1.0	0.684	34.3	0.683	34.3	0.682	34.6
1.5	0.658	34.3	0.657	34.4	0.656	34.6
2.0	0.636	34.3	0.635	34.4	0.634	34.6
2.5	0.616	34.3	0.616	34.4	0.615	34.7
3.0	0.600	34.4	0.600	34.4	0.599	34.7
4.0	0.577	34.4	0.577	34.5	0.576	34.7
5.0	0.566	34.4	0.566	34.5	0.564	34.7
			K2/K1 =20			
0.0	0.711	40.6	0.712	40.5	0.715	40.0
0.5	0.705	40.4	0.705	40.4	0.705	40.5
1.0	0.698	40.5	0.698	40.5	0.698	40.5
1.5	0.692	40.5	0.692	40.5	0.692	40.6
2.0	0.687	40.5	0.687	40.5	0.687	40.6
2.5	0.683	40.5	0.683	40.5	0.682	40.6
3.0	0.679	40.5	0.679	40.6	0.679	40.6
4.0	0.674	40.6	0.673	40.6	0.673	40.6
5.0	0.671	40.6	0.671	40.6	0.670	40.6

Table 1: Fin temperature distribution $(T - T_\infty)/(T_b - T\grave{a})$, and the percent error of the one-dimensional analysis for $L/t = 2$ and $K2/K1 = 1,4,20$.

L/t = 2

Y/t	0.00		0.25		0.50	
	Temp.	1-D error, %	Temp.	1-D error, %	Temp.	1-D error, %
X/t			K2/K1 = 1			
0.00	0.773	29.5	0.775	29.2	0.791	26.6
0.25	0.670	26.7	0.664	27.7	0.641	32.2
0.50	0.576	25.5	0.568	27.1	0.544	32.8
0.75	0.496	24.8	0.489	26.7	0.467	32.6
1.00	0.431	24.4	0.424	26.2	0.405	32.2
1.25	0.378	24.0	0.372	25.9	0.356	31.8
1.50	0.337	23.7	0.332	25.5	0.317	31.5
1.75	0.305	23.4	0.301	25.3	0.287	31.2
2.00	0.283	23.4	0.279	25.2	0.267	31.2
			K2/K1 = 4			
0.00	0.652	53.4	0.654	52.8	0.670	49.4
0.25	0.619	52.0	0.619	52.1	0.615	53.1
0.50	0.587	51.5	0.585	52.0	0.579	53.6
0.75	0.558	51.3	0.556	51.9	0.550	53.6
1.00	0.534	51.3	0.532	51.8	0.525	53.6
1.25	0.513	51.2	0.511	51.8	0.505	53.6
1.50	0.496	51.2	0.494	51.8	0.488	53.6
1.75	0.483	51.2	0.481	51.8	0.476	53.5
2.00	0.474	51.2	0.472	51.7	0.467	53.5
			K2/K1 =20			
0.00	0.601	66.3	0.602	66.1	0.607	64.8
0.25	0.594	65.9	0.594	65.9	0.594	66.0
0.50	0.587	65.8	0.587	65.9	0.586	66.2
0.75	0.581	65.7	0.580	65.9	0.579	66.2
1.00	0.575	65.7	0.574	65.9	0.573	66.3
1.25	0.570	65.7	0.570	65.9	0.568	66.3
1.50	0.566	65.8	0.566	65.9	0.564	66.3
1.75	0.563	65.8	0.563	65.9	0.561	66.3
2.00	0.561	65.8	0.560	65.9	0.559	66.3

fin-to-thickness ratio L/t and the fin-to-wall thermal conductivity ratio $K = K_2/K_1$ are increased. It was further shown that the temperatures predicted by the one-dimensional model were in error, and in the range of parameters considered in this study, the temperature errors varied from 10.6 to 66.3 percent. The largest error occurred at $(L/t, K) = (2, 20)$, while the smallest one corresponded to (10,1).

Two dimensionality of heat transfer also affects the rates of heat transfer. At lower values of L/t, the one-dimensional analysis overestimates the rate of heat transfer, while at higher L/t the converse is true. The errors in Q ranged from +25.9 to -46.1, which corresponded to $(L/t, K) = (2, 20)$ and $(10,20)$, respectively.

When the temperature field within the wall was examined, it was revealed that the presence of the fin was felt in every direction to a distance of about 10t away from the fin base. Moreover, the base temperature experienced a depression, and the magnitude of depression ranged from 10.9 to 39.9 percent of the difference between the base temperature of the one-dimensional model and the temperature of the surrounding fluid, $T_b - T_\infty$.

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to remove the fin.

Efficiencies given in Table 4 are all larger than unity. This indicates that the range of parameters selected for the present investigation were realistic, and the fins actually increased the heat transfer.

With this definition of fin efficiency and under the optimizing condition of equation (4), it is found that longer (and therefore thinner) fins have a better efficiency. It should be emphasized that this conclusion applies strictly to the fins which obey the restricting conditions set forth by equations (4) and (6).

The effect of thermal conductivity ratio K on efficiency is similar to that of L/t. At lower values of K, this effect is stronger, but as K is increased, its improving effect on efficiency is diminished.

The last column in Table 4 shows the errors of one-dimensional analysis in predicting the heat transfer rates. At lower values of L/t, the one-dimensional analysis overestimates the rate of heat transfer, while at higher L/t, the converse is true. In the range of parameters considered in this investigation, the error in Q ranged from +25.9 to −46.1 percent, which corresponded to $(L/t, K) = (2, 20)$ and $(10, 20)$, respectively.

Another aspect of the present problem is concerned with the manner in which the temperature field inside the infinite wall is affected by the presence of the fin. In this regard, Figs. 2 and 3a-c have been prepared to indicate the temperature distribution throughout the wall. However, to simplify the presentation, temperatures are only given along two particular paths, namely, along the convective surface $X = 0,\ Y \geq 0$, and along the symmetry axis of the wall identified by $X \leq 0$ and $Y = 0$.

Attention is first focused on Fig.2, where the wall surface temperature is given along the vertical line $Y \geq 0$, and L/t and K appear as parameters. It is seen that the three family of curves identified by $L/t = 2, 5,$ and 10, have a common behavior. At the fin base $(X = 0, 0 \leq Y/t \leq 0.5)$, the temperature is depressed, with the temperature depression varying from $[T(0,0) - T_\infty]/(T_b - T_\infty) = 0.601$ to $[T(0,0.5) - T_\infty]/(T_b - T_\infty) = 0.891$. These values correspond, respectively, to $(L/t, K) = (2,20)$ and $(10,1)$. Further away from the fin base along the Y, the surface temperature increases and approaches the base temperature of the one-dimensional model T_b.

In general, wall surface temperature depends, in addition to distance from the fin, on two parameters L/t and K. As L/t is increased, the base temperature experiences more depression, since the rate of heat transfer increases with L/t and the temperature gradients at the base become larger. The ratio of thermal conductivities K have also a similar effect.

Figs. 3a-c have been prepared to show the distribution of wall temperature along the line of symmetry. In these figures, the ordinate is the dimensionless temperature $[T(X,0) - T_\infty]/(T_b - T_\infty)$ and the abscissa is the distance from the origin along the negative X direction normalized by the fin thickness, i.e., $-X/t$.

Also shown in these figures are the temperature distributions for the bare wall, which have been identified as "NO FIN". It is clearly seen that the temperature distributions for the no-fin case are exactly linear, they start at $(T - T_\infty)/(T_b - T_\infty) = 1$ and increase continuously as getting away from the surface.

The results of the two-dimensional analysis, which are shown by the lower set of curves, indicate that they are markedly different from those of the no-fin situation. Here, the temperatures depend on both L/t and K. Considering the change of scale on the vertical axis, a careful examination of the curves reveals that the effect of L/t on temperature distributions is negligibly small. However, the effect of thermal conductivity ratio is important, and it appears that the temperature distribution in this region is governed only by this ratio.

CONCLUDING REMARKS

The numerical calculations of the present work have provided a definitive set of results for two-dimensional heat transfer through an isolated rectangular fin attached to an infinite solid wall. It was shown that the predictions of the conventional one-dimensional analysis of heat transfer through fins might involve large errors.

It was found that a temperature gradient exists across the thickness of the fin which decreases as the obtained results indicated that the largest difference was 0.07 percent, and it occurred at a point located on the tip of the fin.

RESULTS AND DISCUSSION

Attention is first focused on Tables 1 to 3 where the fin temperature distributions are presented for a number of selected points. An overall examination of the temperatures indicates that at a fixed X/t, the dimensionless temperatures generally decrease with increasing Y/t. The points located at the fin base, i.e., $X/t = 0$, are exceptions to this rule. In fact, in order to have convective heat transfer from the surface of the fin to the surrounding fluid, there must be a transverse temperature gradient at each cross section across the fin.

The temperature gradients across the fin depend on two parameters, the length of the fin, L/t, and the thermal conductivity ratio, $K = K_2/K_1$. Inspection of the results indicates that as L/t and K are increased, the temperature gradients are decreased. It should be noted that the results reported here are for optimum fins which obey the optimizing condition expressed by equation (4). According to this equation, larger values of L/t imply a smaller Biot number and, thus, a smaller temperature gradient across the thickness of the fin.

As mentioned earlier, the one-dimensional treatment of fins involves an oversimplified model which ignors certain basic facts about heat transfer. These facts are (1) two dimensionality of heat transfer, (2) depression of the base temperature, and (3) the effect of the different thermal conductivity of the parent wall. In this connection, it would be highly desirable to explore the extent of errors caused by this simplified model. Tables 1 to 3 compare the two situations and present the percent of errors for the one-dimensional model. A careful examination of the results indicates that the errors caused by the one-dimensional analysis range from 10.6 to 66.3 percent which correspond, respectively, to $(X/t, Y/t, L/t, K) = (10,0,10,1)$ and $(1-2, 0.5, 2, 20)$. The significant 66.3 percent temperature error becomes especially important when designing a finned surface from the standpoint of the highest permissible temperature. In this regard, any unrealistic prediction of temperature may lead to a partial or complete burnout of the device.

Also from these tables, it is observed that, at a fixed K, the errors of the one-dimensional analysis decrease with L/t, which indicates that at higher L/t the heat transfer behavior becomes one-dimensional. Moreover, for a given L/t, as K is increased a larger difference is observed between one- and two-dimensional predictions. It is believed that higher values of K reduce the thermal resistance along the fin, enhance the rate of heat transfer, and thus depress the temperature at the fin base. Since the one-dimensional model does not account for this temperature depression, it is expected to see larger errors at higher values of K.

Table 4 presents a concise comparison of the results of this study with those of the common one-dimensional analysis. As seen from the values given in the third column of the table, this maximum temperature error ranges from 13.7 to 66.3 percent. Also it is noticed that, in general, the maximum errors decrease with L/t, indicating that at higher values of L/t the one-dimensional heat transfer is dominant. However, the thermal conductivity ratio K has the opposite effect. It appears that the two dimensionality of heat transfer through fins is governed by two conflicting factors; one is the effect of L/t, and the other is that of K. The fourth column of the table specifies the locations of maximum errors.

Also shown in Table 4 are the values of fin efficiency for both one-dimensional and two-dimensional models. The efficiencies were calculated from the defining equation:

$$\eta = Q/[hA(T_b - T_\infty)] \qquad (6)$$

where Q is the real rate of heat transfer from the fin, and A is the cross sectional area of the fin for heat conduction. Clearly, the expression in the denominator is the rate of heat transfer from the base area when the fin is absent. Thus, the above definition of efficiency is particularly useful when we want to know whether the attachment of fin to the surface of the wall would increase heat transfer. A value of η larger than unity means that the presence of the fin is introducing more resistance on the path of heat flow, and it would be better

Thus, the two parameters Bi and L/t are dependent, and the present solutions depend only on L/t and K, with the corresponding values of Bi obtained from equation (4).

Equations (1) to (3f) were solved numerically. Referring to Fig.1, the finite difference equation for any internal point in the solution domain such as point 12 can be written as :

$$\theta_{m,n} = 0.25\ (\theta_{m+1,n} + \theta_{m-1,n} + \theta_{m,n+1} + \theta_{m,n-1}) \qquad (5a)$$

The subscripts m and n indicate, respectively, the horizontal and vertical positions of the nodal points.

For the boundary nodal points 1 to 8, the finite-difference equations are derived separately as follows:

points 1 and 2:

$$\theta_{m,n} = 1 - Bi\xi_{1m,n} \qquad (5b)$$

point 3:

$$\theta_{m,n} = [1\ /(\ 2 + Bi_3\Delta X/t\ )](\ \theta_{m,n+1} + \theta_{m-1,n}) \qquad (5c)$$

point 4:

$$\theta_{m,n} = 0.5[1\ /(\ 2 + Bi\Delta_3 X/t\ )](\ 2\theta_{m-1,n} + \theta_{m,n-1} + \theta_{m,n+1}\ ) \qquad (5d)$$

point 5:

$$\theta_{m,n} = \{1/[2 + (\ Bi_2 + Bi_3\ )\ \Delta X/t\ ]\}\ (\ \theta_{m,n-1} + \theta_{m-1,n}\ ) \qquad (5e)$$

point 6:

$$\theta_{m,n} = 0.5[1\ /(\ 2 + Bi_2\Delta X/t\ )] \times (\ \theta_{m-1,n} + \theta_{m+1,n} + 2\theta_{m,n-1}\ ) \qquad (5f)$$

point 7:

$$\theta_{1,n} = \{1\ /[\ 2 + K + 0.5\ (\ Bi_1 + KBi_2\ )\ \Delta X/t\ ]\}\ [\theta_{-2,n} + 0.5\ (1 + K)\theta_{1,n-1} + 0.5\theta_{1,n+1} + 0.5K\theta_{2,n}\ ] \qquad (5g)$$

point 8:

$$\theta_{m,n} = 0.5\ [1/(2 + Bi_1\Delta X\ /\ t)]\ (\ 2\theta_{m+1,n} + \theta_{m,n-1} + \theta_{m,n+1}) \qquad (5h)$$

For the internal points like 9, we take advantage of symmetry and obtain,

point 9:

$$\theta_{m,n} = 0.25\ (\theta_{m+1,n} + \theta_{m-1,n} + 2\theta_{m,n+1}) \qquad (5i)$$

Similarly, for interface points 10 and 11 we have,

point 10:

$$\theta_{1,1} = 0.5[1/(1 + K)]\ (\theta_{-2,1} + K\theta_{2,1}) + 0.5\theta_{1,n+1} \qquad (5j)$$

point 11:

$$\theta_{1,n} = 0.5\ [1/(1 + K)]\ (\theta_{-2,n} + K\theta_{2,n}) + 0.25\ (\theta_{1,n+1} + \theta_{1,n-1}) \qquad (5k)$$

In these equations, the Biot numbers are defined as $Bi_1 = h_1 t/K_1$, $Bi_2 = h_2 t/K_2$, and $Bi_3 = h_3 t/K_2$. In equation (5b), the variable $\xi_{m,n}$ is the value of X/t for the nodal point (m,n) at points like 1 and 2, and it is a negative number. Also, the problem was simplified by assuming that $h_1 = h_2 = h_3$ and, therefore, $Bi_1 = KBi_2 = KBi_3$.

The solutions of equations (5a-k) were performed on a UNIVAC computer, using the Gauss-Seidel iterative method. The total number of grid points was around 7000, which depended on the selected values of L/t. These grid points were dispersed throughout the solid in the region $Y \geq 0$, encompassing the upper half of the fin and the wall. The computer runs were carried out for $L/t = 2, 5$, and 10 with $K = 1, 4$, and 20.

In order to examine the effect of the number of grid points on the accuracy of the solutions, the values of temperatures for 1708− and 6646−grid point runs were compared. It was found that the maximum difference in temperatures did not exceed 1.03 percent.

Another source of error is associated with the termination of the iterative procedure. In this connection, two additional computer runs were performed. In the first run, the computations were terminated when the temperature difference between two consecutive iterations did not exceed 10^{-6}, while in the other run the maximum difference was $0.5x10^{-7}$. The

The above-mentioned references have all been concerned with the behavior of isolated fins and have totally ignored the effect of any neighboring fin that might be present. The subject of multifin arrays has been treated by Sparrow and Lee [11], and also by Suryanarayana [12].

Despite of the rather simple appearance of the problem, research in the area of two-dimensional heat transfer still continues. Perhaps one of the most recent papers on the subject is that of Look [13]. In his work, Look has provided additional insight into the effect of two dimensionality on fin performance and has studied the effects of unequal top, bottom, and tip surface convection coefficients.

The rectangular fin of the present investigation is shown schematically in Fig.1. As seen, the surfaces of the infinite wall and the fin are exposed to a convective environment with heat transfer coefficient h. In general, the convective coefficients are different for various surfaces, and the symbols h_1, h_2, and h_3 indicate the convective coefficients for the wall, top (or bottom),and tip surfaces of the fin, respectively.

In real problems, the thermal conductivity of the wall k_1 is often quite different from that of the fin material k_2. As will be seen later in this paper, the effects of employing fin materials differing from that of the wall can be significant.

The problem was solved by a numerical procedure to be explained shortly. The ratio of thermal conductivities, $k = k_2/k_1$, ranged from 1 to 20, and the length-to-thickness ratio of the fin,L/t, was equal to 2, 5, and 10.

FORMULATION OF THE PROBLEM

Heat conduction in the fin and in the wall is governed by Laplace's equation:

$$\frac{\partial^2\theta}{\partial^2\xi} + \frac{\partial^2\theta}{\partial^2\eta} = 0 \qquad (1)$$

where the dimensionless temperature θ and the coordinates ξ and η are defined as:

$$\theta = \frac{T - T_\infty}{T_b - T_\infty},\ \xi = \frac{X}{t},\ \eta = \frac{Y}{t} \qquad (2)$$

The coordinates X and Y are shown in Fig.1. In equation (2), T_∞ is the temperature of the surrounding fluid, and T_b is the temperature of the exposed surface of the wall in the absence of the fin.

Considering the symmetry in Fig.1, the boundary conditions to be satisfied by equation (2) are written as:

$$\frac{\partial\theta}{\partial\xi} + Bi\theta = 0 \text{ at } (\ \xi = 0,\ \eta \geq 0.5\ ) \text{ and } (\xi = L/t, 0 \leq \eta \leq 0.5\ ) \qquad (3a)$$

$$\frac{\partial\theta}{\partial\eta} + Bi\theta = 0 \text{ at } (\ 0 \leq \xi \leq L/t,\ \eta = 0.5\ ) \qquad (3b)$$

$$\frac{\partial\theta}{\partial\eta} = 0 \text{ at } (\ -\infty < \xi \leq 0,\ \eta = 0\ ) \text{ and } (\ 0 \leq \xi \leq L/t,\ \eta = 0\ ) \qquad (3c)$$

$$\theta = 1 - Bi\xi \text{ at } (\ -\infty < \xi \leq 0,\ \eta = \infty\ ) \text{ and } (\ \xi = -\infty, 0 \leq \eta < \infty\ ) \qquad (3d)$$

$$\theta|_1 = \theta|_2 \text{ at } (\ \xi = 0,\ 0 \leq \eta \leq 0.5\ ) \qquad (3e)$$

$$\frac{\partial\theta}{\partial\xi}\Big|_1 = K\frac{\partial\theta}{\partial\xi}\Big|_2 \text{ at } (\ \xi = 0,\ 0 \leq \eta \leq 0.5\ ) \qquad (3f)$$

In these equations, the Biot number is defined as $Bi = ht/k$, and K is the ratio of thermal conductivities, namely, $K = K_2/K_1$.

Equation (3d), which was derived from a one-dimensional consideration of heat conduction, is applicable to the points located very far from the fin where the presence of the fin does not disturb the temperature field. In this problem, the distant points were approximated by the points located along the boundary lines *ab* and *bc* (Fig.1) with $x/t = 10$ and $Y/t = 10$. To examine the validity of this approximation, the calculations were repeated for X/t and $Y/t = 20$, and the results indicated that the maximum difference in temperatures was only 0.6 percent.

From equations (1) to (3f), it is revealed that the solutions, $\theta(\xi,\eta)$, depend on the values of three parameters, Biot number Bi, dimensionless length of the fin L/t, and thermal conductivity ratio K. In fin design, however, it is a usual practice to optimize the straight rectangular fins by the correlation [5]

$$Bi(L/t)^2 = 1 \qquad (4)$$

(or bottom) surface of the fin

h_3 Convective heat transfer coefficient for the tip of the fin

k Thermal conductivity in the definition of Biot number

K Thermal conductivity ratio, $K = K_2/K_1$

K_1 Thermal conductivity of the wall material

K_2 Thermal conductivity of the fin material

L Fin lenght

Q Real rate of heat transfer from the fin

t Fin thickness

T Temperature

T_b Wall surface temperature in the absence of fin, base temperature in the one-dimensional analysis

T_∞ Temperature of the surrounding fluid

X Horizontal coordinate (Fig.1)

Y Vertical coordinate (Fig.1)

ΔX Increment in X

Greek Symbols

η Dimensionless vertical coordinate, $\eta = Y/t$,fin efficiency

θ Dimensionless temperature,

$$\theta = \frac{(T - T_\infty)}{(T_b - T_\infty)}$$

$\theta_{m,n}$ Dimensionless temperature at the nodal point *(m,n)*

ξ Dimensionless horizontal coordinate, $\xi = X/t$

$\xi_{m,n}$ Value of ξ for the nodal point *(m,n)*

INTRODUCTION AND BACKGROUND

Fins are usually employed on hot surfaces where heat transfer coefficient is poor and the bare surfaces are unable to transfer enough heat to the surrounding fluid. The main function of a fin is to increase the surface area and, thus, to enhance heat transfer.

There are a large variety of applications where fins are employed to improve heat transfer. Some common examples are the automobile radiators, air conditioners, refrigeration units, electric transformers, air-cooled aero-engines, and forced-draft air-fin coolers [1]. In designing these devices, fins are treated by a simplified model where heat transfer is assumed to be one-dimensional. The assumption of one-dimensionality may not be always justified and, in certain situations, it might result in serious errors in the predicted temperatures and heat transfer rates.

Another common assumption in fin analysis, which is encountered in almost every heat transfer textbook [2-5], is that the temperature at the fin base is constant and is equal to the temperature of the surface when the fin is absent. In reality, when a fin is attached to a surface in order to enhance heat transfer, a temperature gradient appears in the vicinity of the fin base, and the base temperature is depressed. This temperature depression can seriously affect the distribution of temperature throughout the fin and alter the rate of heat transfer.

The present work is concerned with the study of two-dimensional effects on heat transfer from a single rectangular fin attached to the surface of an infinite solid wall. In this connection, the effect of three important factors is considered: (1) two-dimensionality of heat transfer, (2) depression of the base temperature, and (3) the thermal conductivity of the fin and that of the wall to which the fin is attached. Moreover, the solution procedure is such that the convective heat transfer coefficient around the fin and that of the wall could be treated as variable. However, to simplify the problem and to reduce the number of variables, we employed a single constant convective coefficient throughout this investigation. Furthermore, the solution domain was extended to encompass the portion of the solid wall where the temperatures have been affected by the presence of the fin.

There are a number of related studies in the published literature[6-12]. One of the earliest investigations were carried out by Avrami and Little [6] who studied the cooling and insulating effect of rectangular fins. Irey [7], and Wah Lau and Tan [10] have reported their results regarding the errors that might occur in a one-dimensional analysis of pin fins and annular fins. Apparently, temperature depression at the base of a single rectangular fin was first addressed by Sparrow and Hennecke [8] and, later, by Klett and McCulloch [9].

TWO-DIMENSIONAL EFFECTS ON HEAT TRANSFER FROM AN ISOLATED RECTANGULAR FIN

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ABSTRACT

An analysis was made of the two-dimensional heat transfer in a straight rectangular fin which was attached to an infinite solid wall. Temperature distributions in the fin and in the wall, and also the rate of heat transfer were obtained numerically. In the course of calculations, the length-to-thickness ratio of the fin, L/t, ranged from 2 to 10,while the fin-to-wall thermal conductivity ratio, K = K2/K1 ,took on values of 1,4, and 20. A comparison was made between the one-dimensional and the two-dimensional analyses, and the errors of the first were discussed. It was found that the temperatures predicted by the one- dimensional model could be in error by as much as 66.3 percent. Further, it was shown that the one-dimensional model either overestimated or underestimated the rates of heat transfer, with the numerical values of errors ranging from +25.9 to -46.1 percent.

NOMENCLATURE

A Cross sectional area of the fin normal to X axis

Bi Biot number, $Bi = ht/k$

Bi_1 Biot number defined as h_1t/K_1

Bi_2 Biot number defined as h_2t/K_2

Bi_3 Biot number defined as h_3t/K_3

h Convective heat transfer coefficient

h_1 Convective heat transfer coefficient for the wall surface

h_2 Convective heat transfer coefficient for the top