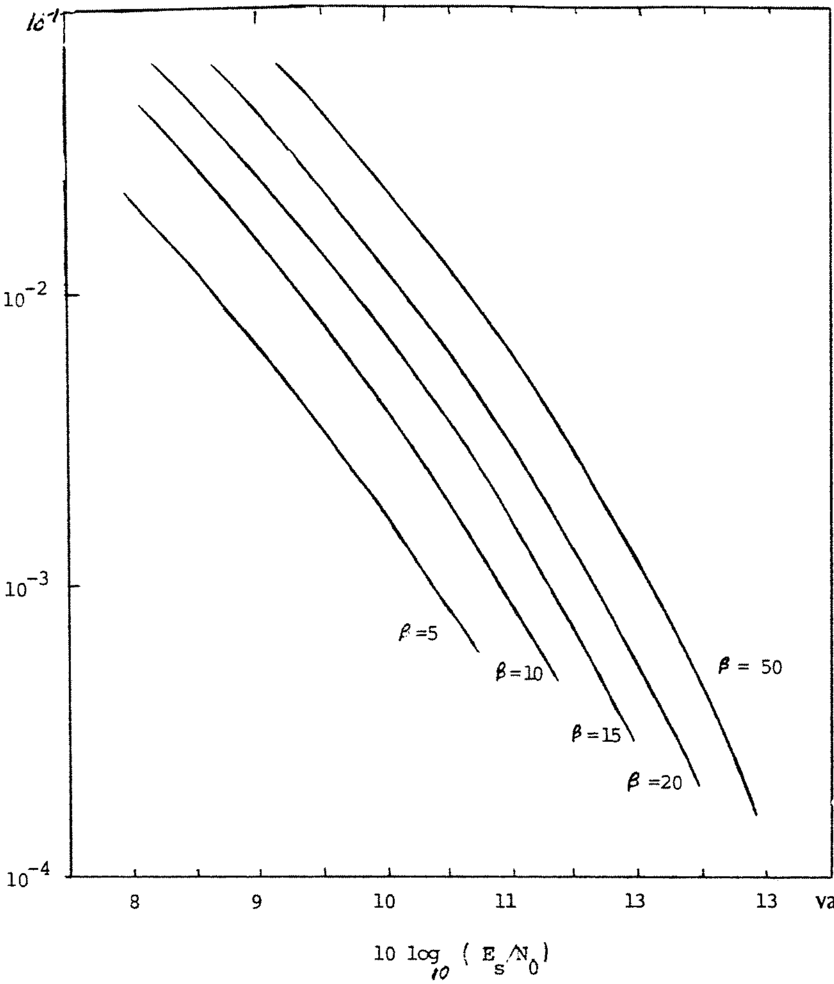

Fig. 5,
Relative frequency of anomaly variaton respect to S/N for different β equally likely orthogonal signals.

REFERENCES

1 J.Wozencraft & I.Jacob: Principles of communication engineering, P. 589, John Wiley & sons, 1965.

2 J. Wozencraft & I.Jacob: Principles of communication engineering, P. 563, John Wiley & sons, 1965.

3 J. Wozencraft & I.Jacob: Principles of communication engineering, P. 612, John wiley & sons, 1965.

4 J. Wozencraft & I. Jacob: Principles of communication engineering, P. 616, John wiley & sons, 1965.

5 D. Barton & H. word: Handbook of Radar Measurement Artech – House, Dedham, MA, 1984.

6 D. C Schleher: Introduction to Electronic Warfare Artech – House, 1986.

7 J. Wozencraft & I. Jacob: Principles of communication engineering, P. 258, John Wiley & sons, 1965.

8 J. Wozencraft & I. Jacob: Principles of communication engineering, P. 626, John Wiley & sons, 1965.

9 H. Nikokar, M.S. Thesis, Fall 1987, E.E. Dept, Sharif Univ. of Technology.

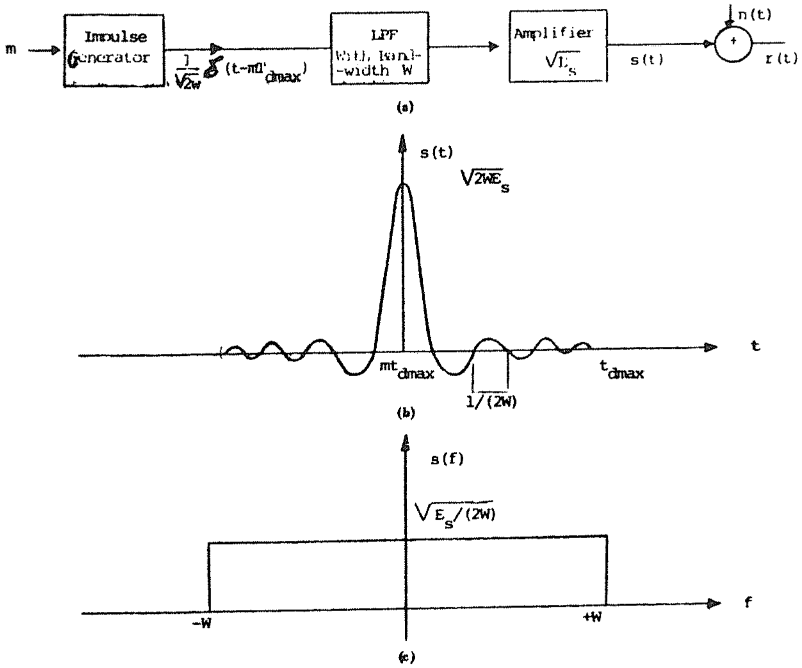

Fig 2 PPM Model and Signal, (a) Ideal PPM Transmitter, (b) PPM Signal in Time Domain, (c) PPM signal in Frequency Domain.

Fig. 3 Maximum Likelihood Receiver.

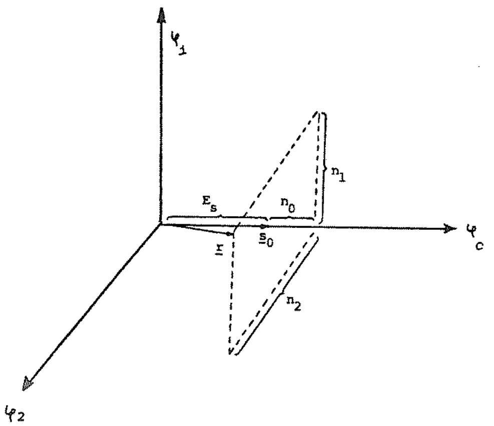

Fig. 4, Three Orthogonal Signals.

ın can also be proved that the dimension number can be written as,[8]

$$\beta = 2\,T_0\,W = 2\,t_{dmax}w \qquad (41)$$

in PPM system where T_0 is the maximum delay and W is the bandwidth of the filter, and in FPM system, [8].

$$\beta = 2\,T_s\,W_0 = 2\,T_s\,f_{dmax} \qquad (42)$$

inwhich T_s is duration of pulse and 2W is the effective bandwidth.

Therefore Eq. (40) showes the measure of S/N which is neccesary to makes P(A) small enough in order to be sure that range and Doppler performance resulted as Eq. (29) and Eq.(34) be valid. P(A) depandace with S/N has been showin in Fig.5. as a parameter β.

V. Conclusion

MIT Radar performance has been modeled by pulse and Frequency Position Modulation. The model is valid for weak noise detection of range and Doppler i.e. probability of anomalous condition has been found to show the measure of S/N ratio to make sure that the weak noise model is valid.

The result showes that rms error in estimating Doppler shift (and range) is inversly proportional to pulse duration (and bandwidth) and is inversly pro– portional to square of signal to noise ratio. These results are valid while anomalous condition does not occur.

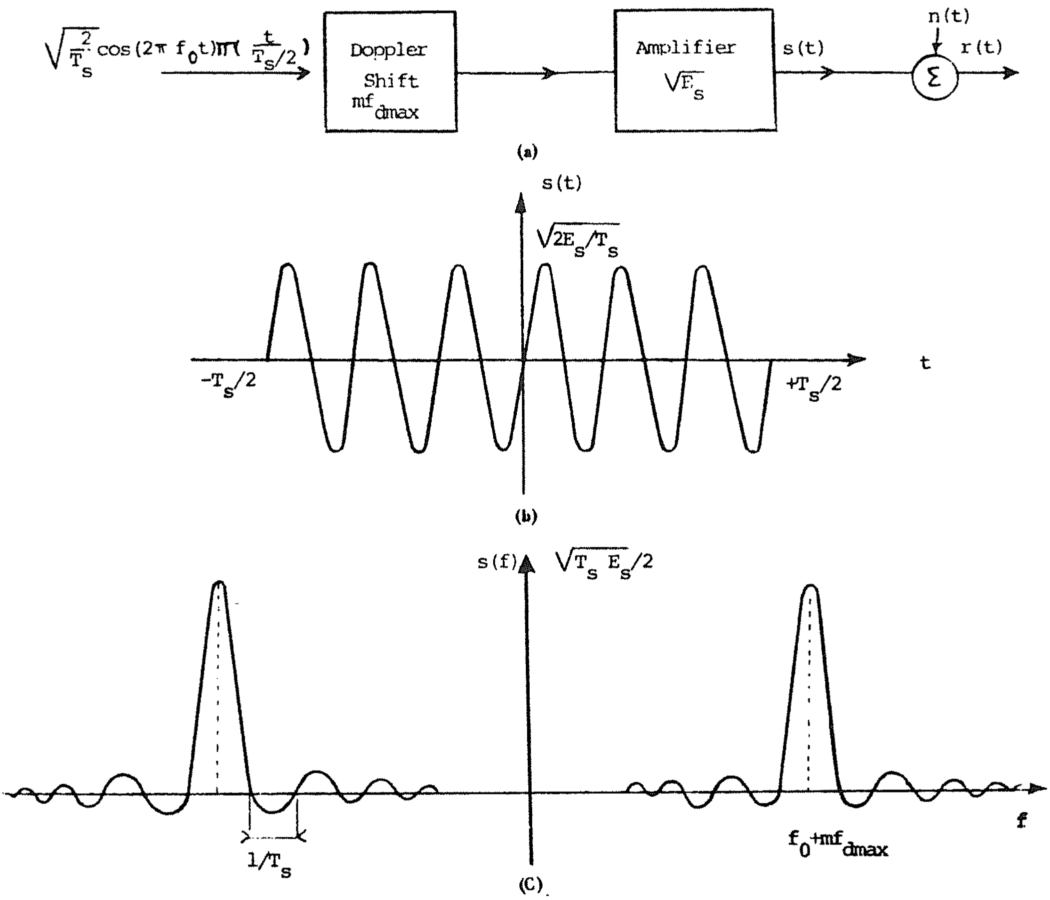

Fig 1, FPM Model and Signal, (a) Doppler Shift Model, (b) Signal in Time Domain, (C) Signal in Frequency Domain.

$$\overline{\epsilon_{t_d}^{\,2}} = \overline{(t_d - \hat{t}_d)^2} = 3N_0 / [2\,E_s\,(2\,\pi W)^2] \qquad (33)$$

By replacing, [8]

$\beta = 4\,W\,t_{dmax}$

in PPM, equation 32, & 33 will become as following

$$\overline{\epsilon_m^{\,2}} = \frac{12}{\pi 2}\,\frac{N_0}{2\,E_s}\,\frac{1}{\beta^2} \qquad (32)$$

$$\overline{\epsilon_{td}^{\,2}} = 12\,N_0\,t^2_{dmax} / (\pi^2 2\,E_s\,\beta^2) \qquad (33)$$

hence,

$$\sqrt{\overline{\epsilon_{t_d}^{\,2}}} = \sqrt{3N_0 / 2E_s} / (2\,\pi W) = \frac{12}{\pi^2}\,\frac{N_0}{2E_s}\,\frac{t_{dmax}}{\beta} \quad (34)$$

i.e. by increasing signal bandwidth, time delay or range estimate will become more accurate. This accuracy will increase by increasing signal to noise ratio, too. This result is in the case of weak noise.

IV. Probability of Anomalous Case

Mean square error of range and Doppler estimation were found in the case of weak noise assumption which we could generalize the result of linear modulation to nonlinear modulations such as PPM and FPM. But when noise increase, we say that the received signal is anomalous. In an anomalous situation it is not possible for any receiver to make a meaningful estimate of m; this follows from the fact the a posteriori density function $P_{m/r}$, which contains all data relevant to any estimate of m, is either fundamentally ambiguous or is misleading. Therefor we have to calculate the probability of anomalous condition.

Assume s_0 corresponding to m_0 is transmitted. Also assume s_0 can be shown by β orthogonal signals. which is given by Eq 41 and Eq. 42. with no loss of generality 3 dimension is shown in Fig.4. when s_{-0} is transmitted the decision will be correct if n_1 and n_2 both be smaller than $\sqrt{E_s} + n_0$

$$\begin{cases} r_0 = \sqrt{E_s} + n_0 \\ r_i = n_i \quad i > 0 \end{cases} \qquad (35)$$

If we assume that all n_i are statistically independent and identically distributed, we will have;

$$P_{r_0}(a) = P_n(a - \sqrt{E_s}) \qquad (36)$$

Where a is a parameter that we want to estimate. $p_n(\varphi)$ has a Gaussian form and independant from each other.

The probability of correct with the condition that m_0 has been transmitted given $r_0 = a$ will be [7]

$$P(C/m_0, r_0 = a) = P(n_1 < a, n_2 < a, \ldots, n_{\beta-1} < a) = [P(n_1 < a)]^{\beta-1} \qquad (37)$$

or

$$P(C/m_0) = \int_{-\infty}^{+\infty} P_n(a - \sqrt{E_s})\,d a \left[\int_{-\infty}^{+\infty} P_n(\gamma)\, d\gamma\right]^{\beta-1} = P(C/m_j) = P(C) \qquad (38)$$

probability of anomalous condition is

$$P(A) = 1 - q\,(C/m_j) \qquad (39)$$

therefore,

$$p(A) = 1 - \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi N_0}} \left\{ \exp -(\gamma - \sqrt{E_s})^2 / N_0 \right\} [1 - Q(\gamma / \sqrt{N_0/2})]^{\beta-1}\, d\gamma = \frac{\beta - 1}{\sqrt{2\pi E_s / N_0}} \exp(-E_s/2N_0) \qquad (40)$$

where Q (α) is

$$Q(a) \triangleq \frac{1}{\sqrt{2\pi}} \int_{-a}^{+\infty} \exp(-\theta^2/2)\, d\theta$$

its component on appropriate basic function set. When $m = m_0$ according to $S = S_0$, for weak noise if the receiver r be close to s_0, i.e. if r be in the region with at most several times of the standard deviation of noise we can behave with nonlinear modulation such as linear modulation. By using

$$\underline{s} = s + (m - m_0)\,(ds/dm)\Big|_{m = m_0} \qquad (24)$$

we find the norm of s and using Eq. (23) we can have the performance of nonlinear modulation as extra–polation of linear modulaton case. s^2 the square of norm of is given by [4]

$$s^2 = \int_{-\infty}^{+\infty} [ds(t)/dm]^2\, dt \qquad (25)$$

which will be shown for linear approximation is independent of m.

B. Frequency Position Modulation Consideration

if we apply Eq. (25) with s (t) given by Eq.(7) which is the signal model of FPM case we will get the following result:

$$s^2 = E_s\ (\pi T_s f_{dmax})^2/3$$

substituting in Eq.(20)

$$\overline{\epsilon_m^{\ 2}} = \overline{(m - m)^2} \geqslant 3N_0 / [\, 2E_s\, (\pi T_s f_{dmax})^2]$$

(27)

Using Eq. (5) to find mean square error for f_d we will have

$$\overline{\epsilon_{f_d}^{\ 2}} = \overline{(f_d - \hat{f}_d)^2} \geqslant 3N_0/\ [2E_s\, (\pi T_s)^2] \qquad (28)$$

by replacing, [8]

$$\beta = 2\, T_s\, f_{dmax}$$

in FPM equation 27 & 28 will become

$$\overline{\epsilon_m^{\ 2}} = \frac{12}{\pi^2}\ \frac{N_0}{2\,E_s}\ \frac{1}{\beta 2} \qquad (27')$$

$$\overline{\epsilon_{f_d}^{2}} = \frac{12}{\pi^2}\ \frac{N_0}{2\,E_s}\ \frac{f_{dmax}^2}{\beta 2} \qquad (28)$$

or r.m.s. of f_d is

$$\sqrt{\overline{\epsilon_{f_d}^{\ 2}}} \geqslant \sqrt{3N_0}\,/2E_s/\ (\pi\, T_s) = \sqrt{\frac{12}{\pi^2}\ \frac{N_0}{2\,E_s}}\ \frac{f_{dmax}}{\beta}$$

which says as much as T_s, the signal duration increase the performance gets better, also by increasing signal to noise the accuracy become better. This result is valid while the weak noise assumption be valid as will be discussed in IV.

Besides the theoretical minimum signal to noise ratio required to measure the frequency of a pulse with accuracy Δf can be found from relation where

$$f = K/\ (T_s \sqrt{S}/N\ ) \qquad (30)$$

T_s is the effective pulsewidth and k is a constant which depends on the pulse shape and ranges from0.5 for rectangular pulse to 0.8 for Guassian shape pulse [5], [6].

C. Pulse Position Modulation Consideration

Applying Eq. (25) with s(t) given by Eq. (13) and using Parseval relationship, s^2 will be calculated;

$$s^2 = E_s\ (2\,\pi\, t_{dmax}\ W)^2/3 \qquad (31)$$

Applying the above equation in Eq. (23) will give the mean square error for m;

$$\overline{\epsilon_m^2} = \overline{(m - \hat{m})^2} = 3\ N_0\ /\ 2E_s\ (2\,\pi\, t_{dmax} w)^2 \qquad (32)$$

Using Eq. (10) will result mean square error of $\hat{t}_d$, i.e.

rewriting Eg (8), Eq (10) and Eg (12) will give:

$$S(t) = \sqrt{2WE_s}\ \text{Sinc}\ [2W(t-mt_{dmax}] \quad (13)$$

Time and frequency shape of S(t) in the case of PPM model is given in Fig 2. In a PPM system we have:

$$S(t) = \sqrt{E_s}\ \varphi(t-mT_0) \quad (8')$$

Where

$$\varphi(t) = \sqrt{2W}\ \text{Sinc}\ (2Wt) \quad (12')$$

According to the Eqs (12), (13), (8') and (12') it can be seen that the envelope of Radar echo for range estimation is a Pulse Position Modulation, PPM.

III. Weak Noise Consideration

Since target speed and range appears as nonlinear modulation on the reflected signal, its performance evaluation is rather complicated; therefore we will apply the linear modulation approach approximation for these cases.

A. Linear Modulation Performance and Its Extrapolation.

When parameter m is linear modulated we receive

$$r(t) = S(t) + n(t) \quad (14)$$

with

$$s(t) = mA\ \varphi(t) \quad (15)$$

Where $\varphi(t)$ is unit energy waveform, A is the gain and n(t) is white Gaussian noise with density $N_0/2$. Eq (14) can also be written as s(t) & n(t)'s component on $\varphi(t)$, i.e.

$$r = mA + n \quad (16)$$

$$s = mA \quad (17)$$

If m has any arbitrary distribution then Maximum Liklihood reciver will estimate m, as [1];

$$\hat{m} = r/A \quad (18)$$

As shows in Fig.3. in this case mean square error can easily be calculated by using Eqs (16) and (18), [2];

$$\overline{\epsilon^2} = \overline{(m-\hat{m})^2} = N_0/(2A^2) \quad (19)$$

If m has limited value such as given in Eqs (6) and (11), then the maximum likelihood receiver will have limiter in order that m to stay in the region of m. In this case mean square error will be smaller than what is given in Eq. (19) because some amount of noise will not appear as error [2], i.e.

$$\overline{\epsilon^2} = \overline{(m-\hat{m})^2} \leqslant N_0/(2A^2) \quad (20)$$

in which the equality hold if and only if m is unbounded. Now we extrapolate the above results for nonlinear modulations. In linear modulation we have as Eq. (17);

$$s = m A$$

If we define $\underline{\varphi}_1$ as basic function representation of $\varphi(t)$ Then we can write Eq. (17) as.

$$\underline{s} = m A\ \underline{\varphi}_1 \quad (21)$$

besides, if we define the norm of ds/dm as, [3]

$$s \triangleq \left| \frac{d\underline{s}}{dm} \right| \quad (22)$$

we can write Eq. (20) as

$$\overline{\epsilon^2} = \overline{(m-\hat{m})^2} \leqslant N_0/(2s^2) \quad (23)$$

In nonlinear modulation we may write s in terms of returned pulse pasition.

A. Doppler Estimation As FPM Model

MTI Radar received signal which is given in Eg. (1) Contains s(t) with energy E_s the affect of the target at the receiver in term of $\varphi(t)$ with unit energy such that;

$$s(t) = \sqrt{E_s}\ \varphi(t) \tag{2}$$

$$\varphi(t) = \begin{cases} \sqrt{2/T_s}\ \text{Cos}\ [2\pi (f_0 + f_d)t] & -T_s\ 2 \langle t \langle T_s/2 \\ 0 & \text{Otherwise} \end{cases} \tag{3}$$

The above equations describe the echo signal from target meanwhile these equations represent FPM signal.

With ordinary FPM the transmitted signal is T_s second pulse of sine wave, with carrier frequency f_0 and Doppler Shift mw_0 as following:

$$s(t) = \sqrt{E_s}\ \varphi\ (t) \tag{2}$$

$$\varphi(t) = \begin{cases} \sqrt{2/T_s}\ \text{Cos}\ [2\pi (f_0 + mw_0)\ t] & -T_s/2 \langle t \langle T_s/2 \\ 0 & \text{eleswhere} \end{cases} \tag{3́}$$

We assume that the random variable m is confined to [−1, 1] and $2w_0$ is effective bandwidth. The estima– tion of Doppler frequency f_d is equivalent with target radial velocity, V_r, estimation. i.e.

$$f_d = 2f_0\ V_r/C \tag{4}$$

Where f_0 is the carrier frequency and C is speed of propagation. In Eg. (3) we have neglected the time delay due to target, because it will be dealt later on. We may write,

$$f_d = mf_{dmax} \tag{5}$$

wher f_{dmax} is macimum Doppler frequency and m is a parameter;

$$-1 \leqslant m \leqslant 1 \tag{6}$$

which must be estimated. Then Eg. (2) and Eg. (3) will become as

$$s(t) = \begin{cases} \sqrt{2/T_s}\ \ \text{Cos}\ [2\pi (f_0 + mf_{dmax})\ t] & -T_s/2 \langle t \langle T_s/2 \\ 0 & \text{eleswhere} \end{cases} \tag{7}$$

which can be considered as Frequency position Modulation FPM. With continous Parameter m which must be estimated, in order to estimate f_d and there– fore target speed. S(t) of FPM is shown in Fig. 1. in time and frequency domain.

B. Range Estimation as PPM Model

s(t) of Eg. (1) for range estimation may be written as;

$$s(t) = \sqrt{E_s}\varphi\ (t-t_d) \tag{8}$$

where t_d is related to range, R, by:

$$t_d = 2R/C \tag{9}$$

where C is the speed of propagation. For simplicity we will write:

$$t_d = mt_{dmax} \tag{10}$$

where t_{dmax} is the maximum interest delay & m is a parameter such that;

$$0 \leqslant\ m \leqslant 1$$

φ (t) is normalized to have unit energy and is assumed to be the output of the filter with bandwidth W for narrow impulsive input pulse, i.e

$$\varphi(t) = \sqrt{2W}\ \text{Sinc}\ (2Wt) \tag{12}$$

Range and Doppler Performance Evaluation by FPM and PPM Modeling of Radar

M.Tabiani, Ph,D
&
H.Nikokar, M.Sc.
Elect. Dept Sharif Univ. of Tech. – IRAN

ABSTRACT

MTI Radar is modeled as Pulse and Frequency Position Modulation to evaluate the performance for maximum likelihood estimation of range and Doppler.

Performance is calculated in the presence of weaK white Gaussian noise. Probability of anomalous condition is calculated to show the condition that weak noise model is not valid.

I. Introduction

Exact evaluation of MTI Radar performance is rather complicated since target range and speed information appear as time delay and Doppler frequency shift respectively in the echo signal. In this paper we present rather simple method to evaluate the performance of range and Doppler optimum estimation. Performance is given in presence of additive, weak, white, Gaussian noise. The condition which makes this model valid is examined.

It is assumed that the received signal r(t) be:

$$r(t) = s(t) + n(t) \qquad (1)$$

where s(t) is the effect of signal due to the target with energy E_s , n(t) is assumed to be stationary white Gaussian process with density height $N_0/2$.

To evaluate the performance we will model the target speed or range effect as Frequency Position or Pulse Position Modulation. Based on this model, mean square error of range and Doppler are calculated as a measure of performance in the presence of weak white Gaussian noise. The probability of anomalous case is given as a criteria to check the validity of weak noise model.

II. Modeling of MTI Radar by FPM and PPM.

The affect of target speed and range on Radar signal can be modeled by PPM and FPM as shown in Fig. 1. (a) and Fig. 2.(a), i.e. speed affect of target can be modeled as Doppler change in frequency & range affect of target can be modeled as delay in