where γ_l and γ_g are the specific weights of liquid and vapor respectively. Substitution of equations (5) and (6) into equation (4) gives

$$P_B(o) - P_A(o) + (\gamma_l - \gamma_g)f(r) = \frac{\sigma}{r}\frac{d}{dr}(r\sin\phi) \quad 7$$

where

$$\sin\phi = \frac{f'(r)}{\sqrt{1+[f'(r)]^2}} \quad 8$$

Introducing the dimensionless variables

$$s = \frac{r}{a} \quad , \quad F(s) = \frac{f(r)}{\alpha}$$

equation (7) is converted to the following differential equation

$$\alpha + \beta F = \frac{1}{s}\frac{d}{ds}(s \sin\phi) \quad 9$$

where

$$\alpha = \left[P_B(o) - P_A(o)\right]\alpha/\sigma \quad 10$$

$$\beta = (\gamma_l - \gamma_g)\alpha^2/\sigma \quad 11$$

Equation (9) is a nonlinear equation which can be solved numerically. However , for most purposes, the approximate solution to be obtained is sufficient.

In most instances, the capillary rise H, will greatly exceed the radius of curvature of the meniscus surface, a first approximation to the solution of equation (9) can be obtained by neglecting F on the left-hand side of equation (9). Subject to the boundary conditions,

$$F(o) = o \quad 12$$

$$F'(o) = o \quad 13$$

$$F(1) = \cot\theta \quad 14$$

we obtain

$$F_1(s) = \sec\theta\left[1 - \sqrt{1 - s^2\cos^2\theta}\right] \quad 15$$

An improved solution $F_2(s)$, is obtained by using $F_1(s)$ for F in the left-hand side of equation (9). This substitution together with integration of the resulting equation from s = 0 to s = 1, gives

$$\frac{\alpha}{2} + \frac{\beta}{2}q(\theta) = \cos\theta \quad 16$$

where

$$q(\theta) = \sec^3\theta\left[\cos^2\theta + \frac{2}{3}\sin^3\theta - \frac{2}{3}\right] \quad 17$$

If the variation in the densities of the liquid and vapor over the capillary rise H, are negligible, then

$$\frac{\alpha}{\beta} = \frac{H}{\alpha} \quad 18$$

and the following relation is obtained between the surface tension -.σ the capillary rise H, and the contact angle θ

$$H = \frac{2\sigma\cos\theta}{(\gamma_l - \gamma_g)\alpha} - \alpha q(\theta) \quad 19$$

If $q(\theta)$ is neglected in this last equation, the remaining terms simply balance the vertical component of the surface tension force $2\pi\alpha\sigma\cos\theta$, against the effective weight of the liquid column $\pi\alpha^2H(\gamma_l - \gamma_g)$. Indeed, the term $q(\theta)$ accounts for the small hydrostatic pressure gene rated by the liquid above H between the meniscus and the tube wall. In most applications the contact angle θ, is small enough so that $\cos\theta \approx 1$ and $q(\theta) \approx$

From equation (19) it can be observed that if $\gamma_l \gg \gamma_g$ and if the second term of the right-hand side is neglected, Batchelor's condition for equilibrium of the column is obtained.

REFERENCE

Batchelor, G.K., «An Introduction to Fluid Mechanics», Cambridge University press, London 1967.

On the Theory of Capillary Rise

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ABSTRACT

A Theory has been developed to predict the rise of a liquid within a capillary. This rise can be used for the experimental determination of surface tensions.

A simplified theory for the prediction of the capillary rise is presented by Batchelor[1]. According to this theory the capillary rise is given by $H = (2\sigma\cos\o)$. This rise can be used for experimental determination of surface tensions. In this paper an improved theory developed which gives a better approximation of H.

Fig. 1 shows a column of liquid in thermodynamic equilibrium with its own vapor.

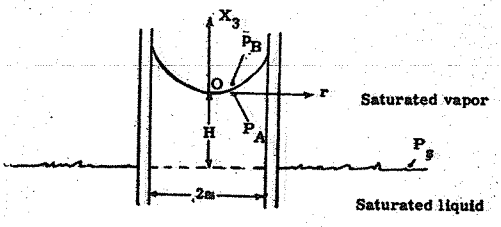

Fig. 1

A necessary thermodynamic condition for such equilibrium to exist is that the temperature be uniform throughout the liquid and vapor phases.

Let the curved interfacial surface at the top of the capillary column be described by $x_3 = f(r)$ **1**

If P_A is the local fluid pressure in the liquid just beneath the interface, and P_B is the local fluid pressure in the vapor just above, the condition for equilibrium is Where R_1 *and* R_2 are *the principal radii of*

$$P_A - P_B = \sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \sigma\nabla\left(\frac{\nabla F}{|\nabla F|}\right) \qquad \mathbf{2}$$

curvature, σ is the surface tension between the two fluids, and $F = x_3 - f(r)$ **3**

Combination of equations (2) and (3) gives

$$P_A - P_B = -\frac{\sigma}{r}\frac{d}{dr}\left\{\frac{r f'(r)}{\sqrt{1+[f'(r)]^2}}\right\} \qquad \mathbf{4}$$

With the origin of the coordinates taken at the low point on the interface, the statical pressure distributions within the vapor and liquid in the vicinity of the origin are given by.

$$P_A = P_A(o) - \gamma_l X_3 \qquad \mathbf{5}$$

$$P_B = P_B(o) - \gamma_g X_3 \qquad \mathbf{6}$$